Math 182: Trigonometry

Practice Quiz 6

This practice quiz will give you an idea of the format and difficulty level of the actual quiz. Like the actual quiz, it consists of 26 questions drawn from the problem sets and modified slightly (e.g. by changing the numbers). Any problem on the problem sets can appear on the actual quiz; it is not enough just to know how to solve the problems on this practice quiz.

No partial credit will be given for wrong answers. You will be graded on the best 25 of the 26 questions, so you can get one answer wrong without penalty. You cannot score more than 100% on the quiz.

Read the instructions carefully and follow them exactly. Your answers on the answer page must exactly match the solutions, or they will be graded as wrong.

If the question asks that you round the answer to a certain number of decimal places, you must do so. Your answer will be considered wrong if it is rounded incorrectly, or to the wrong number of decimal places.

Unless you’re told to round your answer, it should be exact. Some things that can cause you to lose credit for an exact answer are:

- Failure to simplify fractions. $\frac{15}{20}$ is wrong; $\frac{3}{4}$ is correct.

- Failure to rationalize denominators. $\frac{2}{\sqrt{3}}$ is wrong; $\frac{2\sqrt{3}}{3}$ is correct.

- Failure to simplify radicals. $\sqrt{18}$ is wrong; $3\sqrt{2}$ is correct.

- Using decimal approximations instead of exact answers. $1.0472$ is wrong; $\frac{\pi}{3}$ is correct.

On problems where the answer involves units of measure (e.g. “23 ft”), you won’t lose credit for not including the units. However, it’s a good idea to do so. Your answer must be in the correct units: if, for example, the correct answer is “30°”, then “$\pi/6$” will be graded as wrong.

**No partial credit will be given.** If, for example, you get $r$ right and $\theta$ wrong, you will not get half credit.

Your answers must appear in the correct format on the answer sheet. If no answer or a wrong answer appears there, the grader will not check the page with the question to see if you’ve answered it correctly there. That means it’s a good idea to double-check at the end of the quiz and make sure that you’ve copied the answers correctly and in the right places on the answer sheet.
<p>| | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1.</td>
<td>(4.2406, 5.6076)</td>
<td>10.</td>
<td>a</td>
<td>19.</td>
</tr>
<tr>
<td>2.</td>
<td>$9 + 4i$</td>
<td>11.</td>
<td>(11.7047, 199.98°)</td>
<td>20.</td>
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<tr>
<td>3.</td>
<td>$2 (\cos \frac{\pi}{15} + i \sin \frac{\pi}{15})$</td>
<td>12.</td>
<td>$3x - 2y = 5$</td>
<td>21.</td>
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<td>4.</td>
<td>$\frac{1}{5} + \frac{2}{5}i$</td>
<td>13.</td>
<td>(10, 150°)</td>
<td>22.</td>
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<tr>
<td>5.</td>
<td>(-1.3351, 5.8496)</td>
<td>14.</td>
<td>(-3.5166, 8.2845)</td>
<td>23.</td>
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<tr>
<td>6.</td>
<td>125 (cos 2π/3 + i sin 2π/3)</td>
<td>15.</td>
<td>(0, 12)</td>
<td>24.</td>
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<tr>
<td>7.</td>
<td>$(7\sqrt{2}, \frac{3\pi}{4})$</td>
<td>16.</td>
<td>$r = 5$</td>
<td>25.</td>
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<tr>
<td>8.</td>
<td>$\left(\sqrt{29}, \tan^{-1} \frac{5}{2}\right)$</td>
<td>17.</td>
<td>$\sqrt{3} + i, -\sqrt{3} - i$</td>
<td>26.</td>
</tr>
<tr>
<td>9.</td>
<td>$\sqrt{17}$</td>
<td>18.</td>
<td>$6 + 4i$</td>
<td></td>
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This quiz consists of 26 multiple-choice and short-answer questions. There will be no partial credit for wrong answers. You will be graded on the best 25 of the 26, so you can get one question wrong without penalty. You cannot get a higher grade than 100% on this quiz.

Write **only your answers** in the spaces provided on the answer page. Circle the correct answers for multiple-choice questions. Do not do your work on the answer page.

Write your answers in exactly the format that the question asks for. If, for example, you round to the wrong number of decimal places, or fail to rationalize denominators or simplify fractions in exact solutions, your answer will be graded as wrong.

Unless otherwise indicated, your answers should be exact. Rationalize all denominators and simplify all fractions.

Your answer must be in the correct units of measure. If, for example, the problem asks for an angle in degrees, then an answer given in radians would be considered wrong.

**No partial credit will be given.** If, for example, you get \( r \) right and \( \theta \) wrong, you will not get half credit.

Your grade will be based on the answers that you write **on the answer page**. If you have a wrong answer or no answer on the answer page, the grader will not look at the page with the question to see if the correct answer appears there. Illegible or ambiguous answers will be graded as wrong. You are responsible for copying your answers clearly, correctly, and in the appropriate blanks.

You must show your work on the page with the question. Credit will not be given for lucky guesses.
1. Use a calculator or equivalent to convert $(3.309, -2.652)$ from rectangular to polar coordinates. Round $r$ to the nearest 0.0001 and $\theta$ to the nearest 0.0001 rad. Your value of $r$ should be positive; $\theta$ should be in the range $[0, 2\pi)$.

\[
r = \sqrt{3.309^2 + 2.652^2} = 4.2406. \quad \theta \text{ is in quadrant IV and } \tan \theta = -2.652/3.309; \text{ so } \theta = 2\pi - \arctan(2.652/3.309) = 5.6076 \text{ rad. Hence } (r, \theta) = (4.2406, 5.6076)
\]

2. $u = 4 - 3i$. $v = 5 + 7i$. Find $u + v$.

\[
(4 + 5) + (-3 + 7)i = 9 + 4i
\]

3. Find the fifth roots of $z = 32 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$. Give the root with the smallest argument greater than 0. Express your answer in trigonometric form.

The modulus of a fifth root is $32^{1/5} = 2$. The arguments of fifth roots have the form: $\frac{\pi}{15} + \frac{2n\pi}{5}$. The smallest of these greater than 0 is $\pi/15$; so the root asked for is: $2 \left( \cos \frac{\pi}{15} + i \sin \frac{\pi}{15} \right)$.

4. Find the quotient $\frac{2 + i}{4 - 3i}$. Express your answer in standard form.

\[
\frac{2 + i}{4 - 3i} \cdot \frac{4 + 3i}{4 + 3i} = \frac{8 + 11i}{25} = \frac{8}{25} + \frac{11}{25}i
\]

5. Use a calculator or equivalent to convert $(6, 4\pi/7)$ from polar to rectangular coordinates. Round your values to the nearest 0.0001.

\[
(x, y) = (6 \cos 4\pi/7, 6 \sin 4\pi/7) = (-1.3351, 5.8496)
\]

6. $z = 5 \left( \cos 2\pi/9 + i \sin 2\pi/9 \right)$. Find $z^3$. Express your answer in trigonometric form, with the argument in the range $[0, 2\pi)$.

The modulus of $z^3$ is $5^3 = 125$. The argument of $z^3$ is $3(2\pi/9) = 2\pi/3$. Hence in trigonometric form, $z^3 = 125 \left( \cos 2\pi/3 + i \sin 2\pi/3 \right)$.

7. Convert $(-7, 7)$ from rectangular to polar coordinates. Your answer should be exact, with $\theta$ in the range $[0, 2\pi)$. If necessary, express $\theta$ in terms of an arc function of a positive number.

\[
r = \sqrt{(-7)^2 + 7^2} = 7\sqrt{2}. \quad \theta \text{ is in quadrant II and } \tan \theta = -1, \text{ so } \theta = 3\pi/4. \text{ Hence } (r, \theta) = (7\sqrt{2}, 3\pi/4)
\]

8. Convert $(2, 5)$ from rectangular to polar coordinates. Your answer should be exact. If necessary, express $\theta$ in terms of an arc function of a positive number.

\[
r = \sqrt{2^2 + 5^2} = \sqrt{29}. \quad \theta \text{ is in the first quadrant and } \tan \theta = 5/2, \text{ so } \theta = \arctan(5/2). \text{ Hence }
\]

\[
(r, \theta) = \left( \sqrt{29}, \tan^{-1} \frac{5}{2} \right)
\]

9. Find the absolute value of $4 + i$. Your answer should be exact.

\[
|4 + i| = \sqrt{4^2 + 1^2} = \sqrt{17}
\]
10. Which equation corresponds to the graph at right?
(a) \( r \sin \theta = 2 \)
(b) \( r = \cos 2\theta \)
(c) \( r = 2 \)
(d) \( \theta = 2 \)
(e) \( r = 2\theta \)

The graph is \( y = 2 \); in polar coordinates, that’s \( r \sin \theta = 2 \). Hence: (a)

11. Use a calculator or equivalent to convert \((-11, -4)\) from rectangular to polar coordinates. Round \( r \) to the nearest 0.0001 and \( \theta \) to the nearest 0.01°. Your value of \( r \) should be positive; \( \theta \) should be in the range \([0°, 360°)\).

\[
r = \sqrt{11^2 + 4^2} = \sqrt{137} = 11.7047.
\]
\[
\theta = 180° + \frac{180°}{\pi} \tan^{-1} \frac{4}{11} = 199.98°.
\]
Hence \((r, \theta) = (11.7047, 199.98°)\)

12. Convert the equation: \( r(3 \cos \theta - 2 \sin \theta) = 5 \) from polar to rectangular coordinates. \( r \cos \theta = x \) and \( r \sin \theta = y \). Hence: \( 3x - 2y = 5 \)

13. Convert \((-5\sqrt{3}, 5)\) from rectangular to polar coordinates. Your value of \( r \) should be positive; \( \theta \) should be in the range \([0°, 360°)\).

\[
r = \sqrt{(5\sqrt{3})^2 + 5^2} = 10.
\]
\[
\theta = \tan^{-1} \frac{5}{5\sqrt{3}} = \frac{\sqrt{3}}{3}; \text{ so } \theta = 150°.
\]
Hence \((r, \theta) = (10, 150°)\)

14. Use a calculator to convert \((9, 113°)\) from polar to rectangular coordinates. Round your values to the nearest 0.0001.

\((x, y) = (9 \cos 113°, 9 \sin 113°) = (-3.5166, 8.2845)\)

15. Convert \((12, \pi/2)\) from polar to rectangular coordinates. Your answer should be exact.

\((x, y) = (12 \cos \pi/2, 12 \sin \pi/2) = (0, 12)\)

16. Convert the equation: \( x^2 + y^2 = 25 \) from rectangular to polar coordinates. Simplify your answer as much as possible.

\[x^2 + y^2 = r^2 = 25; \text{ so } r = 5. \] [We don’t need to add “or \( r = -5 \),” because \( r = 5 \) and \( r = -5 \) describe the same set of points: \((5, \theta) = (-5, \pi + \theta)\).]
17. Find both square roots of \( z = 2 + 2\sqrt{3}i \). Give your answer in standard form. In trigonometric form, \( z = 4 \left( \cos \frac{\pi}{3} \pm i \sin \frac{\pi}{3} \right) \). The modulus of a square root is \( 4^{1/2} = 2 \); the argument has the form \( \frac{\pi}{6} + n\pi \). In trigonometric form, the two square roots are 
\( 2 \left( \cos \frac{\pi}{6} \pm i \sin \frac{\pi}{6} \right) \) and \( 2 \left( \cos \frac{7\pi}{6} \pm i \sin \frac{7\pi}{6} \right) \). In standard form, these are \( \sqrt{3} + i \) and \( -\sqrt{3} - i \).

18. \( u = 4 + 9i \). \( v = -2 + 5i \). Find \( u - v \).
\( u - v = (4 - (-2)) + (9 - 5)i = 6 + 4i \)

19. Express \( 4 - 4\sqrt{3}i \) in trigonometric form. Your answer should be exact, with the argument in the range \([0, 2\pi)\).
The modulus is \( \sqrt{4^2 + (4\sqrt{3})^2} = 8 \). The argument \( \theta \) is in quadrant IV and 
\( \tan \theta = -\frac{4\sqrt{3}}{4} = -\sqrt{3} \); so \( \theta = 5\pi/3 \). Hence: \( 8 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) \)

20. \( u = 3 - 2i \). \( v = 5 + i \). Find \( uv \).
\( uv = ((3)(5) - (-2)(1)) + ((3)(1) + (5)(-2))i = 17 - 7i \)

21. Find the complex conjugate of \( -4 - 5i \).
\( -4 + 5i \)

22. Find \( i^{42} \).
\( i^{42} = i^{40}i^2 = -1 \)

23. \( u = 15 \left( \cos \frac{9\pi}{7} + i \sin \frac{9\pi}{7} \right) \). \( v = 5 \left( \cos \frac{3\pi}{7} + i \sin \frac{3\pi}{7} \right) \). Find \( u/v \). Express your answer in trigonometric form, with the argument in the range \([0, 2\pi)\).
The modulus of \( u/v \) is \( 15/5 = 3 \). The argument of \( u/v \) is \( 9\pi/7 - 3\pi/7 = 6\pi/7 \). Hence
\( u/v = 3 \left( \cos \frac{6\pi}{7} + i \sin \frac{6\pi}{7} \right) \)

24. Express \( 7\sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \) in standard form. Your answer should be exact.
\( 7\sqrt{2} \left( \frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) = -7 + 7i \)

25. \( u = 3 \left( \cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right) \). \( v = 7 \left( \cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5} \right) \). Find \( uv \). Express your answer in trigonometric form, with the argument in the range \([0, 2\pi)\).
The modulus of \( uv \) is \( (3)(7) = 21 \). The argument of \( uv \) is \( \pi/5 + 7\pi/5 = 8\pi/5 \). Hence
\( uv = 21 \left( \cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5} \right) \)

26. Convert \( (12, 300^\circ) \) from polar to rectangular coordinates. Your answer should be exact.
\((x, y) = (12 \cos 300^\circ, 12 \sin 300^\circ) = \left(\frac{12}{2}, -\frac{12\sqrt{3}}{2}\right) = (6, -6\sqrt{3})\)