Business Mathematics I: Math 173
Sample Final Exam - Solutions

Name:

Print your name neatly. If you forget to write your name, or write so sloppy that I can’t read it, you can lose all of the points! Answer all the questions that you can. Circle your answer.

All problems are worth 1 point each.

Do any 25 of the 26 problems. You may do all 26 if you like, but I will stop grading after you have 25 problems correct. In other words, you cannot score more than 100% on this exam. You must show how you arrived at the answer in order to receive any credit for the problem. Problems for which only an answer is given with no supporting work, will receive a score of zero for that problem.

DO NOT WRITE ON THIS PAGE IN THE SPACE BELOW

1. ______  6. ______  11. ______  16. ______  21. ______
2. ______  7. ______  12. ______  17. ______  22. ______
3. ______  8. ______  13. ______  18. ______  23. ______
5. ______ 10. ______  15. ______  20. ______  25. ______
  26. ______

STUDY GUIDE FOR FINAL EXAM:

The final exam is comprehensive. You should study all of the topics covered on exams 1, 2, 3, and 4. The majority of problems will be taken from homework sets 1-10, however; some problems maybe new to you.

What homework sets should you review for this exam: Homework sets 1-10.

Warning: The actual exam will need not be the same questions, but the number of problems, types of problems, and level of difficulty will be similar.
1. (Compound Interest: Future Value) What is the value of $100 after 10 years at 5% interest, compounded monthly? Round to the nearest dollar.

(a) 1056  (c) 569  (e) 730  (g) 297  (i) 953  (k) 9394
(b) 723  (d) 165  (f) 704  (h) 367  (j) 1  (l) none of the these

**Solution:** Given: \( P = 100, \ t = 10, \ r = .05, \) and \( n = 12. \) Want: \( F. \)

The discrete interest formula is \( F = P \left(1 + \frac{r}{n}\right)^n. \) Upon substituting the given values into the formula we arrive at \( F = 100 \left(1 + \frac{.05}{12}\right)^{12(10)} = 164.7 \approx 165. \)

2. (Compound Interest: yield) What is the effective annual yield \( y \) for problem 3? (Use 3 decimal place accuracy).

(a) .307  (c) .517  (e) .023  (g) .051  (i) .027  (k) .039
(b) .024  (d) .039  (f) .105  (h) .002  (j) .095  (l) none of the these

**Solution:** The formula for effective annual yield in the case of discrete compounding is \( y = \left(1 + \frac{r}{n}\right)^n - 1. \) Substituting the values from problem 3 yields \( y = \left(1 + \frac{.05}{12}\right)^{12} - 1 \approx .051. \)

3. (Compound Interest: time to double investment) You want to invest \( P \) dollars. How long will it take to double your investment at an annual interest rate of 10%, compounded continuously? Round your answer to the nearest year.

(a) 1  (c) 3  (e) 5  (g) 7  (i) 9  (k) 11
(b) 2  (d) 4  (f) 6  (h) 8  (j) 10  (l) none of these

**Solution:** Given: \( F = 2P, \ r = .10, \) and compounding scheme is continuous. Want: \( t_{\text{double}}. \)

Substituting these values into the compound interest formula \( F = Pe^{rt} \) yields

\[
2P = Pe^{rt} \quad \Rightarrow \quad 2 = e^{rt} \quad \text{(divide by P)}
\]

\[
\Rightarrow \quad \ln(2) = rt \quad \text{(Take the natural log of both sides)}
\]

\[
\Rightarrow \quad t = \frac{\ln(2)}{r} \approx \frac{.69}{.1} = 6.9 \quad \text{(Solve for t)}
\]
4. **(Simple Interest)** A total of $12,000 is invested in two funds paying 9% and 11% simple interest. If the interest for the first year is $1,180, how much of the $12,000 is invested at 9%?

**Solution:** Let $I_1$ be the interest earned on the investment at $r_1 = .09$, $I_2$ be the interest earned on the investment at $r_2 = .11$, and $I_1 + I_2 = 1180$ be the total interest from both investments.

Given: time $t = 1$ year and $P = $12,000, we want to find $x$, the amount invested at $12,000.

Using Interest = Principal times rate times time with $t = 1$ we arrive at the equation $I = xr_1 + (P - x)r_2$. Substituting for $r_1$, $r_2$, $I$, and $P$ we have an equation for $x$:

\[
1180 = .09x + .11(12000 - x)
\]

\[
= .11(12000) - .02x
\]

\[
.1(12000) + .01(12000) - .02x
\]

\[
1200 + 120 - .02x
\]

Simplifying yields

\[
.02x = 140 \implies x = 7000.
\]

5. **(Excel-Function notation):** Write $F = P\left(1 + \frac{r}{n}\right)^{nt}$ in Excel-function format.

**Solution:** $F = P\ast(1+r/n)^{(n*t)}$

6. **(Excel-Function notation):** Write $f(x) = 5\ast x^2\ast\text{Exp}(x)$ in standard mathematical notation.

**Solution:** $f(x) = 5x^2e^x$

7. **(Summation)** Expand the following expression written in summation notation. Do not attempt to evaluate the sum!

\[
\sum_{i=1}^{5} x_i^2 f(x_i) =
\]

**Solution:** \[
\sum_{i=1}^{5} x_i^2 f(x_i) = x_1^2 f(x_1) + x_2^2 f(x_2) + x_3^2 f(x_3) + x_4^2 f(x_4) + x_5^2 f(x_5)
\]
For problems 8-9 shade in the appropriate regions for the given diagram

8. 

9. 

10. (Basic Probability) You are promoting an outdoor concert for the coming Saturday (2 days from now). It is a fact that weather records are fairly accurate over 3 day periods. Records indicate that the probability of rain is 0.4, the probability of gale-force winds is 0.5, and the probability of rain or gale-force winds is 0.8. What is the probability that it neither rains nor has high winds?

   (a) 0  (c) .2  (e) .4  (g) .6  (i) .8  (k) 1
   (b) .1  (d) .3  (f) .5  (h) .7  (j) .9  (l) none of the these

   Solution: Answer: (c)

   Step 1: Write down the events. Let
   \( R = \) event of rain
   \( W = \) event of gale-force winds

   Step 2: Write down the given information.
   \( P(R) = .4, P(W) = .5, P(R \text{ or } W) = P(R \cup W) = .8 \)

   Step 3: Write down what you want.
   \( P(\text{neither } R \text{ nor } W) = P(\text{not } R \text{ and not } W) \)
   \[ = P(R^c \cap W^c) \] (convert words into mathematical symbols)
   \[ = P((R \cup W)^c) \] (by De Morgan’s Law)
   \[ = 1 - P(R \cup W) \]
   \[ = 1 - .8 \]
   \[ = .2 \]
11. **(Basic Probability)** You have just opened a restaurant and have applied for a liquor license and a smoking permit (to allow smoking in a closed-off part of the restaurant). You are friends with the city inspector and he has told you that based on past experience you have a 60% chance of getting a liquor license, a 40% chance of getting a smoking permit, and a 20% chance of getting both permits approved. What is the probability that you will be given exactly one of the permits?

(a) 0  (c) .2  (e) .4  (g) .6  (i) .8  (k) 1
(b) .1  (d) .3  (f) .5  (h) .7  (j) .9  (l) none of the these

**Solution: Answer:** (g)

**Step 1: Write down the events.** Let

\[ L = \text{event of obtaining a Liquor license} \]

\[ S = \text{event of obtaining a Smoking license} \]

**Step 2: Write down the given information.**

\[ P(L) = .6, \ P(S) = .4, \ P(L \text{ and } S) = P(L \cap S) = .2 \]

**Step 3: Write down what you want.**

\[ P((L-S) \cup (S-L)) = P(L-S) + P(S-L) \text{ (disjoint sets)} \]

\[ = (P(L) - P(L \cap S)) + (P(S) - P(L \cap S)) \]

\[ = P(L) + P(S) - 2P(L \cap S) \]

\[ = .6 + .4 - 2(.2) \]

\[ = .6 \]

**The idea behind the solution:**

\[ P(\text{Exactly one permit}) = \text{mass of the left moon} + \text{mass of the right moon} = P((A-B) \cup (B-A)) \]. The Venn diagrams below show that these sets are disjoint

![Venn Diagrams](image)

Notice that we must subtract \( A \cap B \) out from the union, because we want to exclude the possibility of being in both sets \( A \) and \( B \). We can also write the probability of “exactly one” as

\[ P((A-B) \cup (B-A)) = P(A \cup B) - P(A \cap B) . \]
12. (Independence) Consider three identical boxes that we shall refer to as box1, box2, and box3. Each box contains 100 parts, one of which is defective. If you randomly choose one part from each box, what is the probability of selecting 1 defective part and 2 non-defective parts?

Solution: We can select one defective part and 2 non-defective parts in 3 different ways:

\[ D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5 \cap D_6 \] (the defective part comes from box 1)

\[ D_4 \cap D_2 \cap D_4 \cap D_2 \cap D_4 \cap D_4 \] (the defective part comes from box 2)

\[ D_3 \cap D_2 \cap D_3 \cap D_3 \cap D_3 \cap D_3 \] (the defective part comes from box 3)

We want the probability of selecting \( D_1 \cap D_2 \cap D_3 \), or \( D_1 \cap D_2 \cap D_3 \), or \( D_1 \cap D_2 \cap D_3 \):

\[
P((D_1 \cap D_2 \cap D_3) \cup (D_1 \cap D_2 \cap D_3) \cup (D_1 \cap D_2 \cap D_3)) = P(D_1 \cap D_2 \cap D_3) + P(D_1 \cap D_2 \cap D_3) + P(D_1 \cap D_2 \cap D_3)
\]

\[
\approx 3(.0098) = .0294,
\]

where the event \( D \) is short-hand for \( D_1, D_2, \) or \( D_3 \).
13. **(Conditional Probability)** In a certain region of the country, atmospheric conditions are classified into a finite number of categories, and weather records involving the events of rain and gale-force winds are kept for each of these categories. Weather records indicate that under category 1 atmospheric condition there is a 40% chance of rain, a 30% chance of gale-force winds, and a 50% chance that it either rains or has gale-force winds. Suppose you wake up on a day with category 1 conditions and find that it is raining. What is the probability of gale-force winds? (Assume that you know the all of the categories and can classify them correctly.)

**Solution:**  
**Step 1:** Write down the events.

Let \( R \) = the event of rain  
\( W \) = the event of gale-force winds

**Step 2:** Write down what you know.

\( P(R) = .4, \quad P(W) = .3, \text{ and } P(R \cup W) = .5. \)

**Step 3:** Write down what you want.

We are told that it is raining. Thus the event \( R \) has occurred. This means that we want to know the probability of gale-force winds given that it is raining. In symbols, we want \( P(W \mid R) \).

**Step 4:** Solve

Using the definition of conditional probability and \( P(R) = .4 \) leads to the following equation:

\[
P(W \mid R) = \frac{P(R \cap W)}{P(R)} = \frac{P(R \cap W)}{.4}.
\]  
(Equation 1)

We cannot evaluate the expression on the right-hand side because we don’t know \( P(R \cap W) \). However, we can use the formula \( P(R \cup W) = P(R) + P(W) - P(R \cap W) \) to solve for \( P(R \cap W) \). Substituting the values found in step 2 into the formula yields

\[
.5 = .4 + .3 - P(R \cap W).
\]

Solving for \( P(R \cap W) \) in the equation gives us \( P(R \cap W) = .2 \). For the last step, we must substitute this value into equation 1 to find the desired result.

\[
P(W \mid R) = \frac{P(R \cap W)}{.4} = \frac{.2}{.4} = \frac{1}{2}.
\]
14. *(Bayes’ Theorem)* An inexpensive blood test can be used to test whether or not a person has a certain type of cancer. The test is not perfect: there is a 12% chance that a person who has the cancer will falsely test negative, and a 15% chance that a person who does not have the cancer will falsely test positive. More accurate (and more expensive) testing has shown that the cancer is present in 8% of the tested population. What is the probability that a person who tests positive has this type of cancer?

**Solution:**

**Step 1:** Define the events: let

\[
\begin{align*}
\Omega &= \text{all people} \\
C &= \text{the person has cancer} \\
C^C &= \text{the person does not have cancer} \\
T^+ &= \text{the person tests positive for cancer} \\
T^- &= \text{the person tests negative for cancer}
\end{align*}
\]

Notice that \((T^+)^C = T^-\). If we assume that there were no inclusive tests, and if all of the population were tested, then \(T^+ \text{ and } T^-\) would partition \(\Omega\). However, the data suggests that we use a different partition. The sets \(C\) and \(C^C\) also partition the sample space. Because of the form of the given information, we will use these sets as our partition.

**Step 2:** Write down the given information:

Given: \(P(C) = 0.08\) \(\Rightarrow\) \(P(C^C) = 1 - 0.08 = .92\).

From the data, we know the probability of testing positive or negative given that the events \(C\) and \(C^C\) have occurred. Thus based on the data, we should use \(C\) and \(C^C\) to partition \(\Omega\).

\[
\begin{align*}
P(T^- | C) &= 0.12 \quad \Rightarrow \quad P(T^+ | C) = 1 - P(T^- | C) = 1 - 0.12 = .88, \\
P(T^+ | C^C) &= 0.15 \quad \Rightarrow \quad P(T^- | C^C) = 1 - P(T^+ | C^C) = 1 - 0.15 = .85,
\end{align*}
\]

**Step 3:** Write down what you are trying to solve for:

Want \(P(C | T^+)\). We can compute this term using Bayes’ theorem with \(C\) and \(C^C\) as our partition.

\[
P(C | T^+) = \frac{P(C)P(T^+ | C)}{P(C)P(T^+ | C) + P(C^C)P(T^+ | C^C)} = \frac{(0.08)(.88)}{(0.08)(.88) + (.92)(.15)} = .3378
\]
15. (Expected Value) Find the missing probability in the table below and use it to compute the expected value of \( X \).

\[
\begin{array}{c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 & 2 \\
 P(X=x) & 0.2 & 0.1 & .3 & .2 & \\
\end{array}
\]

**Solution:** Recall: The sum of all of the probabilities must be one. So if the range of \( X \) is \( \{x_1, x_2, \ldots, x_n\} \), then \( \sum_{i=1}^{n} P(X = x_i) = 1 \). Thus \( .2 + .1 + .3 + P(X = 1) + .2 = 1 \). Solving for \( P(X = 1) \) yields \( P(X = 1) = 1 - .8 = .2 \). The expected value is \( E(X) = -1(.1) + 1(.2) = .1 \).

16. (Expected Value) The range of a random variable \( X \) is \( \{0, 1, 2, 3, 4, 5\} \). Without knowing any of the probabilities associated with the events \( X=x \), determine which of the following values are possible expected values for the random variable \( X \). Circle your answer/answers (there may be more than one). **HINT:** Remember the center of mass analogy for expected value (the weights hanging from the stick). You can’t do any simple calculations here. This is a problem where you know it, or you don’t.

(a.) -13   (b.) 5.1   (c.) -0.1   (d.) 10   (e) 3.2

**Solution:** The expected value of \( X \) must lie somewhere between the minimum range value and the maximum range value. If

Range of \( X = \{x_1, x_2, \ldots, x_n\} \), where \( x_1 < x_2 < \cdots < x_n \), then \( x_1 \leq E(X) \leq x_n \).

In our case, \( x_1 = 0 \leq E(X) \leq 5 = x_6 \). The only possibility is (e).

Problems 17-18 (Intro to Random Variables) Consider the experiment of flipping a coin two times. Let \( H \) be the event that a toss turns up heads and \( T \) be the event that a toss turns up tails. To each outcome in the sample space we assign the number

\[ X = \text{number of heads in two tosses} - \text{number of tails in two tosses}. \]

Notice that \( X \) is a random variable.

17. Write down the sample space \( \Omega \) (the set of the four possible outcomes of the experiment).

**Solution:** \( \Omega = \{(H,H), (H,T), (T,H), (T,T)\} \)
18. Write down all the possible events $X = x$, and give the probabilities associated with each of the events.

**Solution:** To find the range of $X$ it helps to explicitly write out the formula for $X$. If the coin toss results in two heads, then $X((H, H)) = 2 - 0 = 2$. If the coin toss results in one head and one tail in either order, then $X((H, T)) = X((T, H)) = 1 - 1 = 0$. If the coin toss results in two tails, then $X((T, T)) = 0 - 2 = -2$. Thus the range of $X$ is $R_x = \{-2, 0, 2\}$. We now write out the associated probabilities:

\[ f_X(-2) = P(X = -2) = P((T, T)) = \frac{1}{4}. \]

\[ f_X(0) = P(X = 0) = P(\{(H, T), (T, H)\}) = \frac{2}{4} = \frac{1}{2}. \]

\[ f_X(2) = P(X = 2) = P((H, H)) = \frac{1}{4}. \]

Notice that the sum of all of the probabilities is one: $\sum_{i=1}^{3} P(X = x_i) = 1$, where by convention we take $x_1 = -2$, $x_2 = 0$, and $x_3 = 2$. 
Problem 19-21: (Expected Value and Variance) Below is the graph of the probability distribution of the random variable $X$. Notice that the range of $X = \{-3,-2,-1,0,1,2,3\}$.

19. Explicitly write out a table for the probability distribution: $f_X (x_i) = P(X = x_i)$.

**Solution:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_X (x)$</td>
<td>.1</td>
<td>.1</td>
<td>.2</td>
<td>.2</td>
<td>.2</td>
<td>.1</td>
<td>.1</td>
</tr>
</tbody>
</table>

NOTE: the sum of all of the probabilities adds to one, as they must.

20. Compute the mean of the random variable $X$.

**Solution:** $E(X) = (-3)(.1) + (-2)(.1) + (-1)(.2) + (0)(.2) + (1)(.2) + (2)(.1) + (3)(.1) = 0$

21. Compute the variance and standard deviation of the random variable $X$.

**Solution:** First compute the variance.

\[
V(X) = (-3)^2(.1) + (-2)^2(.1) + (-1)^2(.2) + (0)^2(.2) + (1)^2(.2) + (2)^2(.1) + (3)^2(.1)
\]
\[
= 2[(1)^2(.2) + (2)^2(.1) + (3)^2(.1)]
\]
\[
= 2[.2 + .4 + .9] = 3
\]

The standard deviation is

\[
\sigma_X = \sqrt{V(X)} = \sqrt{3}.
\]
22-23. Let $X$ be a finite random variable with the following cumulative probability distribution:

$$
F_X(x) = \begin{cases} 
0 & \text{if } x < -3 \\
.4 & \text{if } -3 \leq x < -1 \\
.7 & \text{if } -1 \leq x < 1 \\
1 & \text{if } 1 \leq x 
\end{cases}
$$

22. What is the range of $X$?

Solution: Range of $X = \{-3, -1, 1\}$.

23. Find all of the values for $f_X(x)$.

Solution: $f_X(-3) = .4$; $f_X(-1) = .3$; $f_X(1) = .3$. Note: the sum of the probabilities is 1.
24. Roll a die 18 times. On each roll, define a success as the event that the face value is one. Find the probability of getting at least 3 ones in 18 rolls of the die.

**Solution:**

Let \( n = 18 \). Then \( \theta = 1/6 \)

\[
P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2) = 1 - F_X(2) = 1 - \text{Binomdist}(2, 18, 1/6, \text{True}) = 0.597346
\]
Formulas for uniform and exponential distributions that you might find useful:

If \( X \) has a uniform distribution, then

\[
P(X \leq x) = \frac{x - a}{b - a} \quad \text{(Formula u1)}
\]

\[
P(X > x) = 1 - P(X \leq 1) = 1 - \frac{x - a}{b - a} = \frac{b - a - x + a}{b - a} = \frac{b - x}{b - a} \quad \text{(Formula u2)}
\]

\[
P(c \leq X \leq d) = F_X(d) - F_X(c) = \frac{d - a}{b - a} - \frac{c - a}{b - a} = \frac{d - c}{b - a} \quad \text{(Formula u3)}
\]

If \( X \) has an exponential distribution, then

\[
P(X \leq x) = 1 - \text{Exp}\left(-\frac{x}{\alpha}\right) \quad \text{(Formula e1)}
\]

\[
P(X > x) = \text{Exp}\left(-\frac{x}{\alpha}\right) \quad \text{(the survival function)} \quad \text{(Formula e2)}
\]

**Proof:** For any range value \( x \) we have

\[
P(X > x) = 1 - P(X \leq x) \quad \text{(by complementation)}
\]

\[
= 1 - F_X(x) \quad \text{(by definition of the cumulative distribution)}
\]

\[
= \text{Exp}\left(-\frac{x}{\alpha}\right) \quad \text{(by definition of the exponential function: see class notes)}
\]

**NOTE:** \( P(X \geq x) = P(X > x) \quad \text{(since } X \text{ is a continuous random variable)} \)

\[
P(c \leq X \leq d) = F_X(d) - F_X(c)
\]

\[
= \left[1 - \text{Exp}\left(-\frac{d}{\alpha}\right)\right] - \left[1 - \text{Exp}\left(-\frac{c}{\alpha}\right)\right] = \text{Exp}\left(-\frac{c}{\alpha}\right) - \text{Exp}\left(-\frac{d}{\alpha}\right) \quad \text{(Formula e3)}
\]

The three most important relations for both discrete and continuous random variables are:

1. \( P(X \leq x) = F_X(x) \)
2. \( P(X > x) = 1 - F_X(x) \)
3. \( P(c < x \leq d) = F_X(d) - F_X(c) \)
Comments:

Property 1 is just the definition of the cumulative distribution function.

Property 2 follows from the fact that \( P(X > x) = 1 - P(X \leq x) \). The probability \( P(X > x) \) is some times expresses as the survival function \( S(x) = P(X > x) \) since if we think of \( X \) as time until death or failure, then \( S(x) = P(X > x) \) is the probability that the individual or part survives to at least time \( x \).

Property 3 is more general than it first appears to be. For example, for a continuous random variable the probability that \( X \) takes on any particular value \( x \) is zero. That is, \( P(X = x) = 0 \) for any \( x \). It then follow that

\[
P(c < x \leq d) = P(c < x < d) = P(c \leq x < d) = P(c \leq x \leq d) = F_x(d) - F_x(c).
\]

Thus, given the three properties above and the formulas for the cumulative distribution function \( F_x(x) \), you can solve any of the following problems by manipulating the definitions to find the formula for the desired probability.

25. (uniform distributions) A 100-ft water pipe runs from your toilet to the alley where it connects with the city’s water line. 20 feet of the pipe lies under your house. By law, you are responsible for repairing any leaks in the pipe. To fix a leak under your house is very expensive. Provided that your house is well constructed, a leak is equally likely to occur anywhere along the pipe. If the pipe springs a leak, what is the probability that it is under your house?

Solution: The location of the break in your pipe can be modeled with a uniform distribution. Let \( X \) be the location of the leak. Then \( X \) is a uniform random variable with \( a = 0 \) and \( b = 100 \), with \( a = 0 \) denoting the starting position under your toilet, and \( b = 100 \) denoting the end of the pipe that connects with the city’s waterline. We want to calculate the probability of a leak in the first 20 feet of the pipe (the part of the pipe under your house). This is given by

\[
P(X \leq 20) = F_x(20) = \frac{20}{100} = \frac{1}{5}.
\]
26. (**exponential distributions**) A new building has just opened on campus. Unfortunately, the administration has forgotten to budget for replacement light bulbs. The lifetime of a bulb has an exponential distribution with a mean of 2000 hours. If there are 500 bulbs in the building, how long will it be before 100 of them have burned out?

**Solution:** Let $X$ = waiting time until the bulbs burn out (out of 100%). For example, $X = 20$ is the time that 20% of the bulbs have burn out. We are given that $\alpha = E(X) = 2000$ hours. We want the time $x$ at which 100 bulbs burn out. This is equivalent to the time that it takes for 100 out of the 500 bulbs to burn out, or 1/5 of the total number of bulbs. Thus, we want the time $x$ such that

$$P(X \leq x) = \frac{1}{5}. $$

This is equivalent to the question: at what time will $\frac{4}{5}$ of the bulbs still be working (surviving)? That is, we want $x$ such that $P(X > x) = \frac{4}{5}$.

Using the definition of the exponential distribution gives

$$P(X > x) = \exp\left(-\frac{x}{\alpha}\right) = \exp\left(-\frac{x}{2000}\right) = \frac{4}{5}. $$

Solving for $x$ yields the desired time:

$$\exp\left(-\frac{x}{2000}\right) = \frac{4}{5} \Rightarrow \ln\left(\exp\left(-\frac{x}{2000}\right)\right) = \ln\left(\frac{4}{5}\right) $$

(taking the natural log of both sides)

$$\Rightarrow -\frac{x}{2000} = \ln\left(\frac{4}{5}\right) $$

(using the property of exponents $\ln(e^a) = a \ln(e) = a$)

$$\Rightarrow x = -2000 \ln\left(\frac{4}{5}\right) = 2000 \ln\left(\frac{5}{4}\right) = 2000 \ln\left(1 + \frac{1}{4}\right) \approx 2000 \cdot \frac{1}{4} = 500 \text{ hours}. $$