Homework set 3: Summation

Elementary-Level Problems involving summation notation

1. (Summation) Express the given sum using summation notation.

\[ 1 + 2 + 3 + 4 + 5 + 6 + 7 = \]

Solution: \[ 1 + 2 + 3 + 4 + 5 + 6 + 7 = \sum_{i=1}^{7} i \]

2. (Summation) Express the given sum using summation notation.

\[ 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = \]

Solution: \[ 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = \sum_{i=1}^{6} i^2 \]

3. (Summation) Express the given sum using summation notation.

\[ 3 \cdot (-1)^2 + 3 \cdot 0^2 + 3 \cdot 1^2 + 3 \cdot 2^2 + 3 \cdot 3^2 + 3 \cdot 4^2 = \]

Solution: \[ 3 \cdot (-1)^2 + 3 \cdot 0^2 + 3 \cdot 1^2 + 3 \cdot 2^2 + 3 \cdot 3^2 + 3 \cdot 4^2 = \sum_{i=-1}^{4} 3 \cdot i^2 = 3 \sum_{i=-1}^{4} i^2 \]

4. (Summation) Express the given sum using summation notation.

\[ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{100^2} = \]

Solution: \[ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{100^2} = \sum_{i=1}^{100} \frac{1}{i^2} \]

5. (Summation) Express the given sum using summation notation.

\[ \frac{1}{1^2 + 1} + \frac{1}{2^2 + 1} + \frac{1}{3^2 + 1} + \cdots + \frac{1}{100^2 + 1} = \]

Solution: \[ \frac{1}{1^2 + 1} + \frac{1}{2^2 + 1} + \frac{1}{3^2 + 1} + \cdots + \frac{1}{100^2 + 1} = \sum_{i=1}^{100} \frac{1}{i^2 + 1} \]

6. (Summation) Expand the following expression written in summation notation.
\[ \sum_{i=1}^{5} i^2 = \]

**Solution:** \[ \sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 \]

7. **(Summation)** Expand the following expression written in summation notation. 
*Do not attempt to evaluate!*

\[ \sum_{i=-1}^{5} \frac{1}{i^2+1} = \]

**Solution:** \[ \sum_{i=-1}^{5} \frac{1}{i^2+1} = \frac{1}{(-1)^2+1} + \frac{1}{0^2+1} + \frac{1}{1^2+1} + \frac{1}{2^2+1} + \frac{1}{3^2+1} + \frac{1}{4^2+1} + \frac{1}{5^2+1} \]

8. **(Summation)** Expand the following expression written in summation notation. 
*Do not attempt to evaluate the sum!*

\[ \sum_{i=1}^{5} x_i P(X = x_i) = \]

**Solution:**

\[ \sum_{i=1}^{5} x_i P(X = x_i) = x_1 P(X = x_1) + x_2 P(X = x_2) + x_3 P(X = x_3) + x_4 P(X = x_4) + x_5 P(X = x_5) \]

9. **(Summation)** Expand the following expression written in summation notation. 
*Do not attempt to evaluate the sum!*

\[ \sum_{i=1}^{5} x_i f(x_i) = \]

**Solution:** \[ \sum_{i=1}^{5} x_i f(x_i) = x_1 f(x_1) + x_2 f(x_2) + x_3 f(x_3) + x_4 f(x_4) + x_5 f(x_5) \]

10. **(Summation)** Expand the following expression written in summation notation. 
*Do not attempt to evaluate the sum!*
\[
\sum_{i=1}^{5} x_i^2 f(x_i) =
\]

**Solution:** \[
\sum_{i=1}^{5} x_i^2 f(x_i) = x_1^2 f(x_1) + x_2^2 f(x_2) + x_3^2 f(x_3) + x_4^2 f(x_4) + x_5^2 f(x_5)
\]

**Elementary-level problems using Excel to evaluate the sum:**

**Note:** On some of these problems we can use the general formulas

\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}
\]

, together with the properties of sums, to find the value of the sum analytically. However, this class is more about learning how to harness the power of Excel than developing your mathematical prowess, so we will resist this temptation and just use Excel to evaluate the sums. Below I just list the answer. For a full solution see the Excel solution file: homework-set2.xls.

11. **(Summation)** Create an Excel worksheet to evaluate the sum \[
\sum_{i=1}^{10} i
\]. Here \(a_i = a(i) = i\). For sum: start value = 1, stop value = 10

**Answer:** 55

12. **(Summation)** Create an Excel worksheet to evaluate the sum \[
\sum_{i=1}^{10} (3i - 5)
\]. Here \(a_i = a(i) = 3i - 5\). For sum: start value = 1, stop value = 10

**Answer:** 115

13. **(Summation)** Create an Excel worksheet to evaluate the sum \[
\sum_{i=-7}^{50} i^2
\]. Here \(a_i = a(i) = i^2\). For sum: start value = -7, stop value = 50

**Answer:** 43,065

14. **(Summation)** Create an Excel worksheet to evaluate the sum \[
\sum_{i=-3}^{100} \frac{1}{i^2 + 1}
\]. Here \(a_i = a(i) = \frac{1}{i^2 + 1}\). For sum: start value = -3, stop value = 100

**Answer:** 2.866724209

15. **(Summation)** Create an Excel worksheet to evaluate the sum
\[
\sum_{n=1}^{20} \frac{1}{n^n}. \quad \text{Here } a_n = a(n) = \frac{1}{n^n}. \quad \text{For sum: start value = 1, stop value = 20}
\]

Answer: 1.291285997