Throughout this problem set, it’s OK to use “arc” and “-1” notation interchangeably. If we write e.g. “cos⁻¹”, and you’ve written it as “arccos”, you’re not wrong.

In problems 1-8, fill in the blank to make the statement true. Assume that θ is in the first quadrant.

1. \( \sin \theta = \frac{2}{3} \) if and only if \( \theta = \arcsin\left(\frac{2}{3}\right) \)
2. \( \cos \theta = 0.6 \) if and only if \( \theta = \arccos(0.6) \)
3. \( \tan \theta = 1.8 \) if and only if \( \theta = \arctan(1.8) \)
4. \( \sin \theta = 0.1 \) if and only if \( \theta = \arcsin(0.1) \)
5. \( \cos \theta = \frac{3}{4} \) if and only if \( \theta = \cos^{-1}\left(\frac{3}{4}\right) \)
6. \( \tan \theta = 9 \) if and only if \( \theta = \tan^{-1}(9) \)
7. \( \sin \theta = 0.4 \) if and only if \( \theta = \sin^{-1}(0.4) \)
8. \( \cos \theta = \frac{2}{7} \) if and only if \( \theta = \cos^{-1}\left(\frac{2}{7}\right) \)

9. \( \sin^{-1}(\frac{1}{4}) = ? \)
   (a) \( \sin(-\frac{1}{4}) \)  (b) \( -\sin(\frac{1}{4}) \)  (c) \( \frac{1}{\sin(\frac{1}{4})} \)
   (d) \( \sin(4) \)  *(e) \( \arcsin(\frac{1}{4}) \)  (f) \( \frac{1}{\arcsin(\frac{1}{4})} \)

This is just to see if you know that \( \sin^{-1} \) and \( \arcsin \) are the same thing.

10. Consider the statements (i) and (ii):
    (i) \( \sin(2\theta) = 2\sin \theta \)
    (ii) \( \cos^2 \theta = \cos(\theta^2) \)

    Choose the correct answer:
    (a) (i) is true, (ii) is true
    (b) (i) is true, (ii) is false
    (c) (i) is false, (ii) is true
    *(d) (i) is false, (ii) is false

    You can easily think of counterexamples to (i). For example, \( \sin 90^\circ = 1 \), but \( \sin 180^\circ = 0 \).
    (ii) is also wrong. \( \cos^2 \theta = (\cos \theta)^2 \neq \cos(\theta^2) \).
In problems 11-20, evaluate the inverse functions. Your answers should be in radians. Give exact values, e.g. $\pi/3$, rather than decimal approximations. You should not use a calculator.

11. $\arccos(1) = \ ?$ We need an angle $\theta$ in the first quadrant for which $\cos \theta = 1$: $\theta = 0$

12. $\sin^{-1}(\sqrt{3}/2) = \ ?$ We need $\theta$ in the first quadrant for which $\sin \theta = \sqrt{3}/2$: $\theta = \pi/3$

13. $\arctan(1) = \ ?$ $\pi/4$

14. $\arcsin(1/2) = \ ?$ $\pi/6$

15. $\cos^{-1}(\sqrt{2}/2) = \ ?$ $\pi/4$

16. $\tan^{-1}(\sqrt{3}) = \ ?$ $\pi/3$

17. $\arccos(1/2) = \ ?$ $\pi/3$

18. $\sin^{-1}(0) = \ ?$ 0

19. $\arctan(0) = \ ?$ 0

20. $\cos^{-1}(\sqrt{3}/2) = \ ?$ $\pi/6$

21. Consider the statements (i) and (ii):

   (i) $\cos\left(\frac{3\theta}{3}\right) = \cos \theta$

   (ii) $\sin^2 \theta = (\sin \theta)^2$

   Choose the correct answer:

   *(a) (i) is true, (ii) is true  
(b) (i) is true, (ii) is false  
(c) (i) is false, (ii) is true  
(d) (i) is false, (ii) is false*

   In (i), the fraction $3\theta/3$ is all in the argument of the cosine function, so it can be simplified before the cosine is evaluated.

   (ii) is just a matter of notation: you have to know that this is what $\sin^2 \theta$ means.

22. $\cos^{-1}(0.3) = \ ?$

   (a) $\cos(-0.3)$  
(b) $-\cos(0.3)$  
(c) $\frac{1}{\cos(0.3)}$

   (d) $\cos(10/3)$  
*(e) arccos(0.3)  
(f) $\frac{1}{\arccos(0.3)}$

   This is just to see if you know what $\cos^{-1}$ means.
In problems 23-32, find \( \theta \) in degrees: \( 0 \leq \theta < 360^\circ \). You should not use a calculator.

23. \( \sin \theta = -\frac{1}{2} \); \( \theta \) is in the third quadrant. Find an angle \( \phi \) in the first quadrant with \( \sin \phi = \frac{1}{2} \); this will be the reference angle. In this case, \( \phi = 30^\circ \). In the third quadrant, \( \theta = 180^\circ + 30^\circ = 210^\circ \).

24. \( \cos \theta = \sqrt{2}/2 \); \( \theta \) is in the fourth quadrant. The reference angle is \( 45^\circ \). In the fourth quadrant, \( \theta = 360^\circ - 45^\circ = 315^\circ \).

25. \( \tan \theta = -1 \); \( \theta \) is in the second quadrant. The reference angle is \( 45^\circ \). In the second quadrant, \( \theta = 180^\circ - 45^\circ = 135^\circ \).

26. \( \sin \theta = -\sqrt{3}/2 \); \( \theta \) is in the fourth quadrant. The reference angle is \( 60^\circ \). \( \theta = 300^\circ \).

27. \( \cos \theta = -\sqrt{3}/2 \); \( \theta \) is in the second quadrant. The reference angle is \( 30^\circ \). \( \theta = 150^\circ \).

28. \( \tan \theta = \sqrt{3} \); \( \theta \) is in the third quadrant. The reference angle is \( 60^\circ \). \( \theta = 240^\circ \).

29. \( \sin \theta = \sqrt{2}/2 \); \( \theta \) is in the second quadrant. The reference angle is \( 45^\circ \). \( \theta = 135^\circ \).

30. \( \cos \theta = -1 \). \( \theta = 180^\circ \); no other angle between \( 0 \) and \( 360 \) has this cosine.

31. \( \tan \theta = -\sqrt{3}/3 \); \( \theta \) is in the fourth quadrant. The reference angle is \( 30^\circ \). \( \theta = 330^\circ \).

32. \( \sin \theta = 0 \); \( \theta \) is in the second quadrant. The reference angle is \( 0^\circ \); \( \theta = 180^\circ \).

33. Consider the statements (i) and (ii):
   (i) \( \frac{\cos(6\theta)}{3} = \cos(2\theta) \)
   (ii) \( \sin^2 \theta = (\sin \theta)^2 \)

   Choose the correct answer:
   (a) (i) is true, (ii) is true
   (b) (i) is true, (ii) is false
   (c) (i) is false, (ii) is true
   (d) (i) is false, (ii) is false
   * (i) is wrong, since \( \cos(6\theta) \neq 3\cos(2\theta) \). For a counterexample, consider \( \theta = 15^\circ \).
   (ii) is correct: you need to know that’s what the \( \sin^2 \) notation means.

34. \( \tan^{-1}(4) = ? \)
   (a) \( \tan(-4) \)
   (b) \( -\tan(4) \)
   (c) \( \frac{1}{\tan(4)} \)
   (d) \( \tan(1/4) \)
   * (e) \( \arctan(4) \)
   (f) \( \frac{1}{\arctan(4)} \)

   This is just to make sure you know that \( \tan^{-1} \) and \( \arctan \) are the same thing.
35. What is the domain of the function: \( y = \arcsin(x) \)?

(a) \((-\infty, \infty)\)  
(b) \([-1,1]\)  
(c) \([0,1]\)  
(d) \([0,\infty)\)  
(e) \((-1,1)\)  
(f) \((0,1)\)

Since \( \sin \theta \) can only take values in the range \([-1,1]\), that must be the domain of the \( \arcsin \) function.

36. What is the domain of the function: \( y = \arctan(x) \)?

*(a) \((-\infty, \infty)\)  
(b) \([-1,1]\)  
(c) \([0,1]\)  
(d) \([0,\infty)\)  
(e) \((-1,1)\)  
(f) \((0,1)\)

tan \( \theta \) can take any value in the range \((-\infty, \infty)\), given the right choice of \( \theta \); so that’s the domain of the \( \arctan \) function.

37. What is the domain of the function: \( y = \cos^{-1}(x) \)?

(a) \((-\infty, \infty)\)  
(b) \([-1,1]\)  
(c) \([0,1]\)  
(d) \([0,\infty)\)  
(e) \((-1,1)\)  
(f) \((0,1)\)

Like \( \sin \theta \), cos \( \theta \) can take any value in \([-1,1]\), and only those values. Hence that’s the domain of the \( \cos^{-1} \) function.
38. Match the the graphs below to the inverse trigonometric functions. Choose one of the answers (a)-(f):

(a) \( y = \sin^{-1}(x) \): Graph 1  \( y = \cos^{-1}(x) \): Graph 2  \( y = \tan^{-1}(x) \): Graph 3

(b) \( y = \sin^{-1}(x) \): Graph 1  \( y = \cos^{-1}(x) \): Graph 3  \( y = \tan^{-1}(x) \): Graph 2

*(c) \( y = \sin^{-1}(x) \): Graph 2  \( y = \cos^{-1}(x) \): Graph 1  \( y = \tan^{-1}(x) \): Graph 3

(d) \( y = \sin^{-1}(x) \): Graph 2  \( y = \cos^{-1}(x) \): Graph 3  \( y = \tan^{-1}(x) \): Graph 1

(e) \( y = \sin^{-1}(x) \): Graph 3  \( y = \cos^{-1}(x) \): Graph 1  \( y = \tan^{-1}(x) \): Graph 2

(f) \( y = \sin^{-1}(x) \): Graph 3  \( y = \cos^{-1}(x) \): Graph 2  \( y = \tan^{-1}(x) \): Graph 1

[See next page for explanation]
Problem 38: You should recognize that Graph 3 is the arc tangent function right away, because of its domain. The domain of both arcsin and arccos is [-1,1], so those functions wouldn’t have any values for x less than -1 or greater than 1.

Notice that Graph 1 includes the point (0, π/2), while Graph 2 includes the point (0,0). Since sin(0) = 0, we know that arcsin(0) = 0; since cos(π/2) = 0, we know that arccos(0) = π/2. Thus Graph 1 is \( y = \arccos(x) \), and Graph 2 is \( y = \arcsin(x) \).

Problem 39: Consider the statements (i) and (ii):

(i) If \( \arccos(x) = \theta \), then \( \cos \theta = x \)
(ii) If \( \cos \theta = x \), then \( \arccos(x) = \theta \)

Choose the correct answer:

(a) (i) is true, (ii) is true
(b) (i) is true, (ii) is false
(c) (i) is false, (ii) is true
(d) (i) is false, (ii) is false

Tricky but important. Remember that arccos is a function, so it can only return one value of \( \theta \) for any value of \( x \). On the other hand, there are many different values of \( \theta \) for which \( \cos(\theta) = x \). For example, \( \cdots = \cos(-\pi/4) = \cos(\pi/4) = \cos(7\pi/4) = \cdots = \sqrt{2}/2 \); but of all these values, we define \( \arccos(\sqrt{2}/2) = \pi/4 \). Thus \( \cos(7\pi/4) = \sqrt{2}/2 \); but \( \arccos(\sqrt{2}/2) \neq 7\pi/4 \)

In problems 40-44, express \( \theta \) as an arc function, e.g. \( \theta = \arcsin(2/5) \). Do not calculate the actual value of \( \theta \).
In problems 45-56, use a calculator or equivalent to evaluate the inverse function. Give your answer in radians; round to four decimal places.

It’s important to use good calculator technique. Don’t copy intermediate results on paper and then re-enter them into your calculator; this can lead to buildup of roundoff errors. If your answer is close to ours but not exactly the same, your calculator technique may need work.

45. \( \sin^{-1}(0.3) = 0.3047 \)

46. \( \cos^{-1}(0.65) = 0.8632 \)

47. \( \tan^{-1}(1.7) = 1.0391 \)

48. \( \sin^{-1}(2/7) = 0.2898 \)

49. \( \cos^{-1}(4/13) = 1.2580 \)

50. \( \tan^{-1}(25/29) = 0.7115 \)

51. \( \arcsin(3/2) = \) no value: \( \sin \theta \) can’t be bigger than 1

52. \( \arccos(4/11) = 1.1986 \)

53. \( \arctan(5/7) = 0.6202 \)

54. \( \arcsin(1/2) = 0.5236 \)

55. \( \arccos(5/13) = 1.1760 \)

56. \( \arctan(5/12) = 0.3948 \)
In problems 57-68, use a calculator or equivalent to find the angle $\theta$ in degrees. Round your answer to two decimal places. Assume that $\theta$ is in the first quadrant.

Here, there’s even more room to go wrong with poor calculator technique, since you’re doing several steps: evaluating the fraction; taking the arc function; then converting from radians to degrees. Again, the key is to store intermediate results in your calculator instead of writing them down and re-entering them.

57. $\cos \theta = 0.7$
   \[ \theta = \frac{180}{\pi} \cos^{-1}(0.7) = 45.57^\circ \]

58. $\sin \theta = 0.55$
   \[ \theta = \frac{180}{\pi} \sin^{-1}(0.55) = 33.37^\circ \]

59. $\tan \theta = 1.2$
   \[ \theta = \frac{180}{\pi} \tan^{-1}(1.2) = 50.19^\circ \]

60. $\cos \theta = \frac{3}{7}$
   \[ \theta = \frac{180}{\pi} \cos^{-1}(\frac{3}{7}) = 64.62^\circ \]

61. $\sin \theta = \frac{5}{13}$
   \[ \theta = \frac{180}{\pi} \sin^{-1}(\frac{5}{13}) = 22.62^\circ \]

62. $\tan \theta = \frac{11}{7}$
   \[ \theta = \frac{180}{\pi} \tan^{-1}(\frac{11}{7}) = 57.53^\circ \]

63. $\cos \theta = \frac{8}{17}$
   \[ \theta = 61.93^\circ \]

64. $\sin \theta = \frac{2}{3}$
   \[ \theta = 41.81^\circ \]

65. $\tan \theta = \frac{29}{23}$
   \[ \theta = 51.58^\circ \]

66. $\cos \theta = \frac{5}{7}$
   \[ \theta = 44.42^\circ \]

67. $\sin \theta = \frac{8}{17}$
   \[ \theta = 28.07^\circ \]

68. $\tan \theta = \frac{22}{21}$
   \[ \theta = 46.33^\circ \]
In problems 69-74, use a calculator or equivalent to determine the angle $\theta$ in radians. Round your answer to four decimal places.

69. $\theta = \arcsin(8/13) = 0.6629$

70. $\theta = \arccos(8/11) = 0.7565$

71. $\theta = \arctan(7/9) = 0.6610$

72. $\theta = \arcsin(4/7) = 0.6082$

73. $\theta = \arccos(19/23) = 0.5987$

74. $\theta = \arctan(31/37) = 0.6974$
In problems 75-80, use a calculator or equivalent to determine the angle $\theta$ in degrees. Round your answer to two decimal places.

75. $\sin \theta = \frac{4}{11}; \theta = 21.32^\circ$

76. $\cos \theta = \frac{7}{12}; \theta = 54.31^\circ$

77. $\tan \theta = \frac{6}{7}; \theta = 40.60^\circ$

78. $\sin \theta = \frac{2}{3}; \theta = 41.81^\circ$

79. $\cos \theta = \frac{3}{5}; \theta = 53.13^\circ$

80. $\tan \theta = \frac{9}{11}; \theta = 39.29^\circ$
In problems 81-89, use a calculator or equivalent to find the value of $\theta$ in radians. Your answers should be in the range $0 \leq \theta < 2\pi$. Round your answers to four decimal places. Remember that an arc function of any positive number in the domain will yield a value in the first quadrant. This can be used for a reference angle.

81. $\sin \theta = 0.4$; $\theta$ is in the second quadrant.
   Reference angle is $\arcsin(0.4);$ $\theta = \pi - \arcsin(0.4) = 2.7301$

82. $\cos \theta = -0.22$; $\theta$ is in the third quadrant.
   Reference angle is $\arccos(0.22);$ $\theta = \pi + \arccos(0.22) = 4.4906$

83. $\tan \theta = -1.56$; $\theta$ is in the fourth quadrant.
   Reference angle is $\arctan(1.56);$ $\theta = 2\pi - \arctan(1.56) = 5.2824$

84. $\sin \theta = -7/13$; $\theta$ is in the third quadrant.
   Reference angle is $\arcsin(7/13);$ $\theta = \pi + \arcsin(7/13) = 3.7102$

85. $\cos \theta = 9/17$; $\theta$ is in the fourth quadrant.
   Reference angle is $\arccos(9/17);$ $\theta = 2\pi - \arccos(9/17) = 5.2703$

86. $\tan \theta = -19/7$; $\theta$ is in the second quadrant.
   Reference angle is $\arctan(19/7);$ $\theta = \pi - \arctan(19/7) = 1.9238$

87. $\sin \theta = -\sqrt{7}/5$; $\theta$ is in the fourth quadrant.
   Reference angle is $\arcsin(\sqrt{7}/5);$ $\theta = 2\pi - \arcsin(\sqrt{7}/5) = 5.7256$

88. $\cos \theta = -\sqrt{6}/6$; $\theta$ is in the second quadrant.
   Reference angle is $\arccos(\sqrt{6}/6);$ $\theta = \pi - \arccos(\sqrt{6}/6) = 1.9913$

89. $\tan \theta = 2\sqrt{5}/3$; $\theta$ is in the third quadrant.
   Reference angle is $\arctan(2\sqrt{5}/3);$ $\theta = \pi + \arctan(2\sqrt{5}/3) = 4.1215$
In problems 90-98, use a calculator or equivalent to find the value of $\theta$ in degrees. Your answers should be in the range $0 \leq \theta < 360$. Round your answers to two decimal places.

We’ll solve these in three steps. First, we’ll take the appropriate arc function of the absolute value of the given number. That will give us the reference angle in radians. We’ll convert that value to get the reference angle $\varphi$ in degrees. Finally, we’ll determine the angle corresponding to $\varphi$ in the given quadrant.

Given the number of steps in this process, good calculator technique is essential. Make sure your answers exactly match ours.

90. $\cos \theta = -0.35$; $\theta$ is in the second quadrant.
Let $\varphi$ be the reference angle. $\cos \varphi = 0.35$; $\theta = 180^\circ \cdot \varphi = 110.49^\circ$

91. $\sin \theta = -0.69$; $\theta$ is in the third quadrant.
Let $\varphi$ be the reference angle. $\sin \varphi = 0.69$; $\theta = 180^\circ + \varphi = 223.63^\circ$

92. $\tan \theta = -1.22$; $\theta$ is in the fourth quadrant.
Let $\varphi$ be the reference angle. $\tan \varphi = 1.22$; $\theta = 360^\circ - \varphi = 309.34^\circ$

93. $\cos \theta = -3/7$; $\theta$ is in the third quadrant.
Let $\varphi$ be the reference angle. $\cos \varphi = 3/7$; $\theta = 180^\circ + \varphi = 244.62^\circ$

94. $\sin \theta = -8/13$; $\theta$ is in the fourth quadrant.
Let $\varphi$ be the reference angle. $\sin \varphi = 8/13$; $\theta = 360^\circ - \varphi = 322.02^\circ$

95. $\tan \theta = -21/13$; $\theta$ is in the second quadrant.
Let $\varphi$ be the reference angle. $\tan \varphi = 21/13$; $\theta = 180^\circ - \varphi = 121.76^\circ$

96. $\cos \theta = \frac{\sqrt{11}}{4}$; $\theta$ is in the fourth quadrant.
Let $\varphi$ be the reference angle. $\cos \varphi = \frac{\sqrt{11}}{4}$; $\theta = 360^\circ - \varphi = 326.01^\circ$

97. $\sin \theta = \frac{2\sqrt{6}}{3}$; $\theta$ is in the second quadrant.
No such value: $\frac{2\sqrt{6}}{3} > 1$

98. $\tan \theta = \frac{3\sqrt{5}}{5}$; $\theta$ is in the third quadrant.
Let $\varphi$ be the reference angle. $\tan \varphi = \frac{3\sqrt{5}}{5}$; $\theta = 180^\circ + \varphi = 233.30^\circ$
99. If $\theta$ is in the first quadrant and $\sec \theta = a$, then $\theta = ?$

(a) $\sin^{-1}(1/a)$
(b) $\frac{1}{\sin^{-1}(a)}$
(c) $\sin^{-1}(1 - a^2)$
*(d) $\cos^{-1}(1/a)$
(e) $\frac{1}{\cos^{-1}(a)}$
(f) $\cos^{-1}(1 - a^2)$

It’s unlikely that your calculator has an “arc secant” button. To solve this, remember that $\sec \theta = \frac{1}{\cos \theta}$. Thus if $\sec \theta = a$, then $\cos \theta = \frac{1}{\sec \theta} = \frac{1}{a}$. That allows you to calculate $\theta = \cos^{-1}(1/a)$.

100. If $\theta$ is in the first quadrant and $\csc \theta = a$, then $\theta = ?$

*(a) $\sin^{-1}(1/a)$
(b) $\frac{1}{\sin^{-1}(a)}$
(c) $\sin^{-1}(1 - a^2)$
(d) $\cos^{-1}(1/a)$
(e) $\frac{1}{\cos^{-1}(a)}$
(f) $\cos^{-1}(1 - a^2)$

To solve this, use the fact that $\csc \theta = \frac{1}{\sin \theta}$. Then if $\csc \theta = a$, $\sin \theta = \frac{1}{\csc \theta} = \frac{1}{a}$. Hence $\theta = \sin^{-1}(1/a)$.

101. If $\theta$ is in the first quadrant and $\cot \theta = a$, then $\theta = ?$

(a) $\frac{\sin^{-1}(a)}{\cos^{-1}(a)}$
(b) $\sin^{-1}(a^2 + 1)$
*(c) $\tan^{-1}(1/a)$
(d) $\frac{\cos^{-1}(a)}{\sin^{-1}(a)}$
(e) $\sin^{-1}(a^2 - 1)$
(f) $\frac{1}{\tan^{-1}(a)}$

Use the fact that $\cot \theta = \frac{1}{\tan \theta}$. Then $\tan \theta = \frac{1}{\cot \theta} = \frac{1}{a}$. Hence $\theta = \tan^{-1}(1/a)$
You will need a calculator or equivalent for word problems 102-112.

102. A flagpole is 80 feet tall. Its shadow is 99 feet long. What is the angle of elevation $\theta$ of the sun at that time? Round your answer to the nearest degree.

$$\tan \theta = \frac{80}{99}; \text{ so } \theta = \frac{180}{\pi} \tan^{-1}(80/99) = 39^\circ$$

103. An airplane pilot thinks he's flying due north at 90 miles per hour. However, there's a wind blowing him eastward at 33 miles per hour. At what angle east of north is he actually flying? Round your answer to the nearest degree.

$$\tan \theta = \frac{33}{90}; \text{ so } \theta = \frac{180}{\pi} \tan^{-1}(33/90) = 20^\circ$$

104. A railroad track crosses two parallel roads at an angle. The distance between the roads is 1100 meters. The length of the railroad track between the two roads is 1234 meters. What is the angle between the roads and the track? Round your answer to the nearest 0.01 radians.

$$\theta = \arcsin(1100/1234) = 1.10 \text{ rad}$$

105. A prisoner is trying to dig a tunnel under a wall 120 feet away. Unfortunately, his compass is inaccurate, and he has to dig 140 feet before he reaches the wall. What angle does his tunnel make to the true direction? Round your answer to the nearest 0.01 radians.

$$\theta = \arccos(120/140) = 0.54 \text{ rad}$$
106. A mountain peak is 4600 feet above you. On the map, the horizontal distance from you to the peak is 2.3 miles. What is the angle of elevation of the peak from where you stand? Round your answer to the nearest 0.01 degrees.

You’ll need to convert miles to feet so that distance and height are in the same units. \( \tan \theta = \frac{4600}{(2.3)(5280)} \); so \( \theta = 20.75^\circ \)

107. Your friend’s house is 7400 feet away from yours. An airplane flies over your friend’s house at an elevation of 5000 feet. What is the angle of elevation of the airplane, as seen from your house? Round your answer to the nearest 0.01 radians.

\( \theta = \arctan\left(\frac{5000}{7400}\right) = 0.59 \text{ rad} \)

108. A pole is supported by a diagonal guy wire. The wire is 17.5 meters long, and is attached to the pole 14.2 meters above the ground. What angle does the wire make with the ground? Round your answer to the nearest 0.1 degrees.

\( \sin \theta = \frac{14.2}{17.5}; \) so \( \theta = \frac{180}{\pi} \arcsin\left(\frac{14.2}{17.5}\right) = 54.2^\circ \)

109. A pole is supported by a diagonal guy wire. The wire is 22.3 meters long, and is anchored in the ground 10.8 meters from the base of the pole. What angle does the wire make with the ground? Round your answer to the nearest 0.01 radians.

\( \theta = \arccos\left(\frac{10.8}{22.3}\right) = 1.07 \text{ rad} \)

110. You are walking up a long slope toward a mountain. Your GPS informs you that you’ve walked a horizontal distance of 5800 meters, during which time your elevation has increased by 550 m. What is the angle of the slope above the horizontal? Round your answer to the nearest 0.01 radians.

\( \theta = \arctan\left(\frac{550}{5800}\right) = 0.09 \text{ rad} \)

111. You are using a ramp to load a truck. The ramp is 13.6 feet long; the bed of the truck is 3.4 feet above the ground. What angle does the ramp make with the ground? Round your answer to the nearest tenth of a degree.

\( \sin \theta = \frac{3.4}{13.6}; \) so \( \theta = 14.5^\circ \)

112. Your new flagpole was 90 ft high and vertical when it was first installed. Unfortunately, the foundation was weak, and the pole is now tilting: the top of the pole is directly above a point 5 feet away from the base. By what angle does the pole deviate from the vertical? Round your answer to the nearest 0.01 radians.

\( \theta = \arcsin\left(\frac{5}{90}\right) = 0.06 \text{ rad} \)
In problems 113-120, evaluate the functions. Give exact values and rationalize all denominators. Do not use a calculator.

113. \( \sin \left( \cos^{-1} \left( \frac{3}{5} \right) \right) = ? \)

You want the sine of an angle whose cosine is 3/5. Sketch a right triangle with sides \(x\), \(y\), and \(r\), with the angle \( \theta \) opposite \( x \). If \( \cos \theta = 3/5 \), then you can label two sides: \( x = 3 \) and \( r = 5 \). You can use the theorem of Pythagoras to calculate \( y = 4 \). Then \( \sin \theta = y/r = 4/5 \).

Alternatively, if you’ve learned fundamental trig identities, you can use: \( \sin^2 \theta + \cos^2 \theta = 1 \). Then \( \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \left( \frac{3}{5} \right)^2} = 4/5 \). (Why not -4/5?)

114. \( \csc \left( \sin^{-1} \left( \frac{13}{19} \right) \right) = ? \)

Use the fact that \( \csc \theta = \frac{1}{\sin \theta} = \frac{19}{13} \)

115. \( \cos \left( \tan^{-1} \left( \frac{\sqrt{5}}{3} \right) \right) = ? \)

Sketch a right triangle as you did for problem 113. If \( \tan \theta = \frac{\sqrt{5}}{3} \), you can label two sides: \( x = 3 \) and \( y = \sqrt{5} \). Then Pythagoras allows you to calculate: \( r = \sqrt{14} \). Now \( \cos \theta = \frac{x}{r} = \frac{3\sqrt{14}}{14} \).

116. \( \sec \left( \cos^{-1} \left( \frac{5}{7} \right) \right) = ? \)

Use the fact that \( \sec \theta = \frac{1}{\cos \theta} = \frac{7}{5} \).

117. \( \cos \left( \sin^{-1} \left( \frac{3}{4} \right) \right) = ? \)

Sketch a right triangle; since \( \sin \theta = 3/4 \), label \( y = 3 \) and \( r = 4 \). Use Pythagoras to calculate \( x = \sqrt{7} \). Then \( \cos \theta = \frac{x}{r} = \frac{\sqrt{7}}{4} \).

If you know trig identities, you can use: \( \sin^2 \theta + \cos^2 \theta = 1 \).
118. \[ \tan \left( \cos^{-1} \frac{\sqrt{21}}{5} \right) = ? \]

Sketch a right triangle. \( x = \sqrt{21} \); \( r = 5 \). By Pythagoras, \( y = 2 \). Then \( \tan \theta = \frac{y}{x} = \frac{2\sqrt{21}}{21} \).

119. \[ \sec \left( \tan^{-1} \left( \frac{12}{5} \right) \right) = ? \]

Sketch a right triangle and label \( x = 5 \) and \( y = 12 \). By Pythagoras, \( r = 13 \), so \( \sec \theta = \frac{r}{x} = \frac{13}{5} \).

If you know trig identities, you can use: \( \sec^2 \theta = \tan^2 \theta + 1 \).

120. \[ \cot \left( \cos^{-1} \left( \frac{2}{3} \right) \right) = ? \]

Sketch a right triangle and label \( x = 2 \) and \( r = 3 \). By Pythagoras, \( y = \sqrt{5} \). Then \( \cot \theta = \frac{x}{y} = \frac{2\sqrt{5}}{5} \).
In problems 121-128, evaluate the functions. All angles are in radians. Give exact values; do not use a calculator.

121. \( \cos^{-1}(\sin(\pi/3)) = ? \)

The key here is to remember that \( \sin \varphi = \cos\left(\frac{\pi}{2} - \varphi\right) \). We want a first-quadrant angle \( \theta \) such that \( \cos \theta = \sin(\pi/3) \). Thus \( \theta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \).

122. \( \tan^{-1}(\cot(\pi/2)) = ? \)

Use the fact that \( \cot \varphi = \tan\left(\frac{\pi}{2} - \varphi\right) \). We want a first-quadrant angle \( \theta \) such that \( \tan \theta = \cot(\pi/2) \). Thus \( \theta = \frac{\pi}{2} - \frac{\pi}{2} = 0 \).

123. \( \sin^{-1}(\cos(\pi/4)) = ? \)

Use the fact that \( \cos \varphi = \sin\left(\frac{\pi}{2} - \varphi\right) \). We want \( \theta \) such that \( \sin \theta = \cos(\pi/4) \). Thus \( \theta = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \).

124. \( \cos^{-1}(\sin(\pi/5)) = ? \)

Use the fact that \( \sin \varphi = \cos\left(\frac{\pi}{2} - \varphi\right) \). We want \( \theta \) such that \( \cos \theta = \sin(\pi/5) \). Thus \( \theta = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10} \).

125. \( \sin^{-1}(\cos 2\pi/7) = ? \)

Use the fact that \( \cos \varphi = \sin\left(\frac{\pi}{2} - \varphi\right) \). We want \( \theta \) such that \( \sin \theta = \cos(2\pi/7) \). Thus \( \theta = \frac{\pi}{2} - \frac{2\pi}{7} = \frac{3\pi}{14} \).

126. \( \tan^{-1}(\cot(3\pi/8)) = ? \)

Use the fact that \( \cot \varphi = \tan\left(\frac{\pi}{2} - \varphi\right) \). We want \( \theta \) such that \( \tan \theta = \cot(3\pi/8) \). Thus \( \theta = \frac{\pi}{2} - \frac{3\pi}{8} = \frac{\pi}{8} \).
127. \( \cos^{-1}(\sin(3\pi/14)) = ? \)

Use the fact that \( \sin \varphi = \cos\left(\frac{\pi}{2} - \varphi\right) \). We want \( \theta \) such that \( \cos \theta = \sin(3\pi/14) \). Thus

\[
\theta = \frac{\pi}{2} - \frac{3\pi}{14} = \frac{2\pi}{7}.
\]

128. \( \sin^{-1}(\cos(5\pi/12)) = ? \)

Use the fact that \( \cos \varphi = \sin\left(\frac{\pi}{2} - \varphi\right) \). We want \( \theta \) such that \( \sin \theta = \cos(5\pi/12) \). Thus

\[
\theta = \frac{\pi}{2} - \frac{5\pi}{12} = \frac{\pi}{12}.
\]