1. **(Compound Interest)** You have $10,000 to invest. You are considering two investment schemes, each offering an annual interest rate of 5%. Scheme A offers 5% interest compounded continuously, while scheme B offers 5% interest compounded quarterly. Which investment scheme will result in the largest future value of your principal?

**Solution:** Choose scheme A. Recall: if $P$, $r$, and $t$ are fixed, then your investment will grow the fastest under continuous compound interest. This scheme will give you the largest future value.

2. **(Compound Interest)** Below are two graphs of the future value of the principal $P = $1000 as a function of time. The interest rate for both graphs is $r = .04$. One graph grows at a simple interest rate, while the other grows at a compound interest rate with $n = 12$. Both graphs are plotted over the time interval starting at time $t = 0$ and ending at time $t = 30$ years. Clearly label which one is the graph of simple interest growth. Warning: If I can’t tell which graph you indicate you will receive a zero for the problem.

**Solution:** Recall: An investment of $P$ dollars under a simple interest rate $r$ grows linearly in time. Hence, the graph of simple interest is linear. Moreover, for fixed $P$ and $r$, it is the slowest-growing interest scheme in time. The lower linear graph is the simple interest graph.
3. (Compound Interest: Future Value) What is the value of $100 after 10 years at 5% interest, compounded monthly? Round to the nearest dollar.

(a) 1056 (c) 569 (e) 730 (g) 297 (i) 953 (k) 9394
(b) 723 (d)* 165 (f) 704 (h) 367 (j) 1 (l) none of the these

Solution: Given: \( P = 100, \ t = 10, \ r = .05, \) and \( n = 12. \) Want: \( F. \)

The discrete interest formula is \( F = P \left(1 + \frac{r}{n}\right)^{nt}. \) Upon substituting the given values into the formula we arrive at \( F = 100 \left(1 + \frac{.05}{12}\right)^{12(10)} = 164.7 \approx 165. \)

4. (Compound Interest: yield) What is the effective annual yield \( y \) for problem 3? (Use 3 decimal place accuracy).

(a) .307 (c) .517 (e) .023 (g)* .051 (i) .027 (k) .039
(b) .024 (d) .039 (f) .105 (h) .002 (j) .095 (l) none of the these

Solution: The formula for effective annual yield in the case of discrete compounding is \( y = \left(1 + \frac{r}{n}\right)^n - 1. \) Substituting the values from problem 3 yields \( y = \left(1 + \frac{.05}{12}\right)^{12} - 1 \approx .051. \)

5. (Compound Interest: Future Value) What is the value of $100 after 10 years at 4%, compounded quarterly (rounded to the nearest dollar)?

(b) 1013 (c) 117 (e) 203 (g) 17 (i) 221 (k) 155
(b) 257 (d) 154 (f)* 149 (h) 196 (j) 6 (l) none of the these

Solution: Given: \( P = 100, \ t = 10, \ r = .04, \) and \( n = 4. \) Want: \( F. \)

The discrete interest formula is \( F = P \left(1 + \frac{r}{n}\right)^{nt}. \) Upon substituting the given values into the formula we arrive at \( F = 100 \left(1 + \frac{.04}{4}\right)^{4(10)} = 148.9 \approx 149. \)

6. (Compound Interest: yield) What is the effective annual yield \( y \) for problem 5?

(b) 0 (c) .5957 (e) .6579 (g)* .0406 (i) .1106 (k) 1
(b) .2535 (d) .3676 (f) .6903 (h) .0824 (j) .9068 (l) none of the these

Solution: The formula for effective annual yield in the case of discrete compounding is \( y = \left(1 + \frac{r}{n}\right)^n - 1. \) Substituting the values from problem 5 yields \( y = \left(1 + \frac{.04}{4}\right)^{4} - 1 \approx .04. \)
7. (Compound Interest: Future Value) What is the value of $1 after 100 years at 10% interest, compounded continuously? Round to the nearest dollar.

(c) 10567      (c) 8569      (e) 48730      (g) 897      (i) 4953      (k) 93904
(b) 900          (d)* 22026           (f) 7094           (h) 33367        (j) 1           (l) none of the these

Solution: Given: $P = 1, t = 100, r = 0.1$, and the compounding scheme is continuous. Want: $F$. The continuous compound interest formula is $F = Pe^{rt}$. Upon substituting the given values into the formula, we arrive at $F = 1e^{100(0.1)} = e^{10} \approx 22026.47$.

8. (Compound Interest: yield) What is the effective annual yield $y$ for problem 7? (Use 3 decimal place accuracy).

(c) .307       (c) .567       (e) .023       (g) .490       (i) .325       (k) .639
(b) .924        (d) .739       (f)* .105       (h) .002       (j) .195       (l) none of the these

Solution: The formula for effective annual yield in the case of continuous compounding is $y = e^r - 1$. Substituting the values from problem 5 yields $y = e^{0.1} - 1 \approx 0.105$.

9. (Compound Interest: Future Value) Find the value of $1,250,000 after 12 years and 6 months, if it is invested at a rate of $\frac{1}{8}$% compounded continuously.

Solution: Given: $P = 1,250,000, r = \frac{41}{(8*100)} = 0.05125, t = 12.5$ years.

$F = Pe^{rt}$
$= 1,250,000e^{(0.05125*12.5)}$
$= 2,372,083.19$

10. (Compound Interest: yield) What is the yield on this investment? Keep your answer accurate to 0.00000001%. You may want to use Excel to compute this.

$y = e^r - 1$
$= e^{(0.05125)} - 1$
$= 5.25860069\%$
11. (Compound Interest: time to double investment) You want to invest P dollars. How long will it take to double your investment at an annual interest rate of 10%, compounded continuously? (round your answer to the nearest year).

(a) 1   (c) 3   (e) 5   (g) 7   (i) 9   (k) 11
(b) 2   (d) 4   (f) 6   (h) 8   (j) 10   (l) none of these

Solution: Given: $F = 2P$, $r = .10$, and compounding scheme is continuous. Want: $t_{\text{double}}$.

Substituting these values into the compound interest formula $F = Pe^{rt}$ yields

\[
2P = Pe^{rt} \Rightarrow 2 = e^{rt} \quad \text{(divide by P)}
\]
\[
\Rightarrow \ln(2) = rt \quad \text{(Take the natural log of both sides)}
\]
\[
\Rightarrow t = \frac{\ln(2)}{r} \approx \frac{.69}{.1} = 6.9 \quad \text{(Solve for $t$)}
\]

12. (Compound Interest: time to double investment) You want to invest P dollars. How long will it take to double your original investment at an annual interest rate of 6.9%, compounded continuously? Round your answer to the nearest year.

(b) 8   (c) 10   (e) 12   (g) 14   (i) 16   (k) 18
(b) 9   (d) 11   (f) 13   (h) 15   (j) 17   (l) none of the these

Solution: Given: $F = 2P$, $r = .069$, and compounding scheme is continuous. Want: $t$.

Substituting these values into the compound interest formula $F = Pe^{rt}$ yields

\[
2P = Pe^{rt} \Rightarrow 2 = e^{rt} \quad \text{(divide by P)}
\]
\[
\Rightarrow \ln(2) = rt \quad \text{(Take the natural log of both sides)}
\]
\[
\Rightarrow t = \frac{\ln(2)}{r} \approx \frac{.69}{.069} = 10 \quad \text{(Solve for $t$)}
\]
13. (Compound Interest: time to double investment) You want to invest $P$ dollars. How long will it take to double your original investment at an annual interest rate of 4%, compounded quarterly? (round your answer to the nearest year).

\[ t \approx \frac{\ln(2)}{n \ln\left(1 + \frac{r}{n}\right)} \approx \frac{\ln(2)}{4 \ln\left(1 + \frac{.04}{4}\right)} \approx \frac{ln(2)}{.04} = 17 \] (Solve for $t$)

14. (Compound Interest) What annual rate, $r$, compounded continuously, would have the same yield as an annual rate of 6%, compounded weekly? Round your answer to the nearest percent (2 decimal place accuracy).

\[ r = \ln\left(1 + \frac{.06}{52}\right)^{52} = .06 \]
Fast way: Recall: \( y \approx r \) for both the discrete and continuous case provided that \( r < 0.2 \).

15. (Compound Interest) Find the annual rate, \( r \), which produces an effective annual yield of 3.75\%, when compounded continuously. Round your answer to the nearest tenth of a percent (three decimal places).

Solution: Given: \( y = 0.0375 \) and the yield formula \( y = e^r - 1 \) for continuous compounding. Want \( r \). We now solve for \( r \).

\[
y = e^r - 1
\]

\[
0.0375 = e^r - 1
\]

\[
1.0375 = e^r \quad \text{(add 1 to both sides)}
\]

\[
\ln(1.0375) = \ln(e^r) = r \ln e \quad \text{(take the log of both sides)}
\]

\[
3.68\% = r
\]

\[
r \approx 0.037
\]

16. (Compound Interest) How long, to the nearest whole month, will it take $40,000 to grow to $65,000 if \( 4 \frac{1}{5} \% \) interest is compounded continuously?

Solution: Substituting the values into the equation \( F = Pe^{rt} \) yields

\[
65000 = 40000e^{0.042t}
\]

\[
\ln\left(\frac{65000}{40000}\right) = \ln e^{0.042t}
\]

\[
\ln\left(\frac{65000}{40000}\right) = 0.042t
\]

\[
11.56 = t
\]

\[
11 + 0.5597 \cdot 12 = t
\]

11 years 7 months \( \geq t \)
17. **(Compound Interest)** How long, to the nearest whole month, will it take $40,000 to grow to $65,000 if $4\frac{1}{5}\%$ interest is compounded monthly?

**Solution:** Substituting the values into the equation $F = P\left(1 + \frac{r}{n}\right)^{nt}$ yields

\[
65000 = 40000 \left(1 + \frac{0.042}{12}\right)^{12t}
\]

\[
1.625 = 1.0035^{12t}
\]

\[
\ln(1.625) = \ln(1.0035)^{12t}
\]

\[
\ln(1.625) = 12 \cdot t \cdot \ln(1.0035)
\]

\[
\frac{\ln(1.625)}{12 \cdot \ln(1.0035)} = t
\]

\[
11.5799 = t
\]

\[
11 + 0.5799 \cdot 12 = t
\]

11 years 7 months $\equiv t$

18. **(Compound Interest)** Two certificates of deposit have the same effective annual yield. The first pays a rate of $r$, compounded monthly. The second pays 6.03%, compounded continuously. What is the value of $r$?

**Solution:** Want $r$ so that

\[
P\left(1 + \frac{r}{n}\right)^{n} = P(1 + y) = Pe^{.0603}
\]

Canceling $P$ from both sides gives

\[
\left(1 + \frac{r}{n}\right)^{n} = e^{.0603}
\]

Since we compound monthly, $n = 12$. The equation becomes

\[
\left(1 + \frac{r}{12}\right)^{12} = e^{.0603}
\]

Taking the 12th root gives

\[
1 + \frac{r}{12} = e^{.0603}^{12}
\]

Solving for $r$ yields

\[
r = 12 \left(e^{.0603}^{12} - 1\right)
\]
19. (Compound Interest) $5000 is invested in an account paying 5.5%, compounded quarterly. How long will it take for the value of the account to reach $6000? Round your answer to the nearest hundredth of a year.

Solution: Given: \( P = 5000 \), \( F = 6000 \), \( r = .055 \), and \( n = 4 \). Solve for \( t \).

\[
F = P \left(1 + \frac{r}{n}\right)^{nt} \quad \Rightarrow \quad \frac{F}{P} = \left(1 + \frac{r}{n}\right)^{nt}
\]

\[
\ln \left(\frac{F}{P}\right) = nt \ln \left(1 + \frac{r}{n}\right)
\]

Solving for \( t \) gives the general formula for time

\[
t = \frac{\ln \left(\frac{F}{P}\right)}{n \ln \left(1 + \frac{r}{n}\right)}
\]

For our problem

\[
t = \frac{\ln \left(\frac{6000}{5000}\right)}{4 \ln \left(1 + \frac{0.55}{4}\right)} \approx 3.34\text{ years}
\]

20. (Compound Interest) $6500 is invested in an account paying 5.15%, compounded continuously. How long will it take for the value of the account to reach $10,000? Round your answer to the nearest hundredth of a year.

Solution: Given: Compounded continuously, \( P = 6500 \), \( F = 10,000 \), \( r = .0515 \). Find \( t \).

Recall that \( F = Pe^{rt} \). Solve for \( t \):

\[
e^{rt} = \frac{F}{P} \quad \text{(divide by } P)\]

Take the natural log of both sides of the equation

\[
\ln[e^{rt}] = \ln \left(\frac{F}{P}\right)
\]

Using the basic property of logarithms \( \ln(x^a) = a \ln x \)

\[
\ln[e^{rt}] = rt = \ln \left(\frac{F}{P}\right)
\]

Dividing by \( r \) yields the formula for time

\[
t = \frac{1}{r} \ln \left(\frac{F}{P}\right)
\]

For our problem, we want

\[
t = \frac{1}{.0515} \ln \left(\frac{10000}{6500}\right) = 8.36\text{ years.}
\]
21. (Simple Interest) A total of $20,000 is invested in two funds paying 8% and 10% simple interest. If the interest for the first year is $2,000, how much of the $20,000 is invested at 10%?

Solution: Let $I_1$ be the interest earned on the investment at $r_1 = 0.08$, $I_2$ be the interest earned on the investment at $r_2 = 0.10$, and $I_1 + I_2 = 2000$ be the total interest from both investments. Given: time $t = 1$ year and $P = 20000$, we want to find $x$, the amount invested at $r_1$.

Using Interest = Principal times rate times time with $t = 1$ we arrive at the equation

$I = xr_1 + (P - x)r_2$. Substituting for $r_1$, $r_2$, $I$, and $P$ we have an equation for $x$:

$2000 = 0.08x + 0.10(20000 - x)$

$= 0.1(20000) - 0.02x$

$= 2000 - 0.02x$

Simplifying yields $.02x = 0 \Rightarrow x = 0$. This answer could have been deduced by observation.

22. (Simple Interest) A total of $P$ dollars is invested in two funds paying $r_1$ and $r_2$ simple interest. If the interest for the first year is $I$, how much of the initial investment $P$ is invested at $r_1$? Warning: your answer should be a formula involving $r_1$, $r_2$, $P$, and $I$.

Solution: Let $I_1$ be the interest earned on the investment at $r_1$, $I_2$ be the interest earned on the investment at $r_2$, and $I_1 + I_2 = I$ be the total interest from both investments. Given: time $t = 1$ year and the initial investment is $P$, we want to find $x$, the amount invested at $r_1$.

Using Interest = Principal times rate times time with $t = 1$ we arrive at the equation

$I = I_1 + I_2 = xr_1 + (P - x)r_2$. Solving this equation for $x$ yields:

$I = xr_1 + (P - x)r_2 = P \cdot r_2 + (r_1 - r_2)x$. Solving for $x$ yields:

$x = \frac{I - P \cdot r_2}{r_1 - r_2}$

23. (Simple Interest) A total of $11,000 is invested in two funds paying 10% and 9% simple interest. If the interest for the first year is $1,000, how much of the $11,000 is invested at 10%?

Solution: Using the formula derived in problem 22 we get

$x = \frac{I - P \cdot r_2}{r_1 - r_2} = \frac{1000 - 11000 \cdot 0.09}{0.1 - 0.09} = \frac{1000 - 990}{0.01} = \frac{10}{0.01} = 1000$
24. (Simple Interest) A total of $11,000 is invested in two funds paying 10% and 9% simple interest. If the interest for the first year is $1,000, how much of the $11,000 is invested at 10%? Solve the problem from scratch without using the formula derived in problem 22.

**Solution:** Let $I_1$ be the interest earned on the investment at $r_1 = .10$, $I_2$ be the interest earned on the investment at $r_2 = .09$, and $I_1 + I_2 = 1000$ be the total interest from both investments.

*Given:* time $t = 1$ year and $P = $11,000, we want to find $x$, the amount invested at $r_1$. Using Interest = Principal times rate times time with $t = 1$ we arrive at the equation $I = xr_1 + (P - x)r_2$. Substituting for $r_1$, $r_2$, $I$, and $P$ we have an equation for $x$:

\[
1000 = .10x + .09(11000 - x)
\]

\[
= .09(11000) - .01x
\]

\[
.09(11000) - .01x
\]

\[
990 - .01x
\]

Simplifying yields

\[
.01x = 10 \implies x = 1000.
\]

25. (Simple Interest) A total of $20,000 is invested in two funds paying 8% and 10% simple interest. If the interest for the first year is $2,000, how much of the $20,000 is invested at 10%?

**Solution:** Let $I_1$ be the interest earned on the investment at $r_1 = .09$, $I_2$ be the interest earned on the investment at $r_2 = .11$, and $I_1 + I_2 = 1180$ be the total interest from both investments.

*Given:* time $t = 1$ year and $P = $12,000, we want to find $x$, the amount invested at $r_1$. Using Interest = Principal times rate times time with $t = 1$ we arrive at the equation $I = xr_1 + (P - x)r_2$. Substituting for $r_1$, $r_2$, $I$, and $P$ we have an equation for $x$:

\[
1180 = .09x + .11(12000 - x)
\]

\[
= .11(12000) - .02x
\]

\[
.1(12000) + .01(12000) - .02x
\]

\[
1200 + 120 - .02x
\]

Simplifying yields

\[
.02x = 140 \implies x = 7000.
\]