In this file, we will use a triangle ABC. The points and the angles will be referred to by the capital letters A, B, and C; the sides will be referred to by the lower-case letters a, b, and c, with each side opposite the corresponding angle. Unless stated otherwise, you should make no assumptions about whether angles are acute or obtuse.

You will need a calculator or equivalent for almost all of these problems.

1. If $A = 20^\circ$ and $B = 100^\circ$, what is $C$?
2. If $A = 50^\circ$ and $C = 60^\circ$, what is $B$?
3. If $B = 140^\circ$ and $C = 20^\circ$, what is $A$?
4. If $A = \pi/5 \text{ rad}$ and $B = 3\pi/5 \text{ rad}$, what is $C$?
5. If $A = 3\pi/7 \text{ rad}$ and $C = 2\pi/7 \text{ rad}$, what is $B$?
6. If $B = \pi/5 \text{ rad}$ and $C = \pi/2 \text{ rad}$, what is $A$?

In problems 7-15, find the missing angles and sides of the triangle ABC. Give exact values for angles; round lengths of sides to four decimal places.

7. $A = 40^\circ$, $C = 70^\circ$, and $b = 10$.
8. $A = 110^\circ$, $B = 20^\circ$, and $c = 12$.
9. $B = 35^\circ$, $C = 75^\circ$, and $a = 6$.
10. $A = 98^\circ$, $B = 34^\circ$, and $c = 5$.
11. $A = 51^\circ$, $C = 128^\circ$, and $b = 20$.
12. $B = 11^\circ$; $C = 23^\circ$; and $a = 8$.
13. $A = \pi/7 \text{ rad}$; $B = 2\pi/7 \text{ rad}$; and $c = 10$.
14. $A = 3\pi/10 \text{ rad}$; $C = \pi/10 \text{ rad}$; and $b = 12$.
15. $B = \pi/5 \text{ rad}$; $C = \pi/4 \text{ rad}$; and $a = 8$.

In problems 16-21, find the missing angles and sides of the triangle ABC. Round all values to four decimal places.

16. $A = 0.35 \text{ rad}$; $B = 0.8 \text{ rad}$; $c = 20$. 
17. A = 0.4 rad; C = 0.25 rad; b = 10.

18. B = 2 rad; C = 0.7 rad; a = 8.

19. A = 1.1 rad; B = 1.1 rad; c = 6.

20. A = 0.85 rad; C = 0.75 rad; b = 12.

21. B = 0.3 rad; C = 2.5 rad; a = 14.

In problems 22-51, find the missing angles and sides of the triangle ABC. If there are two possible triangles, give the side and angles for both. If there is no solution, say so. Round your answers as follows: sides, to four decimal places; angles, to the nearest 0.01° or 0.0001 rad. Use the angle measure that’s used in the statement of the problem.

22. A = 150°; a = 15; c = 10.

23. A = 150°; a = 8; c = 10.


25. A = 30°; a = 4; c = 10.

26. A = 30°; a = 5; c = 10.

27. A = 30°; a = 8; c = 10.

28. A = 30°; a = 12; c = 10.

29. A = 30°; a = 9; c = 18.

30. A = 90°; a = 7; c = 4.

31. A = 126°; a = 12; c = 15.

32. A = 52°; a = 11; c = 12.

33. A = 138°; a = 14; c = 20.

34. A = 22°; a = 25; c = 16.

35. A = 90°; a = 7; c = 9.

36. A = 104°; a = 9; c = 4.
37. \( A = 90^\circ; a = 5; c = 3. \)
38. \( A = 155^\circ; a = 10; c = 13. \)
39. \( A = 30^\circ; a = 8; c = 16. \)
40. \( A = 90^\circ; a = 6; c = 8. \)
41. \( A = 30^\circ; a = 6; c = 9. \)
42. \( A = 49^\circ; a = 20; c = 16. \)
43. \( A = 71^\circ; a = 13; c = 18. \)
44. \( A = \pi/6 \text{ rad}; a = 7; c = 14. \)
45. \( A = 2.6 \text{ rad}; a = 7; c = 9. \)
46. \( A = \pi/5 \text{ rad}; a = 10; c = 7. \)
47. \( A = \pi/2 \text{ rad}; a = 4; c = 3. \)
48. \( A = 5\pi/6 \text{ rad}; a = 9; c = 6. \)
49. \( A = \pi/2 \text{ rad}; a = 6; c = 11. \)
50. \( A = 0.42 \text{ rad}; a = 9; c = 12 \)
51. \( A = 1.2 \text{ rad}; a = 13; c = 15 \)

In problems 52-68, calculate the area of the triangle. Round your answer to four decimal places. If the triangle can’t exist, say so.

52. \( A = 41^\circ, C = 70^\circ, b = 10. \)
53. \( A = 113^\circ, B = 55^\circ, c = 6. \)
54. \( B = 22^\circ, C = 13^\circ, a = 100. \)
55. \( A = 94^\circ; B = 22^\circ; c = 12. \)
56. \( A = 27^\circ; C = 51^\circ; b = 8. \)
57. \( B = 76^\circ; C = 82^\circ; a = 5. \)
58. \( A = 0.81 \text{ rad}; B = 1.02 \text{ rad}; c = 9. \)
59.  A = π/7 rad; C = 0.44 rad; b = 20.

60.  A = 103°; a = 10; c = 4.

61.  B = 53°; b = 18; c = 20; C is acute.

62.  A = 27°; a = 22; b = 36; B is obtuse.

63.  C = 90°; c = 12; a = 5.

64.  B = 30°; b = 7; a = 14.

65.  C = 13°; c = 12; b = 60.

66.  A = 90°; a = 9, c = 15.

67.  A = 115°; a = 3; b = 4.

68.  B = 30°; b = 11; c = 7.

69.  A radio antenna was 240 ft tall and vertical when it was first put up. However, the foundation is weak, and the antenna is now tilted 4.2° from the vertical. To keep it from tilting further, you want to install a diagonal guy wire: the wire is to be attached halfway up the antenna, and should meet the ground at a 50° angle. How long does the wire have to be? Round your answer to the nearest 0.01 ft.

70.  A highway runs directly north-south. A road going in a northeasterly direction crosses the highway at an angle of 78°; 2.3 miles farther north. A railroad going southeasterly crosses the highway at an angle of 63°. What is the distance along the road from the highway to the point where the road and the railroad meet? Round your answer to the nearest 0.1 mile.

71.  You and a friend are standing 1.75 miles apart along Valencia Road, which runs straight east-west. When an airplane flies over Valencia you, facing east, measure its elevation as 0.1132 rad; your friend, facing west, measures its elevation as 0.1504 rad. How high is the airplane? Round your answer to the nearest foot.
In problems 72-79, find the angles of the triangle ABC. Give your answers in the indicated units. Round angles to 0.01° or to 0.0001 rad. If the triangle is impossible, say so.

72. \( a = 4, \ b = 5, \ c = 6 \). Give angles in radians.
73. \( a = 12, \ b = 5, \ c = 9 \). Give angles in radians.
74. \( a = 20, \ b = 6, \ c = 8 \). Give angles in degrees.
75. \( a = 13, \ b = 9, \ c = 11 \). Give angles in degrees.
76. \( a = 10, \ b = 14, \ c = 7 \). Give angles in radians.
77. \( a = 2, \ b = 6, \ c = 5 \). Give angles in degrees.
78. \( a = 10, \ b = 14, \ c = 30 \). Give angles in radians.
79. \( a = 19, \ b = 13, \ c = 9 \). Give angles in degrees.

In problems 80-85, find the missing sides and angles of the triangle ABC. Round sides to 0.0001 and angles to either 0.01° or 0.0001 rad. Use the angle measure that’s used in the statement of the problem.

80. \( A = 72°; \ b = 12; \ c = 10 \).
81. \( a = 30; \ B = 1.8 \text{ rad}; \ c = 19 \).
82. \( a = 8; \ b = 18; \ C = 27° \).
83. \( A = 2.2 \text{ rad}; \ b = 25; \ c = 9 \).
84. \( a = 5; \ B = 14°; \ c = 7 \).
85. \( a = 15; \ b = 11; \ C = 0.6 \text{ rad} \).

In problems 86-95, calculate the area of the triangle ABC. Round your answers to four decimal places. If the triangle is impossible, say so.

86. \( a = 15, \ b = 27, \ C = 1.9 \text{ rad} \).
87. \( a = 8, \ B = 49°, \ c = 11 \).
88. \( a = 13, \ b = 7, \ C = 102° \).
89. \( A = 17°, \ b = 10, \ c = 19 \).
90. \( a = 12, b = 7, c = 9. \)
91. \( a = 12, b = 22, c = 7. \)
92. \( a = 30, b = 19, c = 23. \)
93. \( a = 28, B = 1.35 \text{ rad}, c = 15. \)
94. \( a = 12, b = 14, c = 32. \)
95. \( A = 0.25 \text{ rad}; b = 24; c = 12. \)

96. A drawbridge consists of two unequal sides, each hinged to move up and down. One side is 100 m long; the other is 70 m long. The channel they span when closed is 160 m wide. What angles above the horizontal do the two sides make when they are closed? Give your answer to the nearest 0.0001 rad.

97. Two straight roads meet at an angle of 74°. A car and a motorcycle leave the intersection at the same time: the car going at 45 miles per hour, and the motorcycle going at 65 mph. After one hour, how far apart are the two vehicles? Round your answer to the nearest 0.1 mile.

98. A balloon is tethered by two cables: one 800 m long, the other 850 m long. The cables meet the ground 400 m from one another. If the balloon pulls both cables straight, how high above the ground is it? Round your answer to the nearest meter.

In problems 99-131, find the missing angles and sides of triangle ABC. If the triangle cannot exist, say so. If there are two possibilities for the triangle, find the sides and angles for both. Round sides to the nearest 0.0001; round angles to the nearest 0.01°, or 0.0001 rad. Use the angle measure that’s used in the statement of the problem.

99. \( a = 5.3, b = 7.2, c = 9.4. \) Give angles in radians.

100. \( A = 41°; a = 6.1; c = 6.5 \)
101. \( A = 0.79 \text{ rad}; b = 11; C = 1.14 \text{ rad} \)
102. \( A = 77°; a = 11.8; c = 13.3 \)
103. \( a = 6.2; B = 2.23 \text{ rad}; b = 4.8 \)
104. \( a = 3; C = 90°; c = 5.5 \)
105. \( B = \pi/2 \text{ rad}; b = 10; c = 4.4 \)
106. \( b = 2.4; \ C = 38^\circ; \ c = 3.9 \)

107. \( A = 1.02 \text{ rad}; \ a = 7.5; \ b = 8.3 \)

108. \( a = 11.6; \ B = 48^\circ; \ C = 103^\circ. \)

109. \( a = 43; \ b = 9.6; \ C = \pi/5 \text{ rad} \)

110. \( B = 158^\circ; \ b = 9.7; \ c = 13.3 \)

111. \( a = 31; \ B = 13^\circ; \ c = 27 \)

112. \( a = 15.4; \ B = \pi/6; \ b = 7.7 \)

113. \( a = 3.3; \ B = 0.78 \text{ rad}; \ C = 1.09 \text{ rad} \)

114. \( a = 6.2; \ C = 90^\circ; \ c = 4.6 \)

115. \( B = 58^\circ; \ b = 23.1; \ c = 18.4 \)

116. \( a = 11.4; \ b = 8.8; \ c = 6.7. \) Give angles in degrees.

117. \( A = 137^\circ; \ a = 4.8; \ b = 2.9 \)

118. \( a = 34.4; \ B = 1.31 \text{ rad}; \ C = 1.03 \text{ rad} \)

119. \( a = 16.3; \ B = 2.8 \text{ rad}; \ c = 5.9 \)

120. \( b = 11; \ C = 0.9 \text{ rad}; \ c = 9.2 \)

121. \( a = 3.3; \ b = 8.1; \ c = 6.9. \) Give angles in degrees.

122. \( A = \pi/6 \text{ rad}; \ b = 5.8; \ a = 2.9 \)

123. \( a = 7.9; \ C = 5\pi/7 \text{ rad}; \ c = 4.5 \)

124. \( a = 13; \ B = 74^\circ; \ c = 8.6 \)

125. \( a = 23; \ B = 90^\circ; \ b = 18.5 \)

126. \( A = 2.3 \text{ rad}; \ b = 6.7; \ c = 3 \)

127. \( a = 5.9; \ b = 3.4; \ c = 4.7. \) Give angles in radians.

128. \( A = 54^\circ; \ B = 111^\circ; \ c = 19 \)
129. \(B = 122^\circ; b = 23; c = 17\)

130. \(a = 15.5; C = 46^\circ; c = 12.9\)

131. \(a = 10.7; b = 14.4; c = 17.1\). Give angles in radians.

In problems 132-143, calculate the area of the triangle ABC. Round your answers to four decimal places. If the triangle can’t exist, say so.

132. \(A = 0.78\text{ rad}; a = 13; b = 16; B\) is acute.

133. \(a = 18; b = 12; c = 5\)

134. \(b = 12; C = 2.5\text{ rad}; c = 10\)

135. \(B = 39^\circ; b = 15; c = 20; C\) is obtuse.

136. \(a = 4.9, b = 10.4, c = 8.9\)

137. \(A = 49^\circ; B = 102^\circ; c = 6\)

138. \(b = 12; C = 0.61\text{ rad}; c = 6\)

139. \(a = 11; B = 1.12\text{ rad}; C = 0.98\text{ rad}\)

140. \(A = 102^\circ; b = 25; C = 27^\circ\)

141. \(A = 158^\circ; a = 18; c = 10\)

142. \(B = 18^\circ; b = 10; c = 4\)

143. A road, a railroad track, and a canal form a triangle. The distance along the road between its intersections with the track and with the canal is 1440 m. The distance along the track between its intersection with the road and its crossing of the canal is 790 m. The distance along the canal between the crossings of the road and of the track is 1310 m. What is the area of the piece of land enclosed by the road, the track, and the canal? Round your answer to the nearest 0.1 hectare (1 ha = 10000 m²).

144. To keep a wall from falling over, you brace it with a 5-meter beam. The beam is fixed in the ground 3.77 m from the base of the wall; it contacts the wall 3.29 m high (measured along the wall, not directly up from the ground). What is the wall’s deviation from the vertical? Round your answer to the nearest 0.0001 radian.
145. A highway runs straight east-west. A road intersects it at an angle of 41°, heading northeasterly. You park at the intersection and walk 3.92 miles along the road to its junction with another road. You make a sharp right turn and follow the second road 3.11 miles in a southwesterly direction back to the highway. How far along the highway are you from your car? Round your answer to the nearest 0.01 mile.

146. High-frequency direction-finding (HF/DF) is a technique for finding radio sources. It was used by the Allies during World War II to locate U-boats. Each HF/DF station used a highly directional antenna to determine the direction of a radio source; data from two or more stations could be combined to locate the source.

Station N is 23.3 miles directly north of Station S. The two detect a submarine's radio signal. From Station N, the signal source is 18.1° south of west; from Station S, the sub is 22.5° north of west. How far is the sub from each of the two stations? Round your answers to the nearest 0.1 mile.

147. You are driving along a highway that runs straight east and west. You see a water tower in a northeasterly direction, at an angle of 0.71 radian to the highway. You drive 3.8 km further east and measure the angle again; the water tower is still northeasterly, at an angle of 1.12 radian to the highway. How far north of the highway is the water tower? Round your answer to the nearest 0.1 km.

148. A hill slopes down to the south. When the sun is directly in the south, it is at an elevation of 57.4° above the horizon. A 10-foot rod held vertically casts a shadow 6.15 ft long. What angle does the slope make with the horizontal? The angle is less than 30°. Round your answer to the nearest 0.1°.

149. A crane is being used to drag a very large rock along the ground. The crane is fixed at an angle of 50° to the horizontal, and measures 85 ft from base to top. The cable extends 373 ft from the top of the crane to the rock. What angle does the cable make with the horizontal? Round your answer to the nearest 0.1°.

150. A mountain rises in a long constant slope that makes an angle of 13.6° with the horizontal. At the base of the mountain is a level plain. From the top of the mountain, the angle of depression of a town on the plain is 8.2° below the horizontal. The mountain slope measures 5.71 miles from base to top. How far is it from the base of the mountain to the town? Round your answer to the nearest 0.1 mile.
151. Yet another radio antenna is in danger of falling over; its deviation from the vertical is 8.5°. To keep it from tilting any farther, you guy it with a 300-foot cable, attached 125 ft up from the base of the antenna. How far from the base of the antenna will the cable meet the ground, and at what angle to the horizontal? Round your answers to the nearest 0.1 ft and 0.1°.

152. Two roads intersect at an angle of 77°. Two cars leave the intersection at the same time: one going at 58 miles per hour, the other at 67 mph. Give the distance \( s \) in miles from one car to the other as a function of the time \( t \) in hours since departure from the intersection. Round all constants in your formula to three significant figures.

153. A bridge spans a V-shaped canyon, measuring 129 m from one edge to the other. One side of the canyon makes an angle of 0.209 radians to the vertical; the other side makes an angle of 0.122 radians to the vertical. If the two sides meet at the bottom, how deep is the canyon? Round your answer to the nearest 0.1 meter.

154. A mineshaft slopes downward to the south, making an angle of 68° to the horizontal. The shaft measures 1220 ft from mouth to bottom. You need to dig an airshaft that will start 400 ft south of the mouth of the first shaft, and that will intersect that shaft at its bottom. At what angle to the horizontal do you need to dig? Does the airshaft have to slope downward to the north or to the south? Round your answer to the nearest 0.1°.