Mathematics for Business Decisions, Part I

Problem Set 2: Evaluating and Graphing Functions in Excel

Purpose: To use Excel to evaluate and graph functions that contain parameters.
NOTE: This is a skill that you will use throughout the course.

Evaluating functions in Excel

Before you can start evaluating functions in Excel, you must learn the basics of how Excel computes mathematics.

Example 1: Suppose we want to evaluate the function \( f(x) = 6x^2 - 7x + 8 \) at \( x = 2 \) in Excel. It is clear that in order to compute \( f(2) = 6 \cdot 2^2 - 7 \cdot 2 + 8 \) we must be able to add, subtract, multiply, and evaluate exponential terms like \( x^2 \) in Excel. When you type numbers or letters into a cell in Excel, Excel treats these entries as data. It does not evaluate them. To put Excel in “math mode” you must use the equal sign = as your first entry in the cell. Next, we must look at how to perform basic math computations in Excel like +, −, ×, ÷ and powers. Below is a list of the conversion symbols in Excel:

For addition use +,
for subtraction use −,
for multiplication use *,
for division use /,
and to raise a term to a power use the ^ symbol on the keyboard.

Using the above information we will now write the function using what we shall refer to as Excel-function format notation: \( f(x) = 6 \cdot x^2 - 7 \cdot x + 8 \). You should compare this to the original function above.

We are still not quite ready to evaluate this function in Excel. We can’t put a variable like \( x \) directly into an Excel formula. Instead, we have to use a cell reference for the variable. We can then put the value of \( x \) for which we want to evaluate the formula into that cell. If we want to evaluate the function for a different value of \( x \), we enter the new value in the cell. Here is a case where we’ve put the value of \( x \) in cell A2, and evaluated \( f(x) \) in cell A3. We’ve used \( x=2 \) and evaluated \( f(2) \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
</tr>
</tbody>
</table>

We now show the code that generated this solution:

\[
\begin{array}{c|c}
\hline
\text{x} & \text{f(x)} \\
\hline
2 & =6*A2^2-7*A2+8 \\
\hline
\end{array}
\]

We will now give more examples using Excel-function format so that the reader can get more familiar with it.
Example 2: Convert the following mathematical expression into Excel-function format.

\[ f(x) = \frac{x + 1}{x^2 - x + 1}. \]

Solution: When expressed in Excel-function format \( f(x) = (x + 1)/(x^2 - x + 1). \) The parentheses around the denominator are necessary. The expression \( f(x) = (x + 1)/x^2 - x + 1 \) when written in mathematical notation is \( f(x) = \frac{x + 1}{x^2 - x + 1}. \) This is a very different result from what was intended.

Now let’s go the other direction.

Example 3: Convert the function \( f(x) = (x^2 - 7x + 1)/(x^2 - 3x + 19) \) written in Excel-function format into a mathematical expression.

Solution: In standard mathematical notation \( f(x) = \frac{x^2 - 7x + 1}{x^2 - 3x + 19}. \)

Let’s look at one more example that’s a little more useful. This one will involve a function that has parameters. Most real world “formulas” involve functions that have parameters.

Example 4: Suppose you want to use Excel to make a “program” that computes discrete compound interest. Let \( P= \) Principal (the amount invested of borrowed), \( r= \) nominal interest rate, \( n= \) number of compounding periods per year, \( t= \) time (in years), and \( F= \) future value.

The following formula gives the future value of the investment: \( F = P \left(1 + \frac{r}{n}\right)^{nt}. \)

Solution: The first step is to write the formula out in Excel notation as \( F = P \cdot (1 + r/n)^{nt}. \) The parentheses after the caret ^ are necessary. Without ( ) after the ^, we would be evaluating the \( n \) power instead of the \( n/t \) power. For example, if you type \( =2^3*5 \) in Excel, then Excel returns 40. This is because only the 3 is raised to the power. Excel reads \( 2^3*5 \) as \( 2^3 \cdot 5 = 8 \cdot 5 = 40. \) However, Excel reads \( 2^\left(3*5\right) \) as \( 2^{15} = 32768, \) a very different result. When Excel reads the combination ^\( ( \) it goes along and looks for the closing parenthesis \( ) \). Excel then raises everything to the power inside the parentheses ( )

Lastly, we must make a designated cell for each parameter \( P, r, n, t, \) and for the formula for \( F. \) Suppose you borrow (spend) $3000 on your credit card at a rate of 21% interest compounded monthly \( n=12 \) (most credit cards compound monthly at the end of each payment period), and do not make any payments for 3 years. Make a spreadsheet to compute this situation in Excel.

We list the solution below:

<table>
<thead>
<tr>
<th>P</th>
<th>r</th>
<th>n</th>
<th>t</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>0.21</td>
<td>12</td>
<td>5</td>
<td>8495.449</td>
</tr>
</tbody>
</table>

We now show the code that generated this solution:

```
=a2*(1+b2/c2)^(c2*d2)
```
Graphing functions in Excel

We’ve seen how to evaluate functions at particular values of \(x\), we’re now ready to take the next step and learn how to graph functions. We will assume that the functions are piecewise continuous and smooth. Recall from college algebra that a function is continuous if you can draw the function on a piece of paper without lifting your pen. Put another way, if we represented the function as a wire on a piece of paper, then the wire would be in one piece (it would not be broken). A function is piecewise continuous if it only has finitely many discontinuities, say \(x_1 < x_2 < \cdots < x_n\), and it is continuous over each interval \((x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n)\). A function is smooth if you could run your hand over its graph without it cutting your hand. For more details see your college algebra book or do a Google search on the web.

We plot graphs of functions in Excel by making two column arrays. In the first column we make a column array of values of the independent variables, call them \(x_i\), over the interval that we wish to graph the function. We usually take these values to be evenly spaced over the interval. We shall refer to the distance between any two points, denoted \(\Delta x = x_i - x_{i-1}\), as the step size. For example, if we want to graph a function over the interval \([a, b]\), and we want \(n\) points, then we take our step size to be \(\Delta x = \frac{b-a}{n}\). One the other hand, if we want the spacing between two points to be \(\Delta x\), then we take \(n\) to be \(n = \frac{b-a}{\Delta x}\), where \(n\) need not be an integer. In the second column, we make an array of the values of the function corresponding to the points \(x_i\). The values look like \(f(x_i)\). Rather than give a formal discussion of this, we will learn how to graph functions by example.

**Example 5:** Graph the function \(f(x) = x^2\) over the interval \([-1, 1]\). Use a step size of \(\Delta x = 0.5\). We will soon see that this step size is too large. As a rule of thumb, you should take your step size \(\Delta x\) to be roughly \(\frac{1}{100}\) th the width of the interval over which you are graphing the function. Start by inserting \(-1\) into the first column in cell A2. **Warning:** Be sure to enter the value into the cell. This should advance you to the next cell. Use the up-arrow to return you to the cell A2. Once you are back in cell A2 you will need to bring up the Fill Series menu.

To get to the Fill Series menu, go through the following drop-down menus:
- From the main toolbar go to Edit \(\rightarrow\) Fill \(\rightarrow\) Series (hotkeys <Alt> E i s).
- Once you’re at the Fill Series menu, you need to change the “Series in” option to “Columns” (not “Rows”). **Warning:** the default value is rows, not columns! Next, under the “Type” option, leave the default setting at “Linear”. Set “Step value” to \(\Delta x = 0.5\), and “Stop value” to 1. The steps are shown in the figures below.
Going to Edit → Fill → Series from the main toolbar brings up the Fill Series menu, shown to the right in its default state.

Under the heading “Series in”, switch from “Rows” to “Columns”. Under the heading “Type”, leave “Linear” alone. Change “Step value” to .5, and “Stop value” to 1. Do not check the “Trend” box.

Now hit the OK button. You should get a series in column A as shown at right. Notice that cells A1 and B1 contain column titles. It is always a good idea to label the columns in your Excel files.

Now that we have all of our $x$ values in a column array, we need to get the corresponding $f(x)$ values in a column array. To do this, we will have to construct the formula for $f(x) = x^2$ in column B. In Excel-function format, $f(x) = x^2$. However, we cannot use the value $x$ directly. Instead, we must use the address of the cell where the value of $x$ is stored. For example, $f(-1) = A2^2$, since $-1$ is in the cell A2. We can now use the fill handle (the dark square located in the lower right hand corner of a cell when you click on the cell) to complete the other values of $f(x)$.
The function is constructed using Excel-function notation. The formulas are displayed using the <ctrl> ~ command. You can toggle back and forth between the formulas and their numerical values by using the key sequence <ctrl> ~.

Next, we want to graph the function \( y = f(x) \). To do this, we use the Chart Wizard found in the tool bar. It looks like \( \) . If the chart wizard does not appear in the tool bar then you can add it, just go to the toolbar and under Tools \( \rightarrow \) Customize, then under “Categories”, choose “Insert”. You will find the Chart Wizard under “Commands”. Just drag the icon into the toolbar.

To graph \( y = f(x) \), highlight the range containing the values of \( f(x) \). In our case, this is the cells B2:B6. Then bring up the Chart Wizard. Select “XY(Scatter)”, and choose a scatter plot with points connected by a smooth lines without markers as shown below. The darkened box is the one that you want to choose.

Hit the Next > button, you should then see the following:
Next, click on the “Series” tab. You should then see:

Now, under “X values”, click on the box with a red arrow in the right-hand corner as shown below.
Highlight the x values in the Excel file found in column A in cells A2:A6. You should now see

Under Name put the name that you want to call your chart. Then hit “Next >” and fill in your axis labels. Hit “Next” one more time, then “Finish”. Lastly, you can delete that annoying in the chart and resize your chart. You should have something like

You are now ready to graph functions.
For problems 1-10, convert the mathematical expressions into Excel-function format. For example, the mathematical function \( f(x) = \frac{x+1}{x^2-x+1} \) when expressed in Excel-function format becomes \( f(x) = (x+1)/(x^2-x+1) \). The parentheses around the denominator are necessary. The expression \( f(x) = (x+1)/x^2-x+1 \) when written in mathematical notation is \( f(x) = \frac{x+1}{x^2} - x + 1 \). This is a very different result from what was intended.

**Problem 1:** Write \( f(x) = e^x \) in Excel-function format.

**Solution:** \( f(x) = \text{Exp}(x) \)

**Problem 2:** Write \( f(x) = e^{-x^2} \) in Excel-function format.

**Solution:** \( f(x) = \text{Exp}(-(x^2)) \)

**Warning:** Excel interprets \(-x^2\) as \(-x^2\), so we must write \(-x^2\) as \(-x^2\). This is a work-around for a defect in the Excel programming language. Live with it!

**Problem 3:** Write \( f(x) = 5x^2 + 3x - 7 \) in Excel-function format.

**Solution:** \( f(x) = 5*x^2 + 3*x - 7 \)

**Problem 4:** Write \( F = P \left( 1 + \frac{r}{n} \right)^{nt} \) in Excel-function format.

**Solution:** \( F = P*(1+r/n)^{n*t} \)

**Problem 5:** Write \( F = Pe^{rt} \) in Excel-function format.

**Solution:** \( F = P*\text{Exp}(r*t) \)

**Problem 6:** Write \( f(x) = \frac{x^2+2x+1}{1+x^2} \) in Excel-function format.

**Solution:** \( (x^2+2*x+1)/(1+x^2) \)

**Note:** We needed to put both numerator and denominator in parentheses.

**Problem 7:** Write \( f(x) = x^5 + x^4 - 7x^3 + 13x^2 - 3x + 19 \) in Excel-function format.

**Solution:** \( f(x) = x^5 + x^4 - 7*x^3 + 13*x^2 - 3*x + 19 \)

**Problem 8:** Write \( f(x) = \ln(x^2+2x+1) \) in Excel-function format.

**Solution:** \( f(x) = \text{ln}(x^2+2*x+1) \)

**Problem 9:** Write \( f(x) = 5x^2e^x \) in Excel-function format. Use Excel to evaluate \( f(x) \) at \( x = 2 \).

**Note:** In Excel the exponential function is written as \( e^x = \text{Exp}(x) \).

**Solution:** \( f(x) = 5*x^2*\text{Exp}(x) \)

**Problem 10:** Write \( f(x) = \frac{x^2-1}{x^2+1} \) in Excel-function format. Use Excel to evaluate \( f(x) \) at \( x = .1 \).

**Solution:** \( f(x) = (x^2-1)/(x^2+1) \)
Next, we try going in reverse. For problems 11-20 convert the following equations written in Excel-function format into mathematical expressions. For example, the function 

\[ f(x) = \frac{x^2 - 7x + 1}{x^2 - 3x + 19} \]

is written in Excel-function format. It becomes 

\[ f(x) = \frac{x^2 - 7x + 1}{x^2 - 3x + 19} \] when it is expressed in standard mathematical notation.

**Problem 11:** \( f(x) = x^2 - 3x + 5 \)

*Solution:* 

\[ f(x) = x^2 - 3x + 5 \]

**Problem 12:** \( f(x) = (1+x)^2 \)

*Solution:* 

\[ f(x) = (1 + x)^2 \]

**Problem 13:** \( F = P(1+r/n)^t(n*t) \)

*Solution:* 

\[ F = P \left( 1 + \frac{r}{n} \right)^{nt} \]

**Problem 14:** \( f(x) = 5x^2*\text{Exp}(x) \)

*Solution:* 

\[ f(x) = 5x^2 e^x \]

**Problem 15:** \( f(x) = \frac{x^2 + x + 1}{x^2 - 2x + 1} \)

*Solution:* 

\[ f(x) = \frac{x^2 + x + 1}{x^2 - 2x + 1} \]

**Problem 16:** \( y = (1+r/n)\text{^n}-1 \)

*Solution:* 

\[ y = \left( 1 + \frac{r}{n} \right)^n - 1 \]

**Problem 17:** \( y = \text{Exp}(r) - 1 \)

*Solution:* 

\[ y = e^r - 1 \]

**Problem 18:** \( y = (1/\sqrt{2\pi})*\text{Exp}(-x^2/2) \)

*Solution:* 

\[ y = \frac{1}{\sqrt{2\pi}} \text{Exp}\left(-\frac{x^2}{2}\right) \]

**Problem 19:** \( y = \frac{x^2 - 1}{x^2 + 1} \)

*Solution:* 

\[ y = \frac{x^2 - 1}{x^2 + 1} \]

**Problem 20:** Let \( A1 \) be a cell reference to a variable \( x \). \( y = -6*A1^3 + 5*A1^2 + 7*A1 + 9 \)

*Solution:* 

\[ y = -6x^3 + 5x^2 + 7x + 9 \]
Intermediate-Level Problems

For problems 21-25 graph the following functions.

**Problem 21: (Graphing)** \( y = 25 - x^2 \), over the interval \( 0 \leq x \leq 5 \). In fill series use time steps of length \( \Delta t = .01 \), start value of 0, and end value of 5.

**Solution:** See the Excel sheet: problem-set2-sols.xls.

**Problem 22: (Graphing)** \( F(t) = P \left( 1 + \frac{r}{n} \right)^{nt} \) for \( P = 1000, r = .05, n = 12 \), over the interval \( 0 \leq t \leq 3 \). In fill series use time steps of length \( \Delta t = .01 \), start value of 0, and end value of 3.

**Solution:** See the Excel sheet: problem-set2-sols.xls.

**Problem 23: (Graphing)** Find the point \((x_0, y_0)\) where the two functions intersect over the given interval. \( y = f(x) = 25 - x^2 \) and \( y = g(x) = x - 1 \) over \( 0 \leq t \leq 5 \).

**Hint:** Graph the function \( h(x) = f(x) - g(x) \) and find the roots (zeros) of \( h(x) \).

**Solution:** See the Excel sheet: problem-set2-sols.xls.

**Problem 24: (Graphing)** Two individuals each invest \( P = $100 \) in two different banks. Bank1 has a nominal interest rate of \( r_1 = .04 \) compounded continuously. Bank2 has an investment scheme of simple interest at a nominal interest rate of \( r_2 = .05 \). The formulas for the two schemes are respectively, \( F_1(t) = Pe^{rt} \) and \( F_2(t) = P(1 + rt) \). Find the time that the compound interest formula over takes the simple interest formula. You may assume that this occurs in less than ten years so that \( 0 \leq t \leq 10 \).

**Solution:** See the Excel sheet: problem-set2-sols.xls.

**Problem 25: (Graphing)** Find a value of \( x_0 > 0 \) so that the functions \( y = g(x) = \frac{1}{2} (x - x_0) \) and \( y = f(x) = \frac{x}{1 + x^2} \) will intersect at 3 distinct points.

**Hint:** Graph \( f(x) \) first.

**Solution:** See the Excel sheet: problem-set2-sols.xls.