Note: Vectors can be indicated either by using boldface, or by drawing an arrow over the top of the letter. We use these two interchangeably; they mean the same thing. You should use the arrow in handwritten answers; boldface may be easier if you’re using text-processing software.

We also use angle-bracket and unit-vector notation interchangeably: \( \langle V_x, V_y \rangle, V_x \hat{i} + V_y \hat{j} \), and \( V_x \hat{i} + V_y \hat{j} \). You should use “hat” notation for standard unit vectors in handwritten work.

When we speak of the “components” of a vector, we refer to scalars. Thus if \( \vec{V} = V_x \hat{i} + V_y \hat{j} \), the \( x \)-component is the scalar \( V_x \). We will use “vector component” when we mean a vector: thus the \( x \)-vector component of \( \vec{V} = V_x \hat{i} + V_y \hat{j} \) is \( V_x \hat{i} \).

Unless you’re told to use a calculator, give exact answers. Simplify all fractions and roots, and rationalize all denominators.

Unless stated otherwise, assume that angles are measured in radians.

In problems 1-24, angles are measured in degrees counterclockwise from the positive \( x \)-axis. Give your answers as exact values and rationalize all denominators. Do not use a calculator.

1. \( \vec{V} \) is a vector with magnitude 12 and direction 30°. What are the \( x \)-component \( V_x \) and the \( y \)-component \( V_y \) of \( \vec{V} \)?

2. \( \vec{V} \) is a vector with magnitude 5 and direction 45°. What are the \( x \)-component \( V_x \) and the \( y \)-component \( V_y \) of \( \vec{V} \)?

3. \( \vec{V} \) is a vector with magnitude 6 and direction 60°. What are the \( x \)-component \( V_x \) and the \( y \)-component \( V_y \) of \( \vec{V} \)?

4. \( \vec{V} \) is a vector with magnitude 10 and direction 90°. What are the \( x \)-component \( V_x \) and the \( y \)-component \( V_y \) of \( \vec{V} \)?

5. \( \vec{V} = \langle 1, \sqrt{3} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

6. \( \vec{V} = \langle 5, 5\sqrt{3} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

7. \( \vec{V} = \langle 6, 2\sqrt{3} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

8. \( \vec{V} = \langle 2\sqrt{3}, 2\sqrt{3} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

9. \( \vec{V} \) is a vector with magnitude 10 and direction 150°. Write \( \vec{V} \) in component form: \( \vec{V} = \langle V_x, V_y \rangle \).
10. \( \vec{V} \) is a vector with magnitude 8 and direction 330°. Write \( \vec{V} \) in component form:
\[
\vec{V} = \langle V_x, V_y \rangle.
\]

11. \( \vec{V} \) is a vector with magnitude 3 and direction 270°. Write \( \vec{V} \) in component form:
\[
\vec{V} = \langle V_x, V_y \rangle.
\]

12. \( \vec{V} \) is a vector with magnitude 9 and direction 225°. Write \( \vec{V} \) in component form:
\[
\vec{V} = \langle V_x, V_y \rangle.
\]

13. \( \vec{V} = \langle -2, 2\sqrt{3} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

14. \( \vec{V} = \langle 12, -4\sqrt{3} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

15. \( \vec{V} = \langle -3\sqrt{3}, -3 \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

16. \( \vec{V} = \langle -5, 5 \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

17. \( \vec{V} = 17\hat{i} - 17\hat{j} \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

18. \( \vec{V} = -\sqrt{3}\hat{i} - \hat{j} \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

19. \( \vec{V} = -2\sqrt{3}\hat{i} + 2\sqrt{3}\hat{j} \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

20. \( \vec{V} = 5\sqrt{3}\hat{i} - 15\hat{j} \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

21. \( \vec{V} \) is a vector with magnitude 4 and direction 210°. Write \( \vec{V} \) in terms of unit vectors:
\[
\vec{V} = V_x\hat{i} + V_y\hat{j}.
\]

22. \( \vec{V} \) is a vector with magnitude 7 and direction 315°. Write \( \vec{V} \) in terms of unit vectors:
\[
\vec{V} = V_x\hat{i} + V_y\hat{j}.
\]

23. \( \vec{V} \) is a vector with magnitude \( \sqrt{3} \) and direction 120°. Write \( \vec{V} \) in terms of unit vectors:
\[
\vec{V} = V_x\hat{i} + V_y\hat{j}.
\]

24. \( \vec{V} \) is a vector with magnitude \( 2\sqrt{5} \) and direction 180°. Write \( \vec{V} \) in terms of unit vectors:
\[
\vec{V} = V_x\hat{i} + V_y\hat{j}.
\]
In problems 25-32, angles are measured in radians counterclockwise from the positive x-axis. Give your answers as exact values and rationalize all denominators. Do not use a calculator.

25. \( \vec{V} \) is a vector with magnitude 13 and direction \( \pi/4 \). Write \( \vec{V} \) in component form:
   \[ \vec{V} = \langle V_x, V_y \rangle. \]

26. \( \vec{V} \) is a vector with magnitude 4 and direction \( 2\pi/3 \). Write \( \vec{V} \) in terms of unit vectors:
   \[ \vec{V} = V_x \hat{i} + V_y \hat{j}. \]

27. \( \vec{V} = \langle 2, 2\sqrt{3} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

28. \( \vec{V} = \sqrt{6} \hat{i} - \sqrt{2} \hat{j} \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \| \vec{V} \| \)?

29. \( \vec{V} \) is a vector with magnitude \( \sqrt{2} \) and direction \( \pi/6 \). Write \( \vec{V} \) in terms of unit vectors:
   \[ \vec{V} = V_x \hat{i} + V_y \hat{j}. \]

30. \( \vec{V} \) is a vector with magnitude \( \sqrt{5} \) and direction \( 3\pi/2 \). Write \( \vec{V} \) in component form:
   \[ \vec{V} = \langle V_x, V_y \rangle. \]

31. \( \vec{V} = \langle \sqrt{2}, -\sqrt{6} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \vec{V} \)?

32. \( \vec{V} = \langle -\frac{\pi}{2}, -\frac{\pi}{2} \rangle \). What are the magnitude \( \| \vec{V} \| \) and direction \( \theta \) of \( \| \vec{V} \| \)?

In problems 33-50, perform the indicated operations. Write the resulting vectors in the same format as in the statements of the problems.

33. \( \vec{U} = 5\hat{i} - 2\hat{j}; \vec{V} = -6\hat{i} + 5\hat{j} \). Find \( \vec{U} + \vec{V} \).

34. \( \vec{U} = -3\hat{i} + 4\hat{j} \). Find \( 6\vec{U} \).

35. \( \vec{U} = 2\hat{i} - 7\hat{j}; \vec{V} = 4\hat{i} + 2\hat{j} \). Find \( \vec{U} - \vec{V} \).

36. \( \vec{U} = \langle 4, \sqrt{3} \rangle; \vec{V} = \langle -1, 3\sqrt{3} \rangle \). Find \( \vec{U} + \vec{V} \).

37. \( \vec{U} = \langle 3, 8 \rangle; \vec{V} = \langle -3, -3 \rangle \). Find \( \vec{U} - \vec{V} \).
38. $\vec{U} = \langle 4, -2 \rangle$. Find $3\vec{U}$.

39. $U = \langle -5, 5 \rangle$; $V = \langle -2, -6 \rangle$. Find $U + V$.

40. $U = \langle -5, -8 \rangle$; $V = \langle -7, 4 \rangle$. Find $U - V$.

41. $V = \langle -4, 3 \rangle$. Find $-5V$.

42. $\vec{U} = 6\hat{i} + 3\hat{j}$; $\vec{V} = -3\hat{i} + 6\hat{j}$. Find $\vec{U} + \vec{V}$.

43. $\vec{U} = 6\hat{i} + 3\hat{j}$; $\vec{V} = -3\hat{i} + 6\hat{j}$. Find $\vec{U} - \vec{V}$.

44. $\vec{V} = -3\hat{i} + 6\hat{j}$. Find $2\vec{V}$.

45. $U = 4\hat{i} - 5\hat{j}$. If $U + V = 8\hat{i} - 2\hat{j}$, what is $V$?

46. $U = -2\hat{i} - 4\hat{j}$. If $U - V = 3\hat{i} - 6\hat{j}$, what is $V$?

47. If $5U = -15\hat{i} + 20\hat{j}$, what is $U$?

48. $\vec{U} = 6\hat{i} + 3\hat{j}$. If $\vec{U} + \vec{V} = 4\hat{i} - 6\hat{j}$, what is $V$?

49. $\vec{U} = -\hat{i} + 5\hat{j}$. If $\vec{U} - \vec{V} = 8\hat{i} - \hat{j}$, what is $\vec{V}$?

50. If $-3\vec{U} = -9\hat{i} + 15\hat{j}$, what is $\vec{U}$?

In problems 51-56, find the unit vector in the direction of the vector $V$.

51. $V = 3\hat{i} + 4\hat{j}$

52. $V = \langle -5, 12 \rangle$

53. $\vec{V} = \hat{i} - 2\hat{j}$

54. $V = -6\hat{i}$

55. $V = \langle 1287, -1287 \rangle$

56. $\vec{V} = 2\hat{i} - \sqrt{5}\hat{j}$
In problems 57-78, give exact answers, using trigonometric functions or inverse trigonometric functions as necessary. Do not use a calculator.

57. If \( \mathbf{V} = 3\mathbf{i} - 5\mathbf{j} \), what are the magnitude and direction of \( \mathbf{V} \)?

58. If \( \mathbf{V} = \langle -4, 5 \rangle \), what are the magnitude and direction of \( \mathbf{V} \)?

59. If \( \mathbf{\hat{V}} = 3\mathbf{i} - \sqrt{7}\mathbf{j} \), what are the magnitude and direction of \( \mathbf{\hat{V}} \)?

60. If \( \mathbf{V} = \langle \sqrt{5}, \sqrt{11} \rangle \), what are the magnitude and direction of \( \mathbf{V} \)?

61. If \( \mathbf{V} = -2\mathbf{i} - 3\mathbf{j} \), what are the magnitude and direction of \( \mathbf{V} \)?

62. \( \mathbf{\hat{V}} = 3\mathbf{i} - 4\mathbf{j} \), what are the magnitude and direction of \( \mathbf{\hat{V}} \)?

63. \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 7 \) and direction \( \pi/5 \). Write \( \mathbf{V} \) in component form: \( \mathbf{V} = \langle V_x, V_y \rangle \).

64. \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 8 \) and direction 0.4. Write \( \mathbf{V} \) in terms of unit vectors: \( \mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} \).

65. \( \mathbf{U} \) is a vector with magnitude \( \| \mathbf{U} \| = 10 \) and direction \( \pi/6 \). \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 10 \) and direction \( 5\pi/6 \). Write \( \mathbf{U} + \mathbf{V} \) in component form.

66. \( \mathbf{U} \) is a vector with magnitude \( \| \mathbf{U} \| = 10 \) and direction \( \pi/4 \). \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 10 \) and direction \( 5\pi/4 \). Write \( \mathbf{U} + \mathbf{V} \) in component form.

67. \( \mathbf{U} \) is a vector with magnitude \( \| \mathbf{U} \| = 10 \) and direction \( \pi/6 \). \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 10 \) and direction \( -\pi/6 \). Write \( \mathbf{U} + \mathbf{V} \) in component form.

68. \( \mathbf{U} \) is a vector with magnitude \( \| \mathbf{U} \| = 10 \) and direction \( \pi/6 \). \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 10 \) and direction \( 5\pi/6 \). Write \( \mathbf{U} - \mathbf{V} \) in component form.

69. \( \mathbf{U} \) is a vector with magnitude \( \| \mathbf{U} \| = 10 \) and direction \( \pi/4 \). \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 10 \) and direction \( 5\pi/4 \). Write \( \mathbf{U} - \mathbf{V} \) in component form.

70. \( \mathbf{U} \) is a vector with magnitude \( \| \mathbf{U} \| = 10 \) and direction \( \pi/3 \). \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 10 \) and direction \( -\pi/3 \). Write \( \mathbf{U} - \mathbf{V} \) in component form.

Use a calculator or equivalent to solve these problems. Unless the problem specifies otherwise, round vector components to four decimal places and angles to either 0.0001 rad or 0.01°.

71. \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 10 \) and direction \( 2\pi/7 \). Write \( \mathbf{V} \) in component form.
72. \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 8 \) and direction \( \frac{5\pi}{7} \). Write \( \mathbf{V} \) in terms of unit vectors.

73. \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 15 \) and direction \( \frac{8\pi}{7} \). Write \( \mathbf{V} \) in component form.

74. \( \mathbf{V} \) is a vector with magnitude \( \| \mathbf{V} \| = 11 \) and direction \( \frac{11\pi}{7} \). Write \( \mathbf{V} \) in terms of unit vectors.

75. \( \mathbf{V} = 6\mathbf{i} - 5\mathbf{j} \). What are the magnitude and direction of \( \mathbf{V} \)?

76. \( \mathbf{V} = (-3, 8) \). What are the magnitude and direction of \( \mathbf{V} \)?

77. \( \mathbf{V} = -2\mathbf{i} - 3\mathbf{j} \). What are the magnitude and direction of \( \mathbf{V} \)?

78. \( \mathbf{V} = (4, -7) \). What are the magnitude and direction of \( \mathbf{V} \)?

79. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 10 \); its direction is \( \frac{2\pi}{9} \). \( \| \mathbf{V} \| = 6 \); its direction is \( \frac{\pi}{5} \). Write \( \mathbf{U} + \mathbf{V} \) in component form.

80. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 7 \); its direction is \( \frac{5\pi}{8} \). \( \| \mathbf{V} \| = 11 \); its direction is \( -\frac{2\pi}{5} \). Write \( \mathbf{U} + \mathbf{V} \) in terms of unit vectors.

81. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 12 \); its direction is 1.2. \( \| \mathbf{V} \| = 8 \); its direction is 3.9. Write \( \mathbf{U} + \mathbf{V} \) in component form.

82. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 20 \); its direction is 0.9. \( \| \mathbf{V} \| = 10 \); its direction is \( -1.1 \). Write \( \mathbf{U} - \mathbf{V} \) in terms of unit vectors.

83. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 5 \); its direction is 1.9. \( \| \mathbf{V} \| = 8 \); its direction is 2.2. Write \( \mathbf{U} - \mathbf{V} \) in component form.

84. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 6 \); its direction is 55°. \( \| \mathbf{V} \| = 5 \); its direction is 142°. Write \( \mathbf{U} + \mathbf{V} \) in terms of unit vectors.

85. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 12 \); its direction is 106°. \( \| \mathbf{V} \| = 15 \); its direction is 27°. Write \( \mathbf{U} - \mathbf{V} \) in component form.

86. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 10 \); its direction is 1.2. \( \| \mathbf{V} \| = 5 \); its direction is \( -1.9 \). Find the magnitude and direction of \( \mathbf{U} + \mathbf{V} \).

87. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 8 \); its direction is 3.9. \( \| \mathbf{V} \| = 5 \); its direction is 2.5. Find the magnitude and direction of \( \mathbf{U} - \mathbf{V} \).

88. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 12 \); its direction is 37°. \( \| \mathbf{V} \| = 4 \); its direction is 229°. Find the magnitude and direction of \( \mathbf{U} + \mathbf{V} \). Give the direction in degrees.
89. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 8 \); its direction is \( 102^\circ \). \( \| \mathbf{V} \| = 11 \); its direction is \( -38^\circ \). Find the magnitude and direction of \( \mathbf{U} - \mathbf{V} \). Give the direction in degrees.

90. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 14 \); its direction is \( 0.2 \). \( \| \mathbf{V} \| = 6 \); its direction is \( 0.5 \). Find the magnitude and direction of \( \mathbf{U} + \mathbf{V} \).

91. \( \mathbf{U} \) and \( \mathbf{V} \) are vectors. \( \| \mathbf{U} \| = 9 \); its direction is \( 20\pi/29 \). \( \| \mathbf{V} \| = 12 \); its direction is \( -2.08 \). Find the magnitude and direction of \( \mathbf{U} + \mathbf{V} \).

You will need a calculator or equivalent for most of these problems.

92. An airplane flies in a straight line for 128 miles on a course \( 32.3^\circ \) east of north. It then flies an additional 161 miles on a course \( 14.5^\circ \) west of north. At the end of this time, how far is it from its starting point, and in what direction? Round your distance to the nearest mile and your direction to the nearest \( 0.1^\circ \).

93. A ship sails for 72.1 km on a course \( 0.922 \) rad north of east. It then sails an additional 113.3 km on a course \( 0.303 \) rad north of east. At the end of this time, how far is it from its starting point, and in what direction? Round your distance to the nearest 0.1 km and your direction to the nearest 0.001 rad.

94. You are walking across the desert, trying to reach a point 22.6 km away in a direction \( 0.589 \) rad west of north. To avoid a mountain, you begin by walking 14.4 km in a direction \( 0.122 \) rad east of north. How far do you have to walk, and in what direction, to reach your intended destination? Round your distance to the nearest 0.1 km and your direction to the nearest 0.001 rad.

95. A tire is hanging from a rope attached to a tree branch. The force of gravity on the tire is 92.2 lbs straight down. The wind exerts a horizontal force of 10.0 lb on the tire. What is the total force pulling on the rope, and at what angle from the vertical is the rope hanging? Round the force to the nearest 0.1 lb and the angle to the nearest \( 0.1^\circ \).

96. A truck is driving parallel to a railroad track, pulling a railroad car with a rope. The rope makes an angle of \( 0.092 \) radians with the track, and the car is pulling with a force of 468 N. (The newton, abbreviated “N”, is a metric unit of force.) What is the component of force in the direction of the track? What is the component of force perpendicular to the track? Round your answers to the nearest newton.

97. Because of the rising price of gas, you have acquired two oxen to pull your SUV. One ox is pulling with a force of 275 lbs in a direction \( 9.2^\circ \) to the right of the road. The other ox is pulling with a force of 258 lbs in a direction \( 7.7^\circ \) to the left of the road. What is the total force on the SUV in the direction of the road? What is the total force perpendicular to the road, and does it pull to the left or to the right? Round your answer to the nearest 0.1 lb.

98. An airplane is flying with an airspeed of 212 knots, pointing \( 22^\circ \) north of west. The wind is blowing at a speed of 38 knots in a direction \( 11^\circ \) north of east. What are the speed and
direction of the plane, relative to the ground? Round your answers to the nearest knot and degree.

99. An airplane flies with an airspeed of 180 knots. The wind is blowing at a speed of 27 knots in a direction 39° south of west. If the pilot wants to fly straight north, in what direction should he point the plane? What will his ground speed be on that course? Round your direction to the nearest 0.01° and your ground speed to the nearest 0.1 knot.