Useful formulas: if a parabola has equation \((x - h) = a(y - k)^2\) or \((y - k) = a(x - h)^2\), then \(1/4a\) is the distance from the vertex to the focus, and also from the vertex to the directrix.

In problems 1-15, you are given the equation of a parabola. Find the vertex, focus, and directrix; and say whether the parabola opens upward, downward, to the left, or to the right. If the equation does not describe a parabola, say so.

1. \(y = x^2\)
2. \(y = -2x^2\)
3. \(x = \frac{1}{4}y^2\)
4. \(x = -\frac{2}{5}y^2\)
5. \(y - 2 = 3(x - 5)^2\)
6. \((x + 3)^2 = \frac{1}{5}(y - 1)^2\)
7. \(y + 4 = -\frac{3}{16}(x + 5)^2\)
8. \(y - 1 = 5(x + 1)^2\)
9. \(x = \frac{5}{12}(y - 2)^2\)
10. \(y + 3 = -\frac{7}{20}x^2\)
11. \(x - 7 = \frac{1}{16}(y + 1)\)
12. \(x + 2 = -(y + 7)^2\)
13. \(y = \frac{2}{3}(x - 3)^2\)
14. \[ y - \frac{2}{5} = -\frac{1}{7} \left( x + \frac{1}{3} \right)^2 \]

15. \[ x + \frac{1}{8} = 2 \left( y - \frac{1}{6} \right)^2 \]

In problems 16–27, find the equation of the parabola with the given features. Your answer should be in the form \((x - h) = a(y - k)^2\) or \((y - k) = a(x - h)^2\).

16. The vertex is (0, 0); the focus is (0, 1).
17. The vertex is (0, 0); the focus is (1, 0).
18. The vertex is (0, 0); the focus is (0, -1).
19. The vertex is (1, 5); the focus is (1/2, 5).
20. The vertex is (0, 0); the directrix is \(y = -2\).
21. The vertex is (1, 3); the directrix is \(x = 1/2\).
22. The vertex is (-2, 6); the directrix is \(y = 25/4\).
23. The vertex is (2, 1); the directrix is \(x = -1\).
24. The focus is (0, 0); the directrix is \(x = 2\).
25. The focus is (-3, 1); the directrix is \(y = 2\).
26. The focus is (2, -1); the directrix is \(x = 3/2\).
27. The focus is (-4, -2); the directrix is \(y = 0\).

In problems 28–36, you are given the equation of a parabola. Rewrite it in the form \((x - h) = a(y - k)^2\) or \((y - k) = a(x - h)^2\). If the equation does not describe a parabola, say so.

28. \[ y = x^2 + 6x + 1 \]
29. \[ x = y^2 + 10y + 32 \]
30. \[ 4y = x^2 - 12x + 8 \]
31. $8x = -y^2 + 2y - 9$

32. $y = 5x^2 + 20x - 15$

33. $3x = y^2 - 8y + 7$

34. $y^2 + 6y = 4x^2 + 9$

35. $x^2 - 10x - 8y + 49 = 0$

36. $y^2 = 6x - 8y + 14$

In problems 37-46, you are given the equation of a parabola. Find the vertex, focus, and directrix, and whether the parabola opens upward, downward, to the left, or to the right. If the equation does not describe a parabola, say so.

37. $12y = x^2 - 4x - 56$

38. $4x = -y^2 - 2y + 43$

39. $y^2 = 20x - 2y + 39$

40. $x = 2y^2 + 3x^2 + 12y - 5$

41. $x^2 = 10x + 2y - 21$

42. $y^2 - 16y + 12x - 20 = 0$

43. $x^2 + 9 = 6x + y$

44. $x^2 + 64 = 8x - 24y$

45. $y^2 + 12y + 8x + 28 = 0$

46. $9y^2 - 36y = 3x - 35$
47. Which equation matches the graph at right?
(a) \( y + 3 = (x - 1)^2 \)
(b) \( y - 3 = (x + 1)^2 \)
(c) \( x + 3 = (y - 1)^2 \)
(d) \( x - 3 = (y + 1)^2 \)

48. Which equation matches the graph at right?
(a) \( y = x^2 \)
(b) \( y = -x^2 \)
(c) \( x = y^2 \)
(d) \( x = -y^2 \)

49. Which equation matches the graph at right?
(a) \( y - 1 = (x + 2)^2 \)
(b) \( y - 1 = -(x + 2)^2 \)
(c) \( y + 1 = (x - 2)^2 \)
(d) \( y + 1 = -(x - 2)^2 \)
50. Which graph matches the equation: \( x - 2 = -\frac{1}{2}(y + 3)^2 \)?

(a) 
\[ \text{Graph} \]
(b) 
\[ \text{Graph} \]
(c) 
\[ \text{Graph} \]
(d) 
\[ \text{Graph} \]

51. Which graph matches the equation: \( y + 1 = 3(x - 2)^2 \)?

(a) 
\[ \text{Graph} \]
(b) 
\[ \text{Graph} \]
(c) 
\[ \text{Graph} \]
(d) 
\[ \text{Graph} \]
52. Which graph matches the equation: \( x + 1 = \frac{1}{2}(y - 3)^2 \)?

(a) \( \text{Image} \)  

(b) \( \text{Image} \)  

(c) \( \text{Image} \)  

(d) \( \text{Image} \)  

53. Which graph matches the equation: \( x^2 = 4x + y - 1 \)?

(a) \( \text{Image} \)  

(b) \( \text{Image} \)  

(c) \( \text{Image} \)  

(d) \( \text{Image} \)
54. A cannon is fired at time $t = 0$. At time $t$, the height of the cannonball above the ground is $y = 160t - 16t^2$. What is the maximum height reached by the cannonball, and at what time $t$ does it reach that height? Distances are given in feet.

55. A golf ball is hit from the point (0, 0). The height of the ball $y$ as a function of the horizontal distance $x$ is: $y = \frac{x}{2} - \frac{x^2}{1600}$. What is the maximum height reached by the golf ball, and at what horizontal distance does it reach that height? Distances are given in feet.

56. The mirror of a telescope has a parabolic cross-section: $y = \frac{1}{60}x^2$. What is the distance from the vertex to the focus? Distance are given in inches.

57. The mirror of a telescope has a parabolic cross section: $y = ax^2$. If the focus is 120 cm from the vertex, what is $a$?