In problems 1-10, you are given the standard form of the equation of an ellipse. Give a complete description of the ellipse: its center, its major and minor axes, and whether the major axis is horizontal or vertical. If the equation does not describe an ellipse, say so.

1. \( \frac{x^2}{9} + \frac{y^2}{25} = 1 \)

The standard form for an ellipse is: \( \frac{(x - h)^2}{a_x^2} + \frac{(y - k)^2}{a_y^2} = 1 \), where the ellipse is centered at the point \((h, k)\), the horizontal axis is \(2a_x\), and the vertical axis is \(2a_y\). In this case, the center is \((0, 0)\); the horizontal axis is \(2 \cdot 3 = 6\); and the vertical axis is \(2 \cdot 5 = 10\). Since the vertical axis is greater, it is the major axis. Hence the major axis is 10 and is vertical; the minor axis is 6.

2. \( \frac{(x - 3)^2}{4} + \frac{(y - 7)^2}{9} = 1 \)

The center is \((3, 7)\); the major axis is \(2 \cdot 3 = 6\), and is vertical; the minor axis is \(2 \cdot 2 = 4\).

3. \( \frac{(x + 1)^2}{36} + \frac{(y - 1)^2}{16} = 1 \)

The center is \((-1, 1)\); the major axis is \(2 \cdot 6 = 12\) and is horizontal; the minor axis is \(2 \cdot 4 = 8\).

4. \( \frac{(x - 9)^2}{6} + \frac{(y + 2)^2}{25} = 1 \)

The center is \((9, -2)\); the major axis is \(2 \cdot 5 = 10\) and is vertical; the minor axis is \(2 \sqrt{6} \).

5. \( \frac{x^2}{4} + (y + 5)^2 = 1 \)

Rewrite this: \( \frac{x^2}{4} + \frac{(y + 5)^2}{1} = 1 \) The center is \((0, -5)\); the major axis is \(2 \cdot 2 = 4\) and is horizontal; the minor axis is \(2 \cdot 1 = 2\).

6. \( 4x^2 + \frac{(y - 3)^2}{4} = 1 \)

We can rewrite: \( \frac{x^2}{\frac{1}{4}} + \frac{(y - 3)^2}{\frac{4}{4}} = 1 \). Then \(a_x = \frac{1}{2}\). The center is \((0, 3)\); the major axis is \(2 \cdot 2 = 4\) and is vertical; the minor axis is \(2 \cdot \frac{1}{2} = 1\).
7. \[
\frac{3(x-2)^2}{5} + 9(y+3)^2 = 1
\]
Rewrite: \[
\frac{(x-2)^2}{5/3} + \frac{(y+3)^2}{1/9} = 1
\]. Then \(a_x = \sqrt{\frac{5}{3}} = \frac{\sqrt{15}}{3}\). The center is \((2, -3)\). The major axis is \(\frac{2\sqrt{15}}{3}\) and is horizontal. The minor axis is \(\frac{2}{3}\).

8. \[
\frac{(x+1)^2}{4} - \frac{(y-7)^2}{9} = 1
\]
This is not an ellipse: there’s a negative sign between the two terms.

9. \[
\frac{(x+2)^2}{25} + \frac{(y+1)^2}{25} = 1
\]
We can rewrite this as: \((x+2)^2 + (y+1)^2 = 25\). Thus it’s a circle with a center at \((-2, -1)\) and a radius of 5; so not an ellipse.

10. \(x^2 + 4y^2 = 1\)
    The center is \((0, 0)\). The major axis is \(2 \cdot 1 = 2\) and is horizontal. The minor axis is \(2 \cdot \frac{1}{2} = 1\).

In problems 11-20, you are given the equation of an ellipse in nonstandard form. Give a complete description of the ellipse: its center, its major and minor axes, and whether the major axis is horizontal or vertical. If the equation does not describe an ellipse, say so.

11. \(4x^2 + 9y^2 = 36\)
Divide by 36 to get this in standard form: \(
\frac{x^2}{9} + \frac{y^2}{4} = 1
\). The center is \((0, 0)\); the major axis is 6 and is horizontal; the minor axis is 4.

12. \((x-2)^2 + 4(y+7)^2 = 100\)
Divide by 100 to get this in standard form: \(
\frac{(x-2)^2}{100} + \frac{(y+7)^2}{25} = 1
\). The center is \((2, -7)\); the major axis is 20 and is horizontal; the minor axis is 10.

13. \[
\frac{x^2 - 4x + 4}{49} + \frac{y^2 + 6y + 9}{81} = 1
\]
We rewrite the numerators to get this in standard form: \(
\frac{(x-2)^2}{49} + \frac{(y+3)^2}{81} = 1
\). The center is \((2, -3)\); the major axis is 18 and is vertical; the minor axis is 14.
14. \(9x^2 + 18x + 25y^2 - 150y + 9 = 0\)
We’ll do this in several steps. Begin by factoring the coefficients of \(x^2\) and \(y^2\) out of the terms involving \(x\) and \(y\):
\[9(x^2 + 2x) + 25(y^2 - 6y) + 9 = 0\].
Next, complete the squares inside the parentheses and subtract the appropriate quantities outside:
\[9(x^2 + 2x + 1) + 25(y^2 - 6y + 9) + 9 - 9(1) - 25(9) = 0\].
Rewrite this: \(9(x + 1)^2 + 25(y - 3)^2 = 225\).
Divide by 225 to put this in standard form: \(\frac{(x + 1)^2}{25} + \frac{(y - 3)^2}{9} = 1\).
The center is \((-1, 3)\); the major axis is 10 and is horizontal; the minor axis is 6.

15. \(4x^2 - 32x + 5y^2 + 10y + 49 = 0\)
Factor out the coefficients of \(x^2\) and \(y^2\): \(4(x^2 - 8x) + 5(y^2 + 2y) + 49 = 0\).
Complete the squares inside the parentheses:
\[4(x^2 - 8x + 16) + 5(y^2 + 2y + 1) + 49 - 4(16) - 5(1) = 0\].
Rewrite this: \(4(x - 4)^2 + 5(y + 1)^2 = 20\).
Divide by 20 to put in standard form: \(\frac{(x - 4)^2}{5} + \frac{(y + 1)^2}{4} = 1\).
The center is \((4, -1)\). The major axis is \(2\sqrt{5}\) and is horizontal. The minor axis is 4.

16. \(36x^2 + 72x + y^2 + 6y + 36 = 0\)
Factor out the coefficient of \(x^2\); the coefficient of \(y^2\) is already 1.
\(36(x^2 + 2x) + (y^2 + 6y) + 36 = 0\).
Complete the squares inside the parentheses:
\[36(x^2 + 2x + 1) + (y^2 + 6y + 9) + 36 - 36(1) - 1(9) = 0\].
Rewrite this: \(36(x + 1)^2 + (y + 3)^2 = 9\).
Divide by 9 to put this in standard form: \(4(x + 1)^2 + \frac{(y + 3)^2}{9} = 1\).
The center is \((-1, -3)\); the major axis is 6, and is vertical; the minor axis is 1.

17. \(3x^2 + 5y^2 + 10y = 10\)
Factor out the coefficients of \(x^2\) and \(y^2\): \(3(x^2) + 5(y^2 + 2y) = 10\).
Complete the squares: \(3(x^2) + 5(y^2 + 2y + 1) - 5(1) = 10\).
Rewrite: \(3(x^2) + 5(y + 1)^2 = 15\).
Divide by 15 to put this in standard form: \(\frac{x^2}{5} + \frac{(y + 1)^2}{3} = 1\).
The major axis is \(2\sqrt{5}\) and is horizontal; the minor axis is \(2\sqrt{3}\).
18. \(5x^2 - 16y^2 = 80\)

This isn’t an ellipse: notice that the \(y^2\) term is negative.

19. \(x^2 + 8x + 4y^2 + 24y + 11 = 0\)

Factor out the coefficients of \(x^2\) and \(y^2\): \((x^2 + 8x) + 4(y^2 + 6y) + 11 = 0\).

Complete the squares: \((x^2 + 8x + 16) + 4(y^2 + 6y + 9) + 11 - 16 - 4(9) = 0\).

Rewrite: \((x + 4)^2 + 4(y + 3)^2 = 41\).

Divide by 41: \(\frac{(x + 4)^2}{41} + \frac{4(y + 3)^2}{41} = 1\).

The center is \((-4, -3)\). The major axis is \(2\sqrt{41}\) and is horizontal. The minor axis is \(\sqrt{41}\).

20. \(x^2 - 6x + y^2 + 10y = 2\)

Group the \(x\) and \(y\) terms; we don’t need to factor: \((x^2 - 6x) + (y^2 + 10y) = 2\).

Complete the squares: \((x^2 - 6x + 9) + (y^2 + 10y + 25) = 2 + 9 + 25\).

Rewrite: \((x - 3)^2 + (y + 5)^2 = 36\). This is not an ellipse; it’s a circle with center \((3, -5)\) and radius 6.

In problems 21-30, you are given a description of an ellipse. Write the equation of the ellipse in standard form.

21. The center is \((0, 0)\); the major axis is 8 and is horizontal; the minor axis is 6.

The standard form for an ellipse is: \(\frac{(x - h)^2}{a_x^2} + \frac{(y - k)^2}{a_y^2} = 1\), where the ellipse is centered at the point \((h, k)\), the horizontal axis is \(2a_x\), and the vertical axis is \(2a_y\). In this case: \(\frac{x^2}{16} + \frac{y^2}{9} = 1\).

22. The center is \((1, 3)\); the major axis is 10 and is vertical; the minor axis is 2.

\(\frac{(x - 1)^2}{1} + \frac{(y - 3)^2}{25} = 1\). It’s OK to write this: \((x - 1)^2 + \frac{(y - 3)^2}{25} = 1\.

23. The center is \((-2, 7)\); the major axis is 16 and is horizontal; the minor axis is 6.

\(\frac{(x + 2)^2}{64} + \frac{(y - 7)^2}{9} = 1\)

24. The center is \((4, -1)\); the major axis is 14 and is horizontal; the minor axis is 12.

\(\frac{(x - 4)^2}{49} + \frac{(y + 1)^2}{36} = 1\)
25. The center is \((0, -4)\); the major axis is \(2\sqrt{2}\) and is vertical; the minor axis is 2.

\[
\frac{x^2}{1} + \frac{(y + 4)^2}{2} = 1. \text{ It's OK to write this: } x^2 + \frac{(y + 4)^2}{2} = 1
\]

26. The center is \((1, -1)\); the major axis is 1 and is vertical; the minor axis is \(\frac{2}{5}\).

\[
\frac{(x - 1)^2}{1/25} + \frac{(y + 1)^2}{1/4} = 1; \text{ so } 25(x - 1)^2 + 4(y + 1)^2 = 1.
\]

27. The center is \((2, 0)\); the major axis is \(\frac{2\sqrt{3}}{3}\) and is horizontal; the minor axis is \(\frac{1}{2}\).

\[
\frac{(x - 2)^2}{1/3} + \frac{y^2}{1/16} = 1; \text{ so } 3(x - 2)^2 + 16y^2 = 1.
\]

28. The center is \((0, 1)\); the major axis is 6 and is vertical; the minor axis is \(\frac{2\sqrt{5}}{5}\).

\[
\frac{x^2}{1/5} + \frac{(y - 1)^2}{9} = 1; \text{ so } 5x^2 + \frac{(y - 1)^2}{9} = 1.
\]

29. The center is \((5, -2)\); the major axis is 14 and is horizontal; the minor axis is 6.

\[
\frac{(x - 5)^2}{49} + \frac{(y + 2)^2}{9} = 1.
\]

30. The center is \((-1, -3)\); the major axis is 4 and is horizontal; the minor axis is \(2\sqrt{3}\).

\[
\frac{(x + 1)^2}{4} + \frac{(y + 3)^2}{3} = 1.
\]
31. Which of the ellipses has the largest eccentricity?

(a) ![Ellipse A](image1)
(b) ![Ellipse B](image2)
(c) ![Ellipse C](image3)
(d) ![Ellipse D](image4)

You don’t need to calculate eccentricity for this. Just remember that eccentricity is a measure of how elongated the ellipse is. If the eccentricity is near zero, then the ellipse is close to being a circle; if the eccentricity is close to 1, then the ellipse is stretched along one axis and flattened along the other. Since (c) has the highest ratio of major to minor axis, it must have the greatest eccentricity.

32. Which of the ellipses has the smallest eccentricity?

(a) ![Ellipse A](image5)
(b) ![Ellipse B](image6)
(c) ![Ellipse C](image7)
(d) ![Ellipse D](image8)

The smaller the eccentricity of an ellipse, the closer it is to being a circle. Here (b) is the most circle-like of the ellipses, so it’s the one with the smallest eccentricity.
In problems 33-41, you are given the equation or the graph of an ellipse. Calculate the distance between the foci. Remember that if the major axis is \(2a\), the minor axis is \(2b\), and the distance between foci is \(2c\), then \(a^2 = b^2 + c^2\).

33. \(\frac{x^2}{9} + \frac{y^2}{25} = 1\)

The semimajor axis of this ellipse is \(a = 5\); the semiminor axis is \(b = 3\). Then \(c = \sqrt{5^2 - 3^2} = 4\); so the distance between the foci is \(2c = 8\).

34. \(\frac{(x + 2)^2}{9} + \frac{(y - 7)^2}{5} = 1\)

The semimajor axis is \(a = 3\); the semiminor axis is \(b = \sqrt{5}\). Hence \(c = \sqrt{3^2 - 5} = 2\). The distance between foci is \(2c = 4\).

35. \((x + 2)^2 + \frac{y^2}{10} = 1\)

The semimajor axis is \(a = \sqrt{10}\); the semiminor axis is \(b = 1\). Hence \(c = \sqrt{10 - 1} = 3\). The distance between foci is \(2c = 6\).

36. \(4x^2 + y^2 = 1\)

The semimajor axis is \(a = 1\); the semiminor axis is \(b = \frac{1}{2}\). Hence \(c = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}\). The distance between foci is \(2c = \sqrt{3}\).

37. \(\frac{(x + 4)^2}{2} + \frac{(y - 8)^2}{5} = 1\)

The semimajor axis is \(a = \sqrt{5}\); the semiminor axis is \(b = \sqrt{2}\). Hence \(c = \sqrt{5 - 2} = \sqrt{3}\). The distance between foci is \(2c = 2\sqrt{3}\).

38. \[\text{Image of an ellipse}\]

From the graph, the semimajor axis is \(a = 4\) and the semiminor axis is \(b = 3\). Then \(c = \sqrt{4^2 - 3^2} = \sqrt{7}\). The distance between foci is \(2c = 2\sqrt{7}\).
39. The semimajor axis is \( a = 5 \) and the semiminor axis is \( b = 3 \). Hence \( c = \sqrt{5^2 - 3^2} = 4 \). The distance between foci is \( 2c = 8 \).

40. The semimajor axis is \( a = 2 \) and the semiminor axis is \( b = 1 \). Hence \( c = \sqrt{2^2 - 1^2} = \sqrt{3} \). The distance between foci is \( 2c = 2\sqrt{3} \).

41. The major axis extends from \(-4\) to \(8\); so \( 2a = 12 \); so the semimajor axis is \( a = 6 \). The minor axis extends from \(-4\) to \(2\); so \( 2b = 6 \); so the semiminor axis is \( b = 3 \). Hence \( c = \sqrt{6^2 - 3^2} = 3\sqrt{3} \). The distance between foci is \( 2c = 6\sqrt{3} \).

In problems 42-50, you are given the equation or the graph of an ellipse. Calculate the eccentricity \( e \). Remember that if the major axis is \( 2a \) and the distance between foci is \( 2c \), then the eccentricity is \( e = c/a \).

42. \( \frac{x^2}{4} + y^2 = 1 \)

The semimajor axis is \( a = 2 \); the semiminor axis is \( b = 1 \). Hence \( c = \sqrt{2^2 - 1^2} = \sqrt{3} \); so \( e = \frac{c}{a} = \frac{\sqrt{3}}{2} \).
43. \( \frac{(x + 2)^2}{25} + \frac{(y - 9)^2}{16} = 1 \)

The semimajor axis is \( a = 5 \); the semiminor axis is \( b = 4 \). Hence \( c = \sqrt{5^2 - 4^2} = 3 \); so \\
\( e = \frac{c}{a} = \frac{3}{5} \).

44. \( \frac{(x + 3)^2}{169} + \frac{y^2}{25} = 1 \)

The semimajor axis is \( a = 13 \); the semiminor axis is \( b = 5 \). Hence \( c = \sqrt{13^2 - 5^2} = 12 \); so \\
\( e = \frac{12}{13} \).

45. \( \frac{x^2}{3} + \frac{y^2}{2} = 1 \)

The semimajor axis is \( a = \sqrt{3} \); the semiminor axis is \( b = \sqrt{2} \). Hence \( c = \sqrt{3 - 2} = 1 \); so \\
\( e = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \).

46. \( x^2 + 4y^2 = 1 \)

The semimajor axis is \( a = 1 \); the semiminor axis is \( b = \frac{1}{2} \). Hence \( c = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} \); so \\
\( e = \frac{c}{a} = \frac{\sqrt{3}}{2} \).

47.

The semimajor axis is \( a = 3 \); the semiminor axis is \( b = 2 \). Hence \( c = \sqrt{3^2 - 2^2} = \sqrt{5} \); so \\
\( e = \frac{\sqrt{5}}{3} \).
48. The major axis extends from $-8$ to $2$; so the semimajor axis is $a = 5$. The minor axis extends from $-4$ to $4$; so the semiminor axis is $b = 4$. Hence $c = \sqrt{5^2 - 4^2} = 3$; so $e = \frac{3}{5}$.

49. The major axis extends from $-6$ to $8$, so the semimajor axis is $a = 7$. The minor axis extends from $0$ to $8$, so the semiminor axis is $b = 4$. Hence $c = \sqrt{7^2 - 4^2} = \sqrt{33}$. Hence $e = \frac{\sqrt{33}}{7}$.

50. The major axis extends from $-14$ to $6$, so the semimajor axis is $a = 10$. The minor axis extends from $-6$ to $4$, so the semiminor axis is $b = 5$. Hence $c = \sqrt{10^2 - 5^2} = 5\sqrt{3}$; so $e = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$. 