Useful formulas: if a parabola has equation $(x - h) = a(y - k)^2$ or $(y - k) = a(x - h)^2$, then $1/4a$ is the distance from the vertex to the focus, and also from the vertex to the directrix.

In problems 1-15, you are given the equation of a parabola. Find the vertex, focus, and directrix; and say whether the parabola opens upward, downward, to the left, or to the right. If the equation does not describe a parabola, say so.

1. \( y = x^2 \)
   Rewrite the equation in the form \( (y - k) = a(x - h)^2 \): \( (y - 0) = 1(x - 0)^2 \). The vertex is (0, 0). Since the equation has the form \( (y - k) = a(x - h)^2 \), the parabola opens either upward or downward; since \( a = 1 \) is positive, it opens upward. This means that the focus is above the vertex and the directrix is below. The distance from the vertex to the focus and from the vertex to the directrix is $1/4a = 1/4$. Hence the focus is (0, 1/4) and the directrix is $y = -1/4$.

2. \( y = -2x^2 \)
   This equation has the form \( (y - k) = a(x - h)^2 \); we rewrite it as \( (y - 0) = -2(x - 0)^2 \). The vertex is (0, 0). Since \( a = -2 \) is negative, the parabola opens downward; the focus is below the vertex and the directrix is above. The distance from the vertex to the focus and from the vertex to the directrix is $1/4a = -1/8$. Hence the focus is (0, -1/8) and the directrix is $y = 1/8$.

3. \( x = \frac{1}{4} y^2 \)
   Rewrite this equation: \( (x - 0) = \frac{1}{4} (y - 0)^2 \). The vertex is (0, 0). Since the \( y \) term is squared, this parabola opens either to the left or to the right; since \( a = 1/4 \) is positive, it opens to the right. This means that the focus is to the right of the vertex, and the directrix is to the left. The distance from the vertex to the focus and from the vertex to the directrix is $1/4a = 1$. Hence the focus is (1, 0) and the directrix is $x = -1$.

4. \( x = -\frac{2}{5} y^2 \)
   Rewrite this equation: \( (x - 0) = -\frac{2}{5} (y - 0)^2 \). The vertex is (0, 0). Since the \( y \) term is squared, the parabola opens to the left or to the right; since \( a = -2/5 \) is negative, it opens to the left. This means that the focus is to the left of the vertex, and the directrix is to the right. The distance from the vertex to the focus and to the directrix is $1/4a = 5/8$. Thus the focus is $(-5/8, 0)$ and the directrix is $x = 5/8$.

5. \( y - 2 = 3(x - 5)^2 \)
   The vertex is (5, 2). Since the \( x \) term is squared, the parabola opens upward or downward; since \( a = 3 \) is positive, it opens upward. The distance from the vertex to the focus and to the directrix is $1/4a = 1/12$. Hence the focus is (5, 25/12) and the directrix is $y = 23/12$. 
6. \((x + 3)^2 = \frac{1}{5}(y - 1)^2\)

This is not a parabola: both the \(x\) and the \(y\) term are squared.

7. \(y + 4 = -\frac{3}{16}(x + 5)^2\)

The vertex is \((-5, -4)\). Since the \(x\) term is squared, the parabola opens upward or downward. Since \(a = -3/16\) is negative, it opens downward. The distance from the vertex to the focus and to the directrix is \(1/4a = 4/3\). The focus is \((-5, -16/3)\) and the directrix is \(y = -8/3\).

8. \(y - 1 = 5(x + 1)^2\)

The vertex is \((-1, 1)\). Since the \(x\) term is squared, the parabola opens upward or downward. Since \(a = 5\) is positive, it opens upward. The distance from the vertex to the focus and to the directrix is \(1/4a = 1/20\). The focus is \((-1, 21/20)\) and the directrix is \(y = 19/20\).

9. \(x = \frac{5}{12}(y - 2)^2\)

The vertex is \((0, 2)\). The \(y\) term is squared, so the parabola opens to the right or to the left; since \(a = 5/12\) is positive, it opens to the right. The distance from the vertex to the focus and to the directrix is \(1/4a = 3/5\). The focus is \((3/5, 2)\) and the directrix is \(x = -3/5\).

10. \(y + 3 = -\frac{7}{20}x^2\)

The vertex is \((0, -3)\). The \(x\) term is squared, so the parabola opens upward or downward; \(a = -7/20\) is negative, so it opens downward. The distance from the vertex to the focus and to the directrix is \(1/a = 5/7\). The focus is \((0, -26/7)\) and the directrix is \(y = -16/7\).

11. \(x - 7 = \frac{1}{16}(y + 1)\)

This is not a parabola, since neither the \(x\) nor the \(y\) term is squared.

12. \(x + 2 = -(y + 7)^2\)

The vertex is \((-2, -7)\). The \(y\) term is squared and \(a = -1\) is negative, so the parabola opens to the left. The distance from the vertex to the focus and to the directrix is \(1/a = 1/4\). The focus is \((-9/4, -7)\) and the directrix is \(x = -7/4\).

13. \(y = \frac{2}{3}(x - 3)^2\)

The vertex is \((3, 0)\). The \(x\) term is squared and \(a = 2/3\) is positive, so the parabola opens upward. The distance from the vertex to the focus and to the directrix is \(1/4a = 3/8\). The focus is \((3, 3/8)\) and the directrix is \(y = -3/8\).
14. \( y - \frac{2}{5} = -\frac{1}{7} \left( x + \frac{1}{3} \right)^2 \)

The vertex is \((-1/3, 2/5)\). The \(x\) term is squared and \(a = -1/7\) is negative, so the parabola opens downward. The distance from the vertex to the focus and to the directrix is \(1/4a = 7/4\). The focus is \((-1/3, -27/20)\) and the directrix is \(y = 43/20\).

15. \( x + \frac{1}{8} = 2 \left( y - \frac{1}{6} \right)^2 \)

The vertex is \((-1/8, 1/6)\). The \(y\) term is squared and \(a = 2\) is positive, so the parabola opens to the right. The distance from the vertex to the focus and to the directrix is \(1/4a = 1/8\). The focus is \((0, 1/6)\) and the directrix is \(x = -1/4\).

**In problems 16-27, find the equation of the parabola with the given features. Your answer should be in the form \((x - h) = a(y - k)^2\) or \((y - k) = a(x - h)^2\).**

16. The vertex is \((0, 0)\); the focus is \((0, 1)\).
The focus is above the vertex, so the parabola has the form \((y - k) = a(x - h)^2\), and \(a\) is positive. The distance from the focus to the vertex is \(p = 1\); so \(a = 1/4p = 1/4\). The equation is:
\[
(y - 0) = \frac{1}{4}(x - 0)^2,
\]
which we can simplify as: \(y = \frac{1}{4}x^2\).

17. The vertex is \((0, 0)\); the focus is \((1, 0)\)
The focus is to the right of the vertex, so the parabola has the form \((x - h) = a(y - k)^2\), and \(a\) is positive. The vertex is \((0, 0)\), so \(h = 0\) and \(k = 0\). The distance from the focus to the vertex is \(p = 1\), so \(a = 1/4p = 1/4\). The equation is: \(x = \frac{1}{4}y^2\).

18. The vertex is \((0, 0)\); the focus is \((0, -1)\)
The focus is below the vertex, so the parabola has the form \((y - k) = a(x - h)^2\), and \(a\) is negative. The vertex is \((0, 0)\), so \(h = 0\) and \(k = 0\). The distance from the focus to the vertex is \(p = -1\), so \(a = 1/4p = -1/4\). The equation is: \(y = -\frac{1}{4}x^2\).

19. The vertex is \((1, 5)\); the focus is \((1/2, 5)\)
The focus is to the left of the vertex, so the parabola has the form \((x - h) = a(y - k)^2\), with \(a\) negative. The vertex is \((1, 5)\), so \(h = 1\) and \(k = 5\). The distance from the vertex to the focus is \(p = -1/2\), so \(a = 1/4p = -1/2\). The equation is: \((x - 1) = -\frac{1}{2}(y - 5)^2\).
20. The vertex is (0, 0); the directrix is \( y = -2 \)

The directrix is below the vertex, so the parabola has the form \( (y - k) = a(x - h)^2 \), and \( a \) is positive. The vertex is (0, 0), so \( h = 0 \) and \( k = 0 \). The distance from the directrix to the vertex is \( p = 2 \), so \( a = 1/4p = 1/8 \). The equation is: \( y = \frac{1}{8} x^2 \).

21. The vertex is (1, 3); the directrix is \( x = 1/2 \)

The directrix is to the left of the vertex, so the parabola has the form \( (x - h) = a(y - k)^2 \) and \( a \) is positive. Since the vertex is (1, 3), \( h = 1 \) and \( k = 3 \). The distance from the directrix to the vertex is \( p = 1/2 \), so \( a = 1/4p = 1/2 \). The equation is: \( (x - 1) = \frac{1}{2} (y - 3)^2 \).

22. The vertex is \((-2, 6)\); the directrix is \( y = 25/4 \)

The directrix is above the vertex, so the parabola has the form \( (y - k) = a(x - h)^2 \), and \( a \) is negative. The vertex is \((-2, 6)\), so \( h = -2 \) and \( k = 6 \). The distance from the directrix to the vertex is \( p = -1/4 \), so \( a = 1/4p = -1 \). The equation is: \( (y - 6) = -(x + 2)^2 \).

23. The vertex is \((2, 1)\); the directrix is \( x = -1 \)

The directrix is to the left of the vertex, so the parabola has the form \( (x - h) = a(y - k)^2 \) with \( a \) positive. The vertex is \((2, 1)\), so \( h = 2 \) and \( k = 1 \). The distance from the directrix to the vertex is \( p = 3 \), so \( a = 1/4p = 1/12 \). The equation is: \( (x - 2) = \frac{1}{12} (y - 1)^2 \).

24. The focus is \((0, 0)\); the directrix is \( x = 2 \)

The vertex is halfway between the focus and the directrix, so is \((1, 0)\). Since the focus is to the left of the directrix, the parabola has the form \( (x - h) = a(y - k)^2 \), with \( a \) negative. The vertex is \((1, 0)\), so \( h = 1 \) and \( k = 0 \). The distance from the vertex to the focus is \( p = -1 \), so \( a = 1/p = -1/4 \). The equation is: \( (x - 1) = -\frac{1}{4} y^2 \).

25. The focus is \((-3, 1)\); the directrix is \( y = 2 \)

The vertex is halfway between the focus and the directrix, so is \((-3, 3/2)\). Since the focus is below the vertex, the parabola has the form \( (y - k) = a(x - h)^2 \), with \( a \) negative. The vertex is \((-3, 3/2)\), so \( h = -3 \) and \( k = 3/2 \). The distance from the vertex to the focus is \( p = -1/2 \), so \( a = 1/p = -1/2 \). The equation is: \( \left( y - \frac{3}{2} \right) = -\frac{1}{2} (x + 3)^2 \).
26. The focus is (2, −1); the directrix is \(x = 3/2\)
The vertex is halfway between the focus and the directrix, so is (7/4, −1). The focus is to the right of the vertex, so the parabola has the form \((x - h) = a(y - k)^2\), with \(a\) positive. The vertex is (7/4, −1), so \(h = 7/4\) and \(k = -1\). The distance from the vertex to the focus is \(p = 1/4\), so
\[ a = 1/4p = 1. \]
The equation is:
\[ \left( x - \frac{7}{4} \right) = (y + 1)^2. \]

27. The focus is (−4, −2); the directrix is \(y = 0\)
The vertex is halfway between the focus and directrix, so is (−4, −1). The focus is below the vertex, so the parabola has the form \((y - k) = a(x - h)^2\), with \(a\) negative. The vertex is (−4, −1), so \(h = -4\) and \(k = -1\). The distance from the vertex to the focus is \(p = -1\), so \(a = 1/4p = -1/4\).
The equation is:
\[ (y + 1) = -\frac{1}{4}(x + 4)^2. \]

In problems 28–36, you are given the equation of a parabola. Rewrite it in the form
\((x - h) = a(y - k)^2\) or \((y - k) = a(x - h)^2\). If the equation does not describe a parabola, say so.

28. \( y = x^2 + 6x + 1 \)
Complete the square:
\[ y = (x + 3)^2 - 9 + 1 = (x + 3)^2 - 8; \]
so \( y + 8 = (x + 3)^2. \)

29. \( x = y^2 + 10y + 32 \)
Complete the square:
\[ x = (y + 5)^2 - 25 + 32 = (y + 5)^2 + 7; \]
so \( x - 7 = (y + 5)^2. \)

30. \( 4y = x^2 - 12x + 8 \)
Complete the square:
\[ 4y = (x - 6)^2 - 36 + 8 = (x - 6)^2 - 28; \]
so \( 4y + 28 = (y - 6)^2. \)
Divide by 4:
\[ y + 7 = \frac{1}{4}(x - 6)^2. \]

31. \( 8x = -y^2 + 2y - 9 \)
Eliminate the negative sign from \(y^2\) on the right side:
\[ -8x = y^2 - 2y + 9. \]
Complete the square:
\[ -8x = (y - 1)^2 - 1 + 9 = (y - 1)^2 + 8; \]
so \(-8x - 8 = (y - 1)^2. \)
Divide by −8:
\[ x + 1 = -\frac{1}{8}(y - 1)^2. \]
32. \( y = 5x^2 + 20x - 15 \)
Divide by 5: \( \frac{1}{5} y = x^2 + 4x - 3 \).
Complete the square: \( \frac{1}{5} y = (x + 2)^2 - 4 - 3 = (x + 2)^2 - 7 \); so \( \frac{1}{5} y + 7 = \frac{1}{5} (y + 35) = (x + 2)^2 \).
Multiply by 5: \( y + 35 = 5(x + 2)^2 \)

33. \( 3x = y^2 - 8y + 7 \)
Complete the square: \( 3x = (y - 4)^2 - 16 + 7 = (y - 4)^2 - 9 \); so \( 3x + 9 = 3(x + 3) = (y - 4)^2 \).
Divide by 3: \( x + 3 = \frac{1}{3} (y - 4)^2 \).

34. \( y^2 + 6y = 4x^2 + 9 \)
Both \( x^2 \) and \( y^2 \) occur in this equation. Not a parabola.

35. \( x^2 - 10x - 8y + 49 = 0 \)
Begin by isolating the \( y \) term: \( 8y = x^2 - 10x + 49 \).
Complete the square: \( 8y = (x - 5)^2 - 25 + 49 = (x - 5)^2 + 24 \); so \( 8y - 24 = 8(y - 3) = (x - 5)^2 \).
Divide by 8: \( y - 3 = \frac{1}{8} (x - 5)^2 \).

36. \( y^2 = 6x - 8y + 14 \)
Isolate the \( x \) term: \( 6x = y^2 + 8y - 14 \).
Complete the square: \( 6x = (y + 4)^2 - 16 - 14 = (y + 4)^2 - 30 \); so \( 6x + 30 = 6(x + 5) = (y + 4)^2 \).
Divide by 6: \( x + 5 = \frac{1}{6} (y + 4)^2 \).

In problems 37-46, you are given the equation of a parabola. Find the vertex, focus, and directrix, and whether the parabola opens upward, downward, to the left, or to the right. If the equation does not describe a parabola, say so.

37. \( 12y = x^2 - 4x - 56 \)
Complete the square: \( 12y = (x - 2)^2 - 4 - 56 = (x - 2)^2 - 60 \); so \( 12y + 60 = (x - 2)^2 \).
Divide by 12: \( y + 5 = \frac{1}{12} (x - 2)^2 \).
The vertex is \( (2, -5) \). \( a = 1/12 \) is positive, so the parabola opens upward. The distance from the vertex to the focus and from the directrix to the vertex is \( p = 1/4a = 3 \); so the focus is \( (2, -2) \) and the directrix is \( y = -8 \).
38. \[4x = -y^2 - 2y + 43\]

Make 1 the coefficient of \(y^2\):  
\[-4x = y^2 + 2y - 43.\]

Complete the square:  
\[-4x = (y + 1)^2 - 1 - 43 = (y + 1)^2 - 44; \text{ so } -4x + 44 = (y + 1)^2.\]

Divide by -4:  
\[x - 11 = \frac{1}{4}(y + 1)^2.\]

The vertex is \((11, -1)\). Since \(a = -1/4\) is negative, the parabola opens to the left. The distance \(p\) from the vertex to the focus and from the directrix to the vertex is \(1/4a = -1/4\); so the focus is \((10, -1)\) and the directrix is \(x = 12\).

39. \[y^2 = 20x - 2y + 39\]

Isolate the \(x\) term on one side of the equation:  
\[20x = y^2 + 2y - 39.\]

Complete the square:  
\[20x = (y + 1)^2 - 1 - 39 = (y + 1)^2 - 40; \text{ so } 20x + 40 = (y + 1)^2.\]

Divide by 20:  
\[x + 2 = \frac{1}{20}(y + 1)^2.\]

The vertex is \((-2, -1)\). Since \(a = 1/20\) is positive, the parabola opens to the right. The distance \(p\) from the vertex to the focus and from the directrix to the vertex is \(1/4a = 5/20\); so the focus is \((3, -1)\) and the directrix is \(x = -7\).

40. \[x = 2y^2 + 3x^2 + 12y - 5\]

This equation has both \(x^2\) and \(y^2\) terms. Not a parabola.

41. \[x^2 = 10x + 2y - 21\]

Isolate the \(y\) term on one side:  
\[2y = x^2 - 10x + 21.\]

Complete the square:  
\[2y = (x - 5)^2 - 25 + 21 = (x - 5)^2 - 4; \text{ so } 2y + 4 = (x - 5)^2.\]

Divide by 2:  
\[y + 2 = \frac{1}{2}(x - 5)^2.\]

The vertex is \((5, -2)\). Since \(a = 1/2\) is positive, the parabola opens upward. The distance \(p\) from the vertex to the focus and from the directrix to the vertex is \(1/4a = 1/2\); so the focus is \((5, -3/2)\) and the directrix is \(y = -5/2\).

42. \[y^2 - 16y + 12x - 20 = 0\]

Isolate the \(x\) term on one side:  
\[-12x = y^2 - 16y - 20.\]

Complete the square:  
\[-12x = (y - 8)^2 - 64 - 20 = (y - 8)^2 - 84; \text{ so } -12x + 84 = (y - 8)^2.\]

Divide by -12:  
\[x - 7 = -\frac{1}{12}(y - 8)^2.\]

The vertex is \((7, 8)\). Since \(a = -1/12\) is negative, the parabola opens to the left. The distance \(p\) from the vertex to the focus and from the directrix to the vertex is \(1/4a = -3\); so the focus is \((4, 8)\) and the directrix is \(x = 10\).
43. \( x^2 + 9 = 6x + y \)
Isolate the \( y \) term on one side: \( y = x^2 - 6x + 9 \).
Complete the square: \( y = (x - 3)^2 - 9 + 9 = (x - 3)^2 \).
The vertex is \((3, 0)\). Since \( a = 1 \) is positive, the parabola opens upward. The distance \( p \) from the vertex to the focus and from the directrix to the vertex is \( 1/4a = 1/4 \); so the focus is \((3, 1/4)\) and the directrix is \( y = -1/4 \).

44. \( x^2 + 64 = 8x - 24y \)
Isolate the \( y \) term: \(-24y = x^2 - 8x + 64\).
Complete the square: \(-24y = (x - 4)^2 - 16 + 64 = (x - 4)^2 + 48 \); so \(-24y - 48 = (x - 4)^2 \).
Divide by \(-24\): \(y + 2 = -\frac{1}{24}(x - 4)^2 \).
The vertex is \((4, -2)\). Since \( a = -1/24 \) is negative, the parabola opens downward. The distance \( p \) from the vertex to the focus and from the directrix to the vertex is \( 1/4a = -6 \); so the focus is \((4, -8)\) and the directrix is \( y = 4 \).

45. \( y^2 + 12y + 8x + 28 = 0 \)
Isolate the \( x \) term: \(-8x = y^2 + 12y + 28 \).
Complete the square: \(-8x = (y + 6)^2 - 36 + 28 = (y + 6)^2 - 8 \); so \(-8x + 8 = (y + 6)^2 \).
Divide by \(-8\): \(x - 1 = -\frac{1}{8}(y + 6)^2 \).
The vertex is \((1, -6)\). Since \( a = -1/8 \) is negative, the parabola opens to the left. The distance \( p \) between the vertex and focus and between the directrix and vertex is \( 1/4a = -2 \); so the focus is \((-1, -6)\) and the directrix is \( x = 3 \).

46. \( 9y^2 - 36y = 3x - 35 \)
To avoid dividing 35 by 9, we’ll begin by isolating the \( y \) terms: \(3x - 35 = 9y^2 - 36y \).
Factor out the coefficient of \( x^2 \): \(3x - 35 = 9(y^2 - 4y) \).
Complete the square within the parentheses: \(3x - 35 = 9[(y - 2)^2 - 4] = 9(y - 2)^2 - 36 \).
Isolate the square term: \(3x + 1 = 9(y - 2)^2 \).
Divide by the coefficient of \( x \): \(x + \frac{1}{3} = 3(y - 2)^2 \).
The vertex is \((-1/3, 2)\). Since \( a = 3 \) is positive, the parabola opens to the right. The distance \( p \) between the vertex and focus and between the directrix and vertex is \( 1/4a = 1/12 \). Hence the focus is \((-1/4, 2)\) and the directrix is \( x = -5/12 \).
47. Which equation matches the graph at right?
(a) \( y + 3 = (x - 1)^2 \)
* (b) \( y - 3 = (x + 1)^2 \)
(c) \( x + 3 = (y - 1)^2 \)
(d) \( x - 3 = (y + 1)^2 \)

The vertex of the graphed parabola is \((-1, 3)\). Only (b) has that vertex. Notice also that the graphs of (c) and (d) would open to the right, not upward.

48. Which equation matches the graph at right?
(a) \( y = x^2 \)
(b) \( y = -x^2 \)
(c) \( x = y^2 \)
* (d) \( x = -y^2 \)

All four equations give vertex \((0, 0)\), so that’s no help. However, (a) opens upward, (b) opens downward, and (c) opens to the right. Only (d) opens to the left.

49. Which equation matches the graph at right?
(a) \( y - 1 = (x + 2)^2 \)
* (b) \( y - 1 = -(x + 2)^2 \)
(c) \( y + 1 = (x - 2)^2 \)
(d) \( y + 1 = -(x - 2)^2 \)

The vertex in the graph is \((-2, 1)\), which excludes (c) and (d): they have their vertex at \((2, -1)\). Of (a) and (b), (a) opens upward and (b) opens downward; so (b) must be the answer.
50. Which graph matches the equation: $x - 2 = -\frac{1}{2} (y + 3)^2$?

The $y$ term is squared, and $a = -1/2$ is negative. The graph must open to the left. Only (b) satisfies that condition. (We can also exclude (c) and (d) because the vertex given by the equation is $(2, -3)$, which is in quadrant IV.)

(a)  

(b)  

*(b)  

(c)  

(d)
51. Which graph matches the equation: $y + 1 = 3(x - 2)^2$?

All of the graphs open upward, so that’s no help. The vertex given by the equation is $(2, -1)$, which is in quadrant IV. Only (a) satisfies that requirement.
52. Which graph matches the equation: \( x + 1 = \frac{1}{2} (y - 3)^2 \) ?

In the equation the \( y \) term is squared, and \( a = \frac{1}{2} \) is positive. The graph must open to the right. Only (a) satisfies that. (All four graphs have the vertex in quadrant II, so that’s no help to us.)

*(a)  

(b)  

(c)  

(d)
53. Which graph matches the equation: $x^2 = 4x + y - 1$?

The equation has $y$ on one side; the coefficient of $x^2$ on the other is positive. That means that the graph opens upward, which rules out (c) and (d). However, we need to put the equation in standard form to decide whether the correct answer is (a) or (b). Isolate the $y$ term: $y = x^2 - 4x + 1$.

Complete the square: $y = (x - 2)^2 - 4 + 1 = (x - 2)^2 - 3; \text{ so } y + 3 = (x - 2)^2$.

The vertex is $(2, -3)$. (a) has its vertex in quadrant I, which is wrong; only (b) opens upward and has its vertex in quadrant IV.
54. A cannon is fired at time \( t = 0 \). At time \( t \), the height of the cannonball above the ground is 
\[ y = 160t - 16t^2. \]
What is the maximum height reached by the cannonball, and at what time \( t \) does it reach that height? Distances are given in feet.
We need to find \( t \) and \( y \) at the vertex of the parabola. To do this, we put the equation in standard form.
Divide by the coefficient of \( t^2 \): \[ \frac{1}{16}y = t^2 - 10t. \]
Complete the square: \[ \frac{1}{16}y = (t - 5)^2 - 25; \] so \[ \frac{1}{16}(y - 400) = (t - 5)^2. \]
This is enough to give us the vertex: \((5, 400)\). The maximum height is 400 ft, and it’s reached at \( t = 5 \) sec.

55. A golf ball is hit from the point \((0, 0)\). The height of the ball \( y \) as a function of the horizontal distance \( x \) is:
\[ y = \frac{x}{2} - \frac{x^2}{1600}. \]
What is the maximum height reached by the golf ball, and at what horizontal distance does it reach that height? Distances are given in feet.
We need to find the vertex \((x, y)\).
Divide by the coefficient of \( x^2 \): \[ -1600y = x^2 - 800x \]
Complete the square: \[ -1600y = (x - 400)^2 - 160000; \] so 
\[ -1600y + 160000 = -1600(y - 100) = (x - 400)^2. \]
This is enough to give us the vertex: \((400, 100)\). The maximum height of the ball is 100 ft, and it’s reached when the horizontal distance is 400 ft.

56. The mirror of a telescope has a parabolic cross-section:
\[ y = \frac{1}{60}x^2. \]
What is the distance from the vertex to the focus? Distance are given in inches.
The equation is already in standard form; the vertex is \((0, 0)\). The distance from the vertex to the focus is \( p = 1/4a \), where \( a \) is the coefficient of \( x^2 \). Hence \( p = 15 \) in.

57. The mirror of a telescope has a parabolic cross section:
\[ y = ax^2. \]
If the focus is 120 cm from the vertex, what is \( a \)?
The equation is already in standard form; the vertex is \((0, 0)\). The distance from the vertex to the focus is \( p = 1/4a \); so \( a = 1/4p = 1/480 \).