In this file, we will use a triangle ABC. The points and the angles will be referred to by the capital letters A, B, and C; the sides will be referred to by the lower-case letters a, b, and c, with each side opposite the corresponding angle. Unless stated otherwise, you should make no assumptions about whether angles are acute or obtuse.

You will need a calculator or equivalent for almost all of these problems.

It’s a good idea to sketch and label a triangle when you solve problems like the ones in this set. Your sketch doesn’t have to be exact, though it should try to represent acute/obtuse angles correctly.

1. If A = 20° and B = 100°, what is C?
   A + B + C = 180°; so C = 60°.

2. If A = 50° and C = 60°, what is B? 70°

3. If B = 140° and C = 20°, what is A? 20°

4. If A = π/5 rad and B = 3π/5 rad, what is C? π/5 rad

5. If A = 3π/7 rad and C = 2π/7 rad, what is B? 2π/7 rad

6. If B = π/5 rad and C = π/2 rad, what is A? 3π/10 rad

7. A = 40°, C = 70°, and b = 10.
   These are all ASA problems. We begin by calculating the third angle. A + B + C = 180°; so
   B = 70°. Now we use the law of sines: \( \frac{a}{\sin 40°} = \frac{10}{\sin 70°} = \frac{c}{\sin 70°} \). That gives us:
   \[ a = \frac{10 \sin 40°}{\sin 70°} = 6.8404 \quad \text{and} \quad c = \frac{10 \sin 70°}{\sin 70°} = 10 \]

8. A = 110°, B = 20°, and c = 12.
   C = 50°. \( a = \frac{12 \sin 110°}{\sin 50°} = 14.7202 \) and \( b = \frac{12 \sin 20°}{\sin 50°} = 5.3577 \)

   A = 70°. \( b = \frac{6 \sin 35°}{\sin 70°} = 3.6623 \) and \( c = \frac{6 \sin 75°}{\sin 70°} = 6.1675 \)
10. A = 98°, B = 34°, and c = 5.
   C = 48°.  \[ a = \frac{5 \sin 98°}{\sin 48°} = 6.6627 \quad \text{and} \quad b = \frac{5 \sin 34°}{\sin 48°} = 3.7623 \]

   B = 1°.  \[ a = \frac{20 \sin 51°}{\sin 1°} = 890.5889 \quad \text{and} \quad c = \frac{20 \sin 128°}{\sin 1°} = 903.0397 \]

12. B = 11°; C = 23°; and a = 8.
   A = 146°.  \[ b = \frac{8 \sin 11°}{\sin 146°} = 2.7298 \quad \text{and} \quad c = \frac{8 \sin 23°}{\sin 146°} = 5.5899 \]

13. A = π/7 rad; B = 2π/7 rad; and c = 10.
   C = 4π/7 rad.  \[ a = \frac{10 \sin(\pi/7)}{\sin(4\pi/7)} = 4.4504 \quad \text{and} \quad b = \frac{10 \sin(2\pi/7)}{\sin(4\pi/7)} = 8.0194 \]

14. A = 3π/10 rad; C = π/10 rad; and b = 12.
   B = 3π/5 rad.  \[ a = \frac{12 \sin(3\pi/10)}{\sin(3\pi/5)} = 10.2078 \quad \text{and} \quad c = \frac{12 \sin(\pi/10)}{\sin(3\pi/5)} = 3.8990 \]

15. B = π/5 rad; C = π/4 rad; and a = 8.
   A = 11π/20 rad.  \[ b = \frac{8 \sin(\pi/5)}{\sin(11\pi/20)} = 4.7609 \quad \text{and} \quad c = \frac{8 \sin(\pi/4)}{\sin(11\pi/20)} = 5.7274 \]

In problems 16-21, find the missing angles and sides of the triangle ABC. Round all values to four decimal places.
These are ASA problems as well; the only difference is that they require you to use a calculator to find the missing angle. Make sure you use good calculator technique: store intermediate values in your calculator rather than writing them down and re-entering them.

16. A = 0.35 rad; B = 0.8 rad; c = 20.
   C = π − (0.35 + 0.8) = 1.9916 rad.
   \[ a = \frac{20 \sin(0.35)}{\sin C} = 7.5134 \quad \text{and} \quad b = \frac{20 \sin(0.8)}{\sin C} = 15.7183 \]

17. A = 0.4 rad; C = 0.25 rad; b = 10.
   B = 2.4916 rad.  \[ a = \frac{10 \sin(0.4)}{\sin B} = 6.4347 \quad \text{and} \quad c = \frac{10 \sin(0.25)}{\sin B} = 4.0881 \]

18. B = 2 rad; C = 0.7 rad; a = 8.
   A = 0.4416 rad.  \[ b = \frac{8 \sin(2)}{\sin A} = 17.0209 \quad \text{and} \quad c = \frac{8 \sin(0.7)}{\sin A} = 12.0589 \]
19. A = 1.1 rad; B = 1.1 rad; c = 6.

C = 0.9416 rad. Since A = B, \( a = \frac{6 \sin(1.1)}{\sin C} = 6.6138 \)

20. A = 0.85 rad; C = 0.75 rad; b = 12.

B = 1.5416 rad. \( a = \frac{12 \sin(0.85)}{\sin B} = 9.0192 \) and \( c = \frac{12 \sin(0.75)}{\sin B} = 8.1832 \)

21. B = 0.3 rad; C = 2.5 rad; a = 14.

A = 0.3416 rad. \( b = \frac{14 \sin(0.3)}{\sin A} = 12.3505 \) and \( c = \frac{14 \sin(2.5)}{\sin A} = 25.0117 \)

In problems 22-51, find the missing angles and sides of the triangle ABC. If there are two possible triangles, give the side and angles for both. If there is no solution, say so. Round your answers as follows: sides, to four decimal places; angles, to the nearest 0.01° or 0.0001 rad. Use the angle measure that’s used in the statement of the problem.

These are SSA problems. It’s especially important that you sketch the triangles, because that can help you decide how many solutions there are.

When thinking about SSA problems, it might help to imagine a crane with a cable stretched from the top to the ground. The angle between the crane and the ground is the given angle; the length of the crane is the side adjacent to the angle; and the length of the cable is the side opposite the angle.

22. A = 150°; a = 15; c = 10.

A is obtuse (the crane is leaning backward). The cable (a) is longer than the crane (c), so there’s one solution. Since A is obtuse, B and C must be acute. By the law of sines,

\[
\frac{15}{\sin 150°} = \frac{10}{\sin C}; \text{ so } \sin C = \frac{10 \sin 150°}{15}. \text{ } C \text{ is acute, so } C = 19.47°. \text{ We store this value in the calculator for future computation. } \text{ Don’t write down this approximate C and re-enter it.}
\]

We know A and C, so we can calculate \( B = 180 – (A + C) = 10.53°. \text{ Store this value.} \)

Finally, \( \frac{15}{\sin 150°} = \frac{b}{\sin B}; \text{ so } b = \frac{15 \sin B}{\sin 150°} = 5.4819 \)

Since this has been a somewhat complicated process, it’s a good idea to check the final answer. Make sure that \( A + B + C = 180° \) (we’re OK there), and that the law of sines holds true for our results (since we rounded off, our ratios aren’t exactly equal, but they’re very close).

23. A = 150°; a = 8; c = 10.

A is obtuse (the crane is leaning backward). The cable (a) is shorter than the crane (c), so there’s no way it can touch the ground in front of the crane. There is no solution.

If we’d blindly plugged numbers into the law of sines, we’d have got what looked like a solution. This is because \( \sin 150° = \sin 30°. \) The law of sines can’t tell the difference between an acute and an obtuse angle. We’d have realized that we were in trouble when the angles A and B added up to more than 180°.
A is a right angle. The hypotenuse (a) is longer than the adjacent side (c), so there is a solution. We can use Pythagoras to get: b = 11.1803. We store this value in the calculator for further use.

\[
\cos B = \frac{c}{a} = \frac{10}{15}, \quad \text{so } B = 48.19°.
\]

\[
\cos C = \frac{b}{a} = \frac{11.1803}{15}; \quad \text{and } C \text{ is acute, so } C = 41.81°.
\]

Our angles add up to 180°, and our sides and angles obey the law of sines (up to a little round-off error).

25. A = 30°; a = 4; c = 10.
We try to use the law of sines to find C.

\[
\frac{4}{\sin 30°} = \frac{10}{\sin C}; \quad \text{so } \sin C = \frac{10 \sin 30°}{4} = \frac{5}{2}.
\]

But sines have to be in the range [-1,1]; so no such C exists. There is no solution.

In our crane analogy, the crane is leaning forward at 30°. The crane’s length is 10; the cable’s length is 4. Since 4 < 10 sin 30°, the cable is too short to reach the ground.

26. A = 30°; a = 5; c = 10.
Using the law of sines gives

\[
\frac{5}{\sin 30°} = \frac{10}{\sin C}; \quad \text{so } \sin C = \frac{10 \sin 30°}{5} = \frac{5}{2} = 1.
\]

Thus C = 90°, and there’s only one possible solution. B = 60°; and b = 10 sin 60° = 8.6603

In the crane analogy, since a = c sin 30°, the cable is just long enough to touch the ground when hanging straight down. There’s no other point at which it can touch the ground.

27. A = 30°; a = 8; c = 10.
Here A is acute: the crane leans forward. a > c sin A; so the cable is long enough to touch the ground in two places. a < c; so the cable isn’t as long as the crane, which means that both places the cable touches the ground will be in front of the crane. Thus there are two solutions: one in which C is acute, the other in which C is obtuse.

The law of sines gives

\[
\frac{8}{\sin 30°} = \frac{10}{\sin C}; \quad \text{so } \sin C = \frac{10 \sin 30°}{8} = \frac{5}{8}.
\]

The two solutions are

\[
C_1 = 38.68° \text{ and } C_2 = 141.32°.
\]

(Store these numbers in your calculator: don’t use the rounded versions in further calculations.) The corresponding values for B are

\[
B_1 = 111.32° \text{ and } B_2 = 8.68°.
\]

We use the law of sines again to get the two values for b:

\[
\frac{8}{\sin 30°} = \frac{b}{\sin B}.
\]

Substituting the values of B gives us

\[
b_1 = 14.9053 \text{ and } b_2 = 2.4153.
\]

Since we’ve gone through a number of steps, we check our answers: our angles add up to 180°, as they should; and our angles and sides obey the law of sines.
28. A = 30°; a = 12; c = 10.
Here A is acute: the crane leans forward. a > c: the cable is longer than the crane. Thus the
cable can touch the ground at two points, but one of those points is behind the crane, so it’s not a
valid solution (it’s associated with a triangle with an angle of 150° instead of 30°). Thus C is
going to be acute.

The law of sines give us $\frac{12}{\sin 30°} = \frac{10}{\sin C}$; so $\sin C = \frac{10 \sin 30°}{12} = \frac{5}{12}$. Since C is acute, we get
C = 24.62°. Subtracting A + C from 180° give us B = 125.38°. Finally, the law of sines yields
$\frac{12}{\sin 30°} = \frac{b}{\sin B}$; so $b = \frac{12 \sin B}{\sin 30°} = 19.5690$. Checking, we find that the angles add up to 180°,
and that the triangle we’ve found does obey the law of sines.

If we’d tried to find an obtuse solution for C, we’d have found that A + C > 180°.

29. A = 30°; a = 9; c = 18.
Here a = c sin A; so we have a right triangle, with C = 90°. Then B = 60°, and
b = 18 sin 60° = 15.5885

30. A = 90°; a = 7; c = 4
We don’t have to use the law of sines with a right triangle like this. sin C = 4/7, and C must be
acute; so C = 34.85°. B = 180° − (A + C) = 55.15°. b = 7 sin B = 5.7446.

31. A = 126°; a = 12; c = 15.
A is obtuse. a < c; so there’s no solution.

32. A = 52°; a = 11; c = 12.
A is acute; c sin A < a < c. Thus there are two solutions: one with C₁ acute, the other with C₂
obtuse. By the law of sines, sin C = $\frac{c \sin A}{a} = \frac{12 \sin 52°}{11}$.
C₁ = 59.28°; C₂ = 180° − C₁ = 120.72°.
B₁ = 180° − (A + C₁) = 68.72°; B₂ = 180° − (A + C₂) = 7.28°
\[b₁ = \frac{11 \sin B₁}{\sin 52°} = 13.0077; \quad b₂ = \frac{11 \sin B₂}{\sin 52°} = 1.7682\]

33. A = 138°; a = 14; c = 20.
A is obtuse; a < c, so there’s no solution.

34. A = 22°; a = 25; c = 16.
A is acute; a > c, so there’s one solution, in which C is acute.
\[\sin C = \frac{16 \sin 22°}{25}; \quad \text{so } C = 13.87°.\]
B = 180° − (A + C) = 144.13°.
\[b = \frac{25 \sin B}{\sin 22°} = 39.1058\]
35. A = 90°; a = 7; c = 9.
This would be a right triangle with hypotenuse a shorter than side c. No solution.

36. A = 104°; a = 9; c = 4.
A is obtuse; c < a; so there’s one solution, with C acute.

\[
\sin C = \frac{4 \sin 104°}{9}; \text{ so } C = 25.55°.
\]

B = 180° − (A + C) = 50.45°.

\[
b = \frac{9 \sin B}{\sin 104°} = 7.1524
\]

37. A = 90°; a = 5; c = 3.
This is a right triangle, so we don’t need to use law of sines. Since c < a, a solution exists.

\[
\sin C = \frac{3}{5}; \text{ so } C = 36.87°.
\]

B = 90° − C = 53.13°.

\[
b = \sqrt{5^2 - 3^2} = 4
\]

38. A = 155°; a = 10; c = 13.
A is obtuse; a < c. No solution.

39. A = 30°; a = 8; c = 16.
A is acute; a = c \sin A; so this is a right triangle, with C = 90°.
Then B = 60°, and b = c \sin 60° = 13.8564

40. A = 90°; a = 6; c = 8.
This is a right triangle with a < c; so there’s no solution.

41. A = 30°; a = 6; c = 9.
A is acute; c \sin A < a < c, so there are two solutions: one with C acute, one with C obtuse.

\[
\sin C = \frac{9 \sin 30°}{6}; \text{ so } C_1 = 48.59° \text{ and } C_2 = 131.41°.
\]

B_1 = 101.41° and B_2 = 18.59°.

b_1 = 11.7629 and b_2 = 3.8256

42. A = 49°; a = 20; c = 16.
A is acute; a > c; so there is one solution, with C acute.

\[
\sin C = \frac{16 \sin 49°}{20}; \text{ so } C = 37.14°
\]

B = 93.86°

b = 26.4402
43. $A = 71^\circ; a = 13; c = 18$.
A is acute; $a < c \sin A$; so there’s no solution.

44. $A = \pi/6 \text{ rad}; a = 7; c = 14$.
A is acute. $a = c \sin A$; so $C = \pi/2 = 1.5708 \text{ rad}$. Then $B = \pi/3 = 1.0472 \text{ rad}$; $b = c \sin B = 12.1244$.

45. $A = 2.6 \text{ rad}; a = 7; c = 9$.
A is obtuse; $a < c$. There is no solution.

46. $A = \pi/5 \text{ rad}; a = 10; c = 7$.
A is acute; $a > c$. There is one solution, with $C$ acute. The law of sines give us
\[
\frac{7 \sin(\pi/5)}{10} = \sin C; \text{ so } C = 0.4240 \text{ rad}.
\]
$B = \pi - (\pi/5 + C) = 2.0892 \text{ rad}$.
\[
b = \frac{10 \sin B}{\sin(\pi/5)} = 14.7774.
\]

47. $A = \pi/2 \text{ rad}; a = 4; c = 3$.
ABC is a right angle, so we don’t have to use the law of sines.
$C = \arcsin(3/4) = 0.8481 \text{ rad}$.
$B = \pi/2 - C = .7227 \text{ rad}$.
\[
b = \sqrt{4^2 - 3^2} = \sqrt{7} = 2.6458
\]

48. $A = 5\pi/6 \text{ rad}; a = 9; c = 6$.
A is obtuse; $a > c$, so there is one solution, with $C$ acute.
\[
\sin C = \frac{6 \sin(5\pi/6)}{9}; \text{ so } C = 0.3398 \text{ rad}.
\]
$B = \pi - (5\pi/6 + C) = 0.1838 \text{ rad}$.
\[
b = \frac{9 \sin B}{\sin(5\pi/6)} = 3.2891
\]

49. $A = \pi/2 \text{ rad}; a = 6; c = 11$.
ABC is a right triangle with $a < c$: that is, the hypotenuse shorter than one side. This is impossible: no solution.

50. $A = 0.42 \text{ rad}; a = 9; c = 12$
A is acute; $c \sin A < a < c$, so there are two solutions: one with $C$ acute, one with $C$ obtuse.
\[
\sin C = \frac{12 \sin(0.42)}{9}; \text{ so } C1 = 0.5748 \text{ rad and } C2 = 2.5668 \text{ rad}.
\]
$B = \pi - (0.42 + C)$; so $B1 = 2.1468 \text{ rad and } B2 = 0.1548 \text{ rad}$.
\[
b_1 = \frac{9 \sin B_1}{\sin(0.42)} = 18.5107 \text{ and } b_2 = \frac{9 \sin B_2}{\sin(0.42)} = 3.4034
51. $A = 1.2 \text{ rad}; a = 13; c = 15$
A is acute; $a < c \sin A$, so there’s no solution.

In problems 52–68, calculate the area of the triangle. Round your answer to four decimal places. If the triangle can’t exist, say so.

To calculate the area of a triangle, we need to know two sides and the angle between them. For example, if we know sides $b$ and $c$ and angle $A$, we can calculate the area as: $\frac{1}{2}bc \sin A$.

52. $A = 41^\circ, C = 70^\circ, b = 10$.
This is an ASA problem. We begin by calculating $B = 69^\circ$. Then $a = \frac{10 \sin 41^\circ}{\sin 69^\circ} = 7.0273$.
(Store this number in your calculator’s memory; do not write it down and re-enter it.)
Area = $\left(\frac{1}{2}\right)ab \sin C = \left(\frac{1}{2}\right)10a \sin 70^\circ = 33.0177$

53. $A = 113^\circ, B = 55^\circ, c = 6$.
Another ASA problem. $C = 12^\circ$. $a = \frac{6 \sin 113^\circ}{\sin 12^\circ} = 26.5643$.
Area = $\left(\frac{1}{2}\right)6a \sin 55^\circ = 65.2806$

54. $B = 22^\circ, C = 13^\circ, a = 100$.
$A = 145^\circ$. $b = \frac{100 \sin 22^\circ}{\sin 145^\circ} = 65.3107$. Area = $\left(\frac{1}{2}\right)100b \sin 13^\circ = 734.5852$

55. $A = 94^\circ; B = 22^\circ; c = 12$.
$C = 64^\circ$. $a = \frac{12 \sin 94^\circ}{\sin 64^\circ} = 13.3187$. Area = $\left(\frac{1}{2}\right)12a \sin 22^\circ = 29.9356$

56. $A = 27^\circ; C = 51^\circ; b = 8$.
$B = 102^\circ$. $a = \frac{8 \sin 27^\circ}{\sin 102^\circ} = 3.7131$. Area = $\left(\frac{1}{2}\right)8a \sin 51^\circ = 11.5424$

57. $B = 76^\circ; C = 82^\circ; a = 5$.
$A = 22^\circ$. $b = \frac{5 \sin 76^\circ}{\sin 22^\circ} = 12.9509$. Area = $\left(\frac{1}{2}\right)5b \sin 82^\circ = 32.0621$

58. $A = 0.81 \text{ rad}; B = 1.02 \text{ rad}; c = 9$.
$C = 1.3116 \text{ rad}$. (Store this value in your calculator; don’t write it down and re-enter it.)
$a = \frac{9 \sin(0.81)}{\sin C} = 6.7439$. Area = $\left(\frac{1}{2}\right)9a \sin(1.02) = 25.8593$

59. $A = \pi/7 \text{ rad}; C = 0.44 \text{ rad}; b = 20$.
$B = 2.2528 \text{ rad}$. $a = \frac{20 \sin(\pi/7)}{\sin B} = 11.1780$. Area = $\left(\frac{1}{2}\right)20a \sin(0.44) = 47.6116$
60. \( A = 103^\circ; a = 10; c = 4. \)
This is an SSA problem. \( A \) is obtuse, and \( a > c \); so there is one solution, with \( C \) acute.
\[
\sin C = \frac{4 \sin 103^\circ}{10}; \text{ so } C = 22.94^\circ. \quad B = 54.06^\circ. \quad \text{Area} = \frac{1}{2}(10)(4) \sin B = 16.1929
\]

61. \( B = 53^\circ; b = 18; c = 20; C \) is acute.
\( B \) is acute; \( c \sin B < b < c \), so there are two solutions. We want the one with \( C \) acute.
\[
\sin C = \frac{20 \sin 53^\circ}{18}; \text{ so } C = 62.54^\circ. \quad A = 64.46^\circ. \quad \text{Area} = \frac{1}{2}(18)(20) \sin A = 162.4045.
\]

62. \( A = 27^\circ; a = 22; b = 36; B \) is obtuse.
\( A \) is acute; \( b \sin A < a < b \), so there are two solutions. We want the one with \( B \) obtuse.
\[
\sin B = \frac{36 \sin 27^\circ}{22}; \text{ so } B = 132.02^\circ. \quad C = 20.98^\circ. \quad \text{Area} = \frac{1}{2}(22)(36) \sin C = 141.7749
\]

63. \( C = 90^\circ; c = 12; a = 5. \)
\( C \) is a right angle, and \( c > a \); so there is a solution. We don’t have to use the law of sines here.
The theorem of Pythagoras gets us \( b = 10.9087 \). \( \text{Area} = \frac{1}{2}ab = 27.2718 \)

64. \( B = 30^\circ; b = 7; a = 14. \)
\( B \) is acute; and \( b = a \sin B \), so the solution is a right triangle with \( C = 60^\circ \).
From Pythagoras, \( c = 12.1244 \). \( \text{Area} = \frac{1}{2}bc = 42.4352 \)

65. \( C = 13^\circ; c = 12; b = 60. \)
\( C \) is acute; \( c < b \sin C \), so there’s no solution.

66. \( A = 90^\circ; a = 9, c = 15. \)
\( A \) is a right angle; \( a < c \), so there’s no solution.

67. \( A = 115^\circ; a = 3; b = 4. \)
\( A \) is obtuse; \( a < b \), so there’s no solution.

68. \( B = 30^\circ; b = 11; c = 7. \)
\( B \) is acute; \( b > c \), so there’s one solution, with \( C \) acute.
\[
\sin C = \frac{7 \sin 30^\circ}{11}; \text{ so } C = 18.55^\circ. \quad A = 131.45^\circ. \quad \text{Area} = \frac{1}{2}(11)(7) \sin A = 28.8584
\]
69. A radio antenna was 240 ft tall and vertical when it was first put up. However, the foundation is weak, and the antenna is now tilted 4.2° from the vertical. To keep it from tilting further, you want to install a diagonal guy wire: the wire is to be attached halfway up the antenna, and should meet the ground at a 50° angle. How long does the wire have to be? Round your answer to the nearest 0.01 ft.

We’ve labelled the corners of the triangle formed by the antenna, the ground, and the wire. The antenna tilts 4.2° from the vertical, so \( A = 94.2° \). \( C = 50° \). We know \( c = 120 \) (since the wire attaches halfway up a 240-ft antenna), and we want \( a \): the length of the wire.

We can use the law of sines without having to solve the whole triangle.

\[
a = \frac{120 \sin 94.2°}{\sin 50°} = 156.23 \text{ ft}.
\]

70. A highway runs directly north-south. A road going in a northeasterly direction crosses the highway at an angle of 78°; 2.3 miles farther north, a railroad going southeasterly crosses the highway at an angle of 63°. What is the distance along the road from the highway to the point where the road and the railroad meet? Round your answer to the nearest 0.1 mile.

This is an ASA problem. We’ve labelled the corners of the triangle for convenience. We know \( A = 63° \), \( B = 78° \), and \( c = 2.3 \) miles. We want the value of \( a \).

\[
C = 39°. \quad a = \frac{2.3 \sin 63°}{\sin 39°} = 3.3 \text{ miles}.
\]

71. You and a friend are standing 1.75 miles apart along Valencia Road, which runs straight east-west. When an airplane flies over Valencia you, facing east, measure its elevation as 0.1132 rad; your friend, facing west, measures its elevation as 0.1504 rad. How high is the airplane? Round your answer to the nearest foot.

This is an ASA problem. We know \( A, c, \) and \( B \). We want to calculate the height \( h \); to do this, we can calculate either \( a \) or \( b \), then use \( h = a \sin B = b \sin A \). We’ll work with \( a \).

\[
C = π - (A + B) = 2.878 \text{ rad}. \quad \text{(Store this number in your calculator; don’t write it down and re-enter it later.) Then } a = \frac{(1.75)(5280)\sin(0.1132)}{\sin C} = 4005.77, \text{ and } h = a \sin B = 600 \text{ ft}.
\]
In problems 72-79, find the angles of the triangle ABC. Give your answers in the indicated units. Round angles to 0.01° or to 0.0001 rad. If the triangle is impossible, say so.

72. $a = 4$, $b = 5$, $c = 6$. Give angles in radians.
This is an SSS problem. We begin by using the law of cosines to find $C$. We choose $C$ because $c$ is the longest side; so if there’s an obtuse angle in the triangle, it’ll have to be $C$. The law of cosines allows us to distinguish between an acute and an obtuse angle. We get

$$\cos C = \frac{4^2 + 5^2 - 6^2}{2(4)(5)} = \frac{1}{8}.$$ Then $C = 1.4455$ rad.

We could use the law of cosines twice more to get $A$ and $B$. However, the law of sines is more convenient; and we know that $A$ and $B$ must be acute.

$$\sin A = \frac{4 \sin C}{6};$$ so $A = 0.7227$ rad. \hspace{0.5cm} \sin B = \frac{5 \sin C}{6};$$ so $B = 0.9734$ rad.

It would have been quicker still to calculate $B = \pi - (A + C)$. We used the law of sines because that allowed us to use the sum of the angles as a quick check on our work: making sure that $A + B + C = \pi$. If not, we know there’s a mistake, and we can find and correct it.

73. $a = 12$, $b = 5$, $c = 9$. Give angles in radians.
a is the longest side, so we’ll use the law of cosines to find $A$.

$$\cos A = \frac{5^2 + 9^2 - 12^2}{2(5)(9)};$$ so $A = 2.0067$ rad.

$$\sin B = \frac{5 \sin A}{12};$$ so $B = 0.3873$ rad. \hspace{0.5cm} \sin C = \frac{9 \sin A}{12};$$ so $C = 0.7476$ rad.

74. $a = 20$, $b = 6$, $c = 8$. Give angles in degrees.
This is an impossible situation: one side of the triangle is longer than the other two put together.

75. $a = 13$, $b = 9$, $c = 11$. Give angles in degrees.

$$\cos A = \frac{9^2 + 11^2 - 13^2}{2(9)(11)};$$ so $A = 80.41^\circ$.

$$\sin B = \frac{9 \sin A}{13};$$ so $B = 43.05^\circ$ \hspace{0.5cm} \sin C = \frac{11 \sin A}{13};$$ so $C = 56.54^\circ$.

76. $a = 10$, $b = 14$, $c = 7$. Give angles in radians.

$$\cos B = \frac{10^2 + 7^2 - 14^2}{2(10)(7)};$$ so $B = 1.9132$ rad.

$$\sin A = \frac{10 \sin B}{14};$$ so $A = 0.7380$ rad. \hspace{0.5cm} \sin C = \frac{7 \sin B}{14};$$ so $C = 0.4904$ rad.
77. \(a = 2, b = 6, c = 5\). Give angles in degrees.

\[
\cos B = \frac{2^2 + 5^2 - 6^2}{2(2)(5)} \text{; so } B = 110.49^\circ.
\]

\[
\sin A = \frac{2 \sin B}{6} \text{; so } A = 18.19^\circ. \quad \sin C = \frac{5 \sin B}{6} \text{; so } C = 51.32^\circ.
\]

78. \(a = 10, b = 14, c = 30\). Give angles in radians.

This is an impossible situation, since \(c\) is longer than \(a\) and \(b\) put together.

79. \(a = 19, b = 13, c = 9\). Give angles in degrees.

\[
\cos A = \frac{13^2 + 9^2 - 19^2}{2(13)(9)} \text{; so } A = 118.32^\circ.
\]

\[
\sin B = \frac{13 \sin A}{19} \text{; so } B = 37.04^\circ. \quad \sin C = \frac{9 \sin A}{19} \text{; so } C = 24.65^\circ.
\]

Notice that our rounded angles add up to \(A + B + C = 180.01^\circ\).

**In problems 80-85, find the missing sides and angles of the triangle \(ABC\). Round sides to 0.0001 and angles to either 0.01° or 0.0001 rad. Use the angle measure that’s used in the statement of the problem.**

80. \(A = 72^\circ; b = 12; c = 10\).

These are SAS problems. We begin by using the law of cosines to calculate \(a\).

\[
a^2 = 12^2 + 10^2 - 2(12)(10) \cos 72^\circ \text{; so } a = 13.0321.
\]

Since \(a\) is the longest side, \(A\) is the largest angle in the triangle; so we can use the law of sines to find \(B\) and \(C\) without having to worry about whether they’re obtuse.

\[
\sin B = \frac{12 \sin 72^\circ}{a} \text{; so } B = 61.13^\circ. \quad \sin C = \frac{10 \sin 72^\circ}{a} \text{; so } C = 46.87^\circ.
\]

81. \(a = 30; B = 1.8 \text{ rad}; c = 19\).

We start with the law of cosines to calculate \(b\).

\[
b^2 = 30^2 + 19^2 - 2(30)(19) \cos (1.8) \text{; so } b = 38.9873.
\]

This is the longest side, so we can use the law of sines to calculate \(A\) and \(C\).

\[
\sin A = \frac{30 \sin(1.8)}{b} \text{; so } A = 0.8471 \text{ rad.} \quad \sin C = \frac{19 \sin(1.8)}{b} \text{; so } C = 0.4945 \text{ rad.}
\]

82. \(a = 8; b = 18; C = 27^\circ\).

\[
c^2 = 8^2 + 18^2 - 2(8)(18) \cos 27^\circ \text{; so } c = 11.4626.
\]

The longest side is \(b\); so we can’t be sure that \(B\) is acute. However, we know that \(A\) is. We’ll use the law of sines: \(\sin A = \frac{8 \sin 27^\circ}{c} \text{; so } A = 18.47^\circ\). Since \(A + C < 90^\circ\), \(B\) must be obtuse.

\[
\sin B = \frac{18 \sin 27^\circ}{c} \text{; so } B = 134.53. \text{ Our quick check gives } A + B + C = 180, \text{ as it should.}
\]
83. A = 2.2 rad; b = 25; c = 9.

\[ a^2 = 25^2 + 9^2 - 2(25)(9)\cos(2.2) \]; so \( a = 31.1581 \). A is obtuse, so B and C are acute.

\[ \sin B = \frac{25\sin(2.2)}{a} \]; so \( B = 0.7059 \). \( \sin C = \frac{9\sin(2.2)}{a} \); so \( C = 0.2357 \).

84. a = 5; B = 14°; c = 7.

\[ b^2 = 5^2 + 7^2 - 2(5)(7)\cos(14°) \]; so \( b = 2.4656 \). C is the largest angle, so A is acute.

\[ \sin A = \frac{5\sin(14°)}{b} \]; so \( A = 29.38° \).

A + B < 90°; so C is obtuse. \( \sin C = \frac{7\sin(14°)}{b} \); so \( C = 136.62° \).

85. a = 15; b = 11; C = 0.6 rad.

\[ c^2 = 15^2 + 11^2 - 2(15)(11)\cos(0.6) \]; so \( c = 8.5813 \).

a is the longest side; so A is the largest angle; so B must be acute.

\[ \sin B = \frac{11\sin(0.6)}{c} \]; so \( B = 0.8093 \) rad. Since B + C < \( \pi/2 \), A is obtuse.

\[ \sin A = \frac{15\sin(0.6)}{c} \]; so \( A = 1.7323 \) rad.

**In problems 86-95, calculate the area of the triangle ABC. Round your answers to four decimal places. If the triangle is impossible, say so.**

86. a = 15, b = 27, C = 1.9 rad

This is an SAS situation. To find the area, we only need two sides and the included angle; so we don’t have to solve the triangle at all. Area = \( \frac{1}{2}(15)(27)\sin(1.9) = 191.6258 \)

87. a = 8, B = 49°, c = 11.

Area = \( \frac{1}{2}(8)(11)\sin(49°) = 33.2072 \)

88. a = 13, b = 7, C = 102°.

Area = \( \frac{1}{2}(13)(7)\sin(102°) = 44.5057 \)

89. A = 17°, b = 10, c = 19

Area = \( \frac{1}{2}(10)(19)\sin(17°) = 27.7753 \)

90. a = 12, b = 7, c = 9.

This is an SSS situation. To calculate the area, we need to know one of the angles. We’ll use the law of cosines to calculate: \( \cos A = \frac{7^2 + 9^2 - 12^2}{2(7)(9)} \); so \( A = 1.6821 \) rad. (Store this number in the calculator; don’t write it down and re-enter it.) Area = \( \frac{1}{2}(7)(9)(\sin A) = 31.3050 \)
91.  $a = 12$, $b = 22$, $c = 7$.
This is an impossible situation: $a + c < b$.

92.  $a = 30$, $b = 19$, $c = 23$.
We use the law of cosines to calculate $A$.
\[
\cos A = \frac{19^2 + 23^2 - 30^2}{2(19)(23)}; \text{ so } A = 1.5822 \text{ rad.}
\]
Area = \(\frac{1}{2})(19)(23)(\sin A) = 218.4857

93.  $a = 28$, $B = 1.35 \text{ rad}$, $c = 15$.
This is an SAS situation. We don’t need to solve the triangle:
Area = $(28)(15)(\sin 1.35) = 204.9019$

94.  $a = 12$, $b = 14$, $c = 32$.
$a + b < c$; so the triangle can’t exist.

95.  $A = 0.25 \text{ rad}$; $b = 24$; $c = 12$.
This is an SAS situation; so we don’t need to solve the triangle.
Area = $(24)(12)(\sin 0.25) = 35.6262
96. A drawbridge consists of two unequal sides, each hinged to move up and down. One side is 100 m long; the other is 70 m long. The channel they span when closed is 160 m wide. What angles above the horizontal do the two sides make when they are closed? Give your answer to the nearest 0.0001 rad.

We’ve labelled the corners of the triangle for convenience. We’ll assume that \( a = 70 \) and \( b = 100 \). We have the three sides; we want the angles \( A \) and \( B \). Since \( c \) is the longest side, \( C \) is the largest angle; so \( A \) and \( B \) must be acute.

\[
\cos A = \frac{100^2 + 160^2 - 70^2}{2(100)(160)}; \quad \text{so } A = 0.2860 \text{ rad.}
\]

\[
\sin B = \frac{100 \sin A}{70}; \quad \text{so } B = 0.4148 \text{ rad.}
\]

97. Two straight roads meet at an angle of 74°. A car and a motorcycle leave the intersection at the same time: the car going at 45 miles per hour, and the motorcycle going at 65 mph. After one hour, how far apart are the two vehicles? Round your answer to the nearest 0.1 mile.

This is an SAS situation. We’ll let \( a = 45 \), \( b = 65 \), and \( C = 74° \). We want \( c \):

\[
c^2 = 45^2 + 65^2 - 2(45)(65) \cos 74°; \quad \text{so } c = 68.1 \text{ mile.}
\]

98. A balloon is tethered by two cables: one 800 m long, the other 850 m long. The cables meet the ground 400 m from one another. If the balloon pulls both cables straight, how high above the ground is it? Round your answer to the nearest meter.

This is an SAS situation. We’ve labelled the corners of the triangle. We’ll assume that \( a = 800 \) and \( b = 850 \). We want the elevation \( h \). To get that, we’ll calculate either \( A \) or \( B \), then use \( h = a \sin B = b \sin A \). We’ll use \( A \):

\[
\cos A = \frac{850^2 + 400^2 - 800^2}{2(850)(400)}; \quad \text{so } A = 1.2062 \text{ rad; and } h = 850 \sin A = 794 \text{ m.}
\]

In problems 99-131, find the missing angles and sides of triangle \( ABC \). If the triangle cannot exist, say so. If there are two possibilities for the triangle, find the sides and angles for both. Round sides to the nearest 0.0001; round angles to the nearest 0.01°, or 0.0001 rad. Use the angle measure that’s used in the statement of the problem.

99. \( a = 5.3 \), \( b = 7.2 \), \( c = 9.4 \). Give angles in radians.

SSS. \( c \) is the longest side; so \( C \) is the only angle that can be obtuse. Use the law of cosines:

\[
\cos C = \frac{5.3^2 + 7.2^2 - 9.4^2}{2(5.3)(7.2)}; \quad \text{so } C = 1.6815 \text{ rad.}
\]

\[
\sin A = \frac{5.3 \sin C}{9.4}; \quad \text{so } A = 0.5948 \text{ rad.} \quad \sin B = \frac{7.2 \sin C}{9.4}; \quad \text{so } B = 0.8653 \text{ rad.}
\]
100. $A = 41^\circ; a = 6.1; c = 6.5$
This is an SSA problem. $A$ is acute; $c \sin A < a < c$, so there are two solutions: $C_1$ acute, and $C_2$ obtuse. From the law of sines, $\frac{c \sin A}{6.5} = \frac{6.1}{41^\circ}$; so $C_1 = 44.35^\circ$ and $C_2 = 135.65^\circ$.
Then $B_1 = 94.65^\circ$ and $B_2 = 3.35^\circ$.

\[ b_1 = \frac{6.1 \sin B_1}{\sin 41^\circ} = 9.2674 \quad b_2 = \frac{6.1 \sin B_2}{\sin 41^\circ} = 0.5438. \]

101. $A = 0.79$ rad; $b = 11; C = 1.14$ rad

\[ a = \frac{11 \sin(0.79)}{\sin B} = 8.3466 \quad c = \frac{11 \sin(1.14)}{\sin B} = 10.6764. \]

102. $A = 77^\circ; a = 11.8; c = 13.3$
SSA problem. $A$ is acute; $a < c \sin A$; so there is no solution.

103. $a = 6.2; B = 2.23$ rad; $b = 4.8$
SSA problem. $A$ is obtuse; $b < a$, so there is no solution.

104. $a = 3; C = 90^\circ; c = 5.5$
SSA problem. Right triangle with $c > a$, so a solution exists. Pythagoras gives us:

\[ b = \sqrt{5.5^2 - 3^2} = 4.6098. \quad \sin A = 3/5.5; \quad \sin B = b/5.5; \quad \sin B = 56.94^\circ \]

105. $B = \pi/2$ rad; $b = 10; c = 4.4$
SSA problem. Right triangle with $b > c$; so a solution exists.

\[ a = \sqrt{10^2 - 4.4^2} = 8.9800. \quad A = \arcsin(a/10) = 1.1152 \text{ rad.} \quad C = \arcsin(4.4/10) = 0.4556 \text{ rad.} \]

106. $b = 2.4; C = 38^\circ; c = 3.9$
SSA problem. $C$ is acute; $c > b$, so one solution with $B$ acute.

\[ \sin B = \frac{2.4 \sin 38^\circ}{3.9}; \quad \sin B = 22.26^\circ. \quad A = 180^\circ - (38^\circ + B) = 119.74^\circ. \]

\[ a = \frac{3.9 \sin A}{\sin 38^\circ} = 5.5005. \]

107. $A = 1.02$ rad; $a = 7.5; b = 8.3$
SSA problem. $A$ is acute; $b \sin A < a < b$, so there are two solutions: $B_1$ acute, $B_2$ obtuse.

\[ \sin B = \frac{8.3 \sin(1.02)}{7.5}; \quad \sin B_1 = 1.2315 \text{ rad and } B_2 = 1.9101 \text{ rad.} \]

$C_1 = 0.8901 \text{ rad and } C_2 = 0.2115 \text{ rad.}$

\[ c_1 = \frac{7.5 \sin C_1}{\sin(1.02)} = 6.8399 \quad \text{and} \quad c_2 = \frac{7.5 \sin C_2}{\sin(1.02)} = 1.8480. \]
108.  \( a = 11.6; B = 48^\circ; C = 103^\circ. \)

ASA.  \( A = 29^\circ. \)  \( b = \frac{11.6 \sin 48^\circ}{\sin 29^\circ} = 17.7812. \)  \( c = \frac{11.6 \sin 103^\circ}{\sin 29^\circ} = 23.3137. \)

109.  \( a = 43; b = 9.6; C = \pi/5 \) rad

SAS.  \( c^2 = 43^2 + 9.6^2 - 2(43)(9.6)\cos(\pi/5); \) so \( c = 35.6824 \)

\( b \) is the shortest side, so \( B \) is the smallest angle, and must be acute.

\( \sin B = \frac{9.6 \sin(\pi/5)}{c}; \) so \( B = 0.1588 \) rad.

Since \( B + C < \pi/2, \) \( A \) must be obtuse.

\( \sin A = \frac{43 \sin(\pi/5)}{c}; \) so \( A = 2.3545 \) rad.

110.  \( B = 158^\circ; b = 9.7; c = 13.3 \)

SSA.  \( B \) is obtuse, and \( b < c; \) so there’s no solution.

111.  \( a = 31; B = 13^\circ; c = 27 \)

SAS.  \( b^2 = 31^2 + 27^2 - 2(31)(27)\cos(13^\circ); \) so \( b = 7.6749. \)

\( c \) is not the longest side, so \( C \) must be acute.

\( \sin C = \frac{27 \sin 13^\circ}{b}; \) so \( C = 52.31^\circ. \)

\( B + C < 90; \) so \( A \) is obtuse.  \( \sin A = \frac{31 \sin 13^\circ}{b}; \) so \( A = 114.69^\circ. \)

112.  \( a = 15.4; B = \pi/6; b = 7.7 \)

SSA.  \( B \) is acute; \( b = a \sin B, \) so \( A \) is a right angle: \( A = \pi/2 = 1.5708 \) rad.

\( C = \pi/3 = 1.0472 \) rad.

\( c = 15.4 \sin(\pi/3) = 13.3368. \)

113.  \( a = 3.3; B = 0.78 \) rad; \( C = 1.09 \) rad

ASA.  \( A = \pi - (0.78 + 1.09) = 1.2716 \)

\( b = \frac{3.3 \sin(0.78)}{\sin A} = 2.4287. \)  \( c = \frac{3.3 \sin(1.09)}{\sin A} = 3.0619 \)

114.  \( a = 6.2; C = 90^\circ; c = 4.6 \)

SSA.  \( C \) is a right angle; \( c < a, \) so no solution.

115.  \( B = 58^\circ; b = 23.1; c = 18.4 \)

SSA.  \( B \) is acute; \( b > c, \) so there’s one solution, with \( C \) acute.

\( \sin C = \frac{18.4 \sin 58^\circ}{23.1}; \) so \( C = 42.49^\circ. \)  \( A = 180^\circ - (B + C) = 79.51^\circ. \)

\( a = \frac{23.1 \sin A}{\sin 58^\circ} = 26.7835 \)
116. a = 11.4; b = 8.8; c = 6.7. Give angles in degrees.  
a is the longest side; so if any angle is obtuse, it’s A.  
\[ \cos A = \frac{8.8^2 + 6.7^2 - 11.4^2}{2(8.8)(6.7)} \]; so A = 93.71°.  
\[ \sin B = \frac{8.8 \sin A}{11.4} \]; so B = 50.38°. \sin C = \frac{6.7 \sin A}{11.4} \]; so C = 35.91°.

117. A = 137°; a = 4.8; b = 2.9  
SSA. A is obtuse; a > b, so there’s one solution, with B acute.  
\[ \sin B = \frac{2.9 \sin 137°}{4.8} \]; so B = 24.33°. \ C = 180° – (A + B) = 18.67°.  
\[ c = \frac{4.8 \sin C}{\sin 137°} = 2.2527 \]

118. a = 34.4; B = 1.31 rad; C = 1.03 rad  
ASA. A = \pi – (B + C) = 0.8016 rad.  
\[ b = \frac{34.4 \sin(1.31)}{\sin A} = 46.2608. \ c = \frac{34.4 \sin(1.03)}{\sin A} = 41.0474. \]

119. a = 16.3; B = 2.8 rad; c = 5.9  
SAS. \ b^2 = 16.3^2 + 5.9^2 - 2(16.3)(5.9)\cos(2.8) \]; so b = 21.9483.  
B is obtuse; so A and C must be acute.  
\[ \sin A = \frac{16.3 \sin(2.8)}{b} \]; so A = 0.2514 rad. \sin C = \frac{5.9 \sin(2.8)}{b} \]; so C = 0.0902 rad.

120. b = 11; C = 0.9 rad; c = 9.2  
SSA. C is acute; b \sin C < c < b, so there are two solutions: B1 acute, B2 obtuse.  
\[ \sin B = \frac{11 \sin(0.9)}{9.2} \]; so B1 = 1.2128 rad and B2 = 1.9288 rad.  
A = \pi – (B + C); so A1 = 1.0288 and A2 = 0.3128.  
\[ a = \frac{9.2 \sin A}{\sin(0.9)} \]; so a1 = 10.0617 and a2 = 3.6137.

121. a = 3.3; b = 8.1; c = 6.9. Give angles in degrees.  
SSS. b is the longest side; so if there’s an obtuse angle, it’s B.  
\[ \cos B = \frac{3.3^2 + 6.9^2 - 8.1^2}{2(3.3)(6.9)} \]; so B = 98.98°.\[ \sin A = \frac{3.3 \sin B}{8.1} \]; so A = 23.73°. \sin C = \frac{6.9 \sin B}{8.1} \]; so C = 57.29°.
122. $A = \pi/6 \text{ rad}; b = 5.8; a = 2.9$
SSA. $a = b \sin A; \text{ so } B \text{ is a right angle: } B = \pi/2 = 1.5708 \text{ rad. } C = \pi/3 = 1.0472 \text{ rad. }$
$c = \sqrt{5.8^2 - 2.9^2} = 5.0229.$

123. $a = 7.9; C = 5\pi/7 \text{ rad}; c = 4.5$
SSA. $C \text{ is obtuse; } c < a; \text{ so there is no solution.}$

124. $a = 13; B = 74^\circ; c = 8.6$
SAS. $b^2 = 13^2 + 8.6^2 - 2(13)(8.6)\cos 74^\circ; \text{ so } b = 13.4658. \ b \text{ is the longest side; so } B \text{ is the largest angle; so } A \text{ and } C \text{ are both acute.}$
$
\sin A = \frac{13\sin 74^\circ}{b}; \text{ so } A = 68.13^\circ. \ \sin C = \frac{8.6\sin 74^\circ}{b}; \text{ so } C = 37.87^\circ.$

125. $a = 23; B = 90^\circ; b = 18.5$
SSA. $B \text{ is a right angle; } b < a; \text{ so there's no solution.}$

126. $A = 2.3 \text{ rad}; b = 6.7; c = 3$
SAS. $a^2 = 6.7^2 + 3^2 - 2(6.7)(3)\cos 2.3^\circ; \text{ so } a = 8.9819. \ A \text{ is obtuse, so } B \text{ and } C \text{ are acute.}$
$
\sin B = \frac{6.7\sin 2.3^\circ}{a}; \text{ so } B = 0.5899 \text{ rad. } \sin C = \frac{3\sin 2.3^\circ}{a}; \text{ so } C = 0.2517 \text{ rad.}$

127. $a = 5.9; b = 3.4; c = 4.7. \text{ Give angles in radians.}$
SSS. $a \text{ is the longest side; so if there is an obtuse angle, it's } A.$
$
\cos A = \frac{3.4^2 + 4.7^2 - 5.9^2}{2(3.4)(4.7)}; \text{ so } A = 1.6071 \text{ rad.}$
$
\sin B = \frac{3.4\sin A}{5.9}; \text{ so } B = 0.6137 \text{ rad. } \sin C = \frac{4.7\sin A}{5.9}; \text{ so } C = 0.9208 \text{ rad.}$

128. $A = 54^\circ; B = 111^\circ; c = 19$
ASA. $C = 15^\circ. \ a = \frac{19\sin 54^\circ}{\sin 15^\circ} = 59.3902. \ b = \frac{19\sin 111^\circ}{\sin 15^\circ} = 68.5345.$

129. $B = 122^\circ; b = 23; c = 17$
SSA. $B \text{ is obtuse; } b > c, \text{ so there is one solution, with } C \text{ acute.}$
$
\sin C = \frac{17\sin 122^\circ}{23}; \text{ so } C = 38.82^\circ. \ A = 180^\circ - (B + C) = 19.18^\circ.$
$
a = \frac{23\sin A}{\sin 122^\circ} = 8.9122.$
130.  \(a = 15.5; \ C = 46^\circ; \ c = 12.9\)
SSA.  \(C\) is acute; \(a \sin C < c < a\), so there are two solutions: \(A_1\) acute, \(A_2\) obtuse.
\[
\sin A = \frac{15.5 \sin 46^\circ}{12.9}; \text{ so } A_1 = 59.81^\circ \text{ and } A_2 = 120.19^\circ.
\]
\(B = 180^\circ - (A + C); \text{ so } B_1 = 74.19^\circ \text{ and } B_2 = 13.81^\circ.\)
\[
b = \frac{12.9 \sin B}{\sin 46^\circ}; \text{ so } b_1 = 17.2551 \text{ and } b_2 = 4.2793.
\]

131.  \(a = 10.7; \ b = 14.4; \ c = 17.1. \)  Give angles in radians.
SSS.  \(c\) is the longest side; so if there’s an obtuse angle, it’ll be \(C.\)
\[
\cos C = \frac{10.7^2 + 14.4^2 - 17.1^2}{2(10.7)(14.4)}; \text{ so } C = 1.4751 \text{ rad.}
\]
\[
\sin A = \frac{10.7 \sin C}{17.1}; \text{ so } A = 0.6724 \text{ rad.} \quad \sin B = \frac{14.4 \sin C}{17.1}; \text{ so } B = 0.9941 \text{ rad.}
\]

In problems 132-143, calculate the area of the triangle \(ABC.\)  Round your answers to four decimal places.  If the triangle can’t exist, say so.

Area is the product of two sides and the sine of the included angle.
Area = \(ab \sin C = ac \sin B = bc \sin A.\)

132.  \(A = 0.78 \text{ rad}; \ a = 13; \ b = 16; \ B\) is acute.
SSA.  \(b \sin A < a < b; \) so there are two solutions.  We want the one with \(B\) acute.
\[
\sin B = \frac{16 \sin(0.78)}{13}; \text{ so } B = 1.0463 \text{ rad.} \quad C = \pi - (A + B) = 1.3153 \text{ rad.}
\]
We don’t need \(c\) to calculate the area: Area = \((13)(16)(\sin C) = 100.6238.\)

133.  \(a = 18; \ b = 12; \ c = 5\)
SSS.  This triangle can’t exist, since \(a > b + c\)

134.  \(b = 12; \ C = 2.5 \text{ rad}; \ c = 10\)
SSA.  \(C\) is obtuse, and \(c < b; \) so the triangle can’t exist.

135.  \(B = 39^\circ; \ b = 15; \ c = 20; \ C\) is obtuse.
SSA.  \(c \sin B < b < c; \) so there are two solutions.  We want the one with \(C\) obtuse.
\[
\sin C = \frac{20 \sin 39^\circ}{15}; \text{ so } C = 122.96^\circ. \quad A = 180^\circ - (B + C) = 18.04^\circ.
\]
Area = \((15)(20) \sin 18.04^\circ = 46.4635.\)

136.  \(a = 4.9, \ b = 10.4, \ c = 8.9\)
SSS.  \(b\) is the longest side; so if there’s an obtuse angle, it’s \(B.\)
\[
\cos B = \frac{4.9^2 + 8.9^2 - 10.4^2}{2(4.9)(8.9)}; \text{ so } B = 1.6275 \text{ rad.}
\]
Area = \((4.9)(8.9) \sin B = 21.7700\)
137. $A = 49^\circ; B = 102^\circ; c = 6$
ASA. $C = 180^\circ - (A + B) = 29^\circ$.
\[
a = \frac{6 \sin 49^\circ}{\sin 29^\circ} = 9.3403.
\]
Area = $6a \sin 102^\circ = 27.4085$.

138. $b = 12; C = 0.61 \text{ rad}; c = 6$
SSA. $c < b \sin C$; so the triangle can’t exist.

139. $a = 11; B = 1.12 \text{ rad}; C = 0.98 \text{ rad}$
ASA. $A = \pi - (1.12 + 0.98) = 1.0416 \text{ rad}$.
\[
b = \frac{11 \sin(1.12)}{\sin A} = 11.4701.
\]
Area = $11b \sin(0.98) = 52.3924$.

140. $A = 102^\circ; b = 25; C = 27^\circ$
ASA. $B = 180^\circ - (102 + 27) = 51^\circ$.
\[
a = \frac{25 \sin 102^\circ}{\sin 51^\circ} = 31.4660.
\]
Area = $25a \sin 27^\circ = 178.5659$

141. $A = 158^\circ; a = 18; c = 10$
SSA. $A$ is obtuse; $a > c$, so there’s one solution, with $C$ acute.
\[
\sin C = \frac{10 \sin 158^\circ}{18}; \text{ so } C = 12.01^\circ. \quad B = 180^\circ - (158 + C) = 9.99^\circ.
\]
Area = $(18)(10) \sin B = 15.6099$.

142. $B = 18^\circ; b = 10; c = 4$
SSA. $B$ is acute; $b > c$, so there’s one solution, with $C$ acute.
\[
\sin C = \frac{4 \sin 18^\circ}{10}; \text{ so } C = 7.1^\circ. \quad A = 180^\circ - (B + C) = 154.9^\circ.
\]
Area = $(10)(4) \sin A = 8.4841$

143. A road, a railroad track, and a canal form a triangle. The distance along the road between its intersections with the track and with the canal is 1440 m. The distance along the track between its intersection with the road and its crossing of the canal is 790 m. The distance along the canal between the crossings of the road and of the track is 1310 m. What is the area of the piece of land enclosed by the road, the track, and the canal? Round your answer to the nearest 0.1 hectare (1 ha = 10000 m²).

SSS. Let $r$ be the length of the road, and $R$ the opposite angle. $\cos R = \frac{790^2 + 1310^2 - 1440^2}{2(790)(1310)}$; so $R = 1.4416 \text{ rad}$. Area = $(790)(1310) \sin R = 513140 \text{ m}^2 = 51.3 \text{ ha}$. 
144. To keep a wall from falling over, you brace it with a 5-meter beam. The beam is fixed in the ground 3.77 m from the base of the wall; it contacts the wall 3.29 m high (measured along the wall, not directly up from the ground). What is the wall’s deviation from the vertical? Round your answer to the nearest 0.0001 radian.

You should sketch this situation. Let $a = 5$ be the length of the beam; $b = 3.77$ be the length along the ground from the beam to the base of the wall; and let $c = 3.29$ be the distance from the base of the wall to the upper end of the beam. Then $A$ is the angle that the wall makes with the ground. Careful! We asked for the angle the wall makes with the vertical, which will be $\pi/2 - A$.

$$\cos A = \frac{3.77^2 + 3.29^2 - 5^2}{2(3.77)(3.29)}; \text{ so } A = 1.5693 \text{ rad. } \pi/2 - A = 0.0015 \text{ rad.}$$

145. A highway runs straight east-west. A road intersects it at an angle of 41°, heading northeasterly. You park at the intersection and walk 3.92 miles along the road to its junction with another road. You make a sharp right turn and follow the second road 3.11 miles in a southwesterly direction back to the highway. How far along the highway are you from your car? Round your answer to the nearest 0.01 mile.

We’ve labelled the triangle for convenience. We need to find $b$.

This is an SSA situation; we want the solution with $C$ obtuse.

$$\sin C = \frac{3.92 \sin 41^\circ}{3.11}; \text{ so } C = 124.22^\circ. \quad B = 180 - (A + C) = 14.78^\circ.$$ \[b = \frac{3.11 \sin B}{\sin 41^\circ} = 1.21 \text{ miles.}\]

146. High-frequency direction-finding (HF/DF) is a technique for finding radio sources. It was used by the Allies during World War II to locate U-boats. Each HF/DF station used a highly directional antenna to determine the direction of a radio source; data from two or more stations could be combined to locate the source.

Station N is 23.3 miles directly north of Station S. The two detect a submarine’s radio signal. From Station N, the signal source is 18.1° south of west; from Station S, the sub is 22.5° north of west. How far is the sub from each of the two stations? Round your answers to the nearest 0.1 mile.

We’ll use N, S, and U (for “U-boat”) to label our triangle. Be careful with your angles! The values given are relative to an east-west line. In our triangle, $N = 90 - 18.1 = 71.9^\circ$ and $S = 90 - 22.5 = 67.5^\circ$. We want to find the sides $n$ and $s$.

$$U = 180 - (N + S) = 40.6^\circ. \quad n = \frac{23.3 \sin 71.9^\circ}{\sin 40.6^\circ} = 34.0 \text{ mile. } \quad s = \frac{23.3 \sin 67.5^\circ}{\sin 40.6^\circ} = 33.1 \text{ mile.}$$
147. You are driving along a highway that runs straight east and west. You see a water tower in a northeasterly direction, at an angle of 0.71 radian to the highway. You drive 3.8 km further east and measure the angle again; the water tower is still northeasterly, at an angle of 1.12 radian to the highway. How far north of the highway is the water tower? Round your answer to the nearest 0.1 km.

We’ve labelled the triangle for convenience. Again, you need to be careful with your angles: the second measurement, 1.12 rad, is on the outside of the triangle. The correct value for $B$ is $\pi - 1.12 = 2.0216$ rad.

$C = \pi - (0.71 + B) = 0.41$ rad. $a = \frac{3.8 \sin(0.71)}{\sin C} = 6.214$ km. Then $h = a \sin(1.12) = 5.6$ km.

148. A hill slopes down to the south. When the sun is directly in the south, it is at an elevation of 57.4° above the horizon. A 10-foot rod held vertically casts a shadow 6.15 ft long. What angle does the slope make with the horizontal? The angle is less than 30°. Round your answer to the nearest 0.1°.

Do we need to say it? Be careful with your angles! The elevation of the sun and the angle of the slope are measured relative to a horizontal line; but the rod that forms one side of angles $B$ and $C$ is vertical. Thus $B = 90 - 57.4 = 32.6°$; and the slope angle is $90 - C$. (Draw a bigger sketch and work it out if you’re having trouble figuring out why.)

This is an SSA situation. $B$ is acute and $\sin B < b < a$, so there are two solutions: one with $A$ acute, one with $A$ obtuse. $\sin A = \frac{10 \sin 32.6°}{6.15}$; so $A_1 = 61.2°$ and $A_2 = 118.8°$. Then $C = 180 - (A + C)$; so $C_1 = 86.2°$ and $C_2 = 28.6°$. Finally, the slope angle is $90° - C$; so it’s either 3.8° or 61.4°. Since the problem specified that it’s less than 30°, it must be 3.8°.

149. A crane is being used to drag a very large rock along the ground. The crane is fixed at an angle of 50° to the horizontal, and measures 85 ft from base to top. The cable extends 373 ft from the top of the crane to the rock. What angle does the cable make with the horizontal? Round your answer to the nearest 0.1°.

This is an SSA problem. $A$ is acute, and $a > b$, so there’s one solution, with $B$ acute. $\sin B = \frac{85 \sin 50°}{373}$; so $B = 10.1°$. 
150. A mountain rises in a long constant slope that makes an angle of 13.6° with the horizontal. At the base of the mountain is a level plain. From the top of the mountain, the angle of depression of a town on the plain is 8.2° below the horizontal. The mountain slope measures 5.71 miles from base to top. How far is it from the base of the mountain to the town? Round your answer to the nearest 0.1 mile.

We’ve labelled the triangle for convenience. The angles will take a little calculating; we can’t just read them from the problem. C = 180 – 13.6 = 166.4°. A = 13.6 – 8.2 = 5.4°. (Draw a larger version of the diagram and work these angles out for yourself if you’re not sure how we got them.) This is an ASA situation: we know A, b = 5.71, and C. Then

\[ B = 180 - (A + C) = 8.2°. \]
\[ a = \frac{5.71 \sin 5.4°}{\sin 8.2°} = 3.8 \text{ miles}. \]

151. Yet another radio antenna is in danger of falling over; its deviation from the vertical is 8.5°. To keep it from tilting any farther, you guy it with a 300-foot cable, attached 125 ft up from the base of the antenna. How far from the base of the antenna will the cable meet the ground, and at what angle to the horizontal? Round your answers to the nearest 0.1 ft and 0.1°.

We’ve sketched the figure; you should have done the same before trying to solve this problem. We know A = 98.5°, b = 125, and a = 300. We want to know c and B. This is an SSA situation; since A is obtuse and a > b, there is one solution, with B acute.

\[ \sin B = \frac{125 \sin 98.5°}{300}; \text{ so } B = 24.3°. \]
\[ c = \frac{300 \sin C}{\sin 98.5°} = 254.9 \text{ ft}. \]

152. Two roads intersect at an angle of 77°. Two cars leave the intersection at the same time: one going at 58 miles per hour, the other at 67 mph. Give the distance s in miles from one car to the other as a function of the time t in hours since departure from the intersection. Round all constants in your formula to three significant figures.

At time t, the 58-mph car is at C; the 67-mph car is at B. Then \( b = 58t \) and \( c = 67t \).

\[ a^2 = (58t)^2 + (67t)^2 - (58t)(67t) \cos 77°. \]

We can factor out \( t^2 \) from each of the three terms. Hence \( a = 78.1t \).
153. A bridge spans a V-shaped canyon, measuring 129 m from one edge to the other. One side of the canyon makes an angle of 0.209 radians to the vertical; the other side makes an angle of 0.122 radians to the vertical. If the two sides meet at the bottom, how deep is the canyon? Round your answer to the nearest 0.1 meter.

This is an ASA problem. Be careful with your angles—again! The angles described in the problem are measured with respect to the vertical; but for the triangle, you need them with respect to the horizontal. Thus

\[ A = \frac{\pi}{2} - 0.209 = 1.3618 \text{ rad}; \quad B = \frac{\pi}{2} - 0.122 = 1.4488 \text{ rad}. \]

We want the elevation \( h \).

\[ C = \pi - (A + B) = 0.331 \text{ rad}. \]

\[ b = \frac{129 \sin B}{\sin C} = 393.99. \quad h = b \sin A = 385.4 \text{ m}. \]

154. A mineshaft slopes downward to the south, making an angle of 68° to the horizontal. The shaft measures 1220 ft from mouth to bottom. You need to dig an airshaft that will start 400 ft south of the mouth of the first shaft, and that will intersect that shaft at its bottom. At what angle to the horizontal do you need to dig? Does the airshaft have to slope downward to the north or to the south? Round your answer to the nearest 0.1°.

This is an SAS problem.

\[ a^2 = 1220^2 + 400^2 - (1220)(400) \cos 68°; \quad a = 1132.6 \text{ ft}. \]

Since \( b \) is the longest side of the triangle, \( B \) could be acute or obtuse. We’ll use the law of cosines to calculate it:

\[ \cos B = \frac{a^2 + 400^2 - 1220^2}{2(a)(400)}; \quad B = 92.9°. \]

This means that it slopes downward to the south, making an angle of \( 180 - B = 87.1° \) with the horizontal.

Another approach would be to calculate the horizontal distance from \( A \) to a point directly above \( C \). That distance would be 1220 \( \cos 68° = 457 \) ft. Since this is greater than 400 ft, the airshaft would have to incline downward to the south. (We did that calculation before we drew the sketch, so we’d know which way to put the airshaft.) Then we’d use the law of sines to calculate the angle with the horizontal:

\[ \sin \beta = \frac{1220 \sin 68°}{a}; \quad \beta = 87.1°. \]