Answer key for exam 1 (mathematical prerequisites)

1. a b c d e 21. a b c d e 41. a b c d e
2. a b c d e 22. a b c d e 42. a b c d e
3. a b c d e 23. a b c d e 43. a b c d e
4. a b c d e 24. a b c d e 44. a b c d e
5. a b c d e 25. a b c d e 45. a b c d e
6. a b c d e 26. a b c d e 46. a b c d e
7. a b c d e 27. a b c d e 47. a b c d e
8. a b c d e 28. a b c d e 48. a b c d e
9. a b c d e 29. a b c d e 49. a b c d e
10. a b c d e 30. a b c d e 50. a b c d e
11. a b c d e 31. a b c d e 51. a b c d e
12. a b c d e 32. a b c d e 52. a b c d e
13. a b c d e 33. a b c d e 53. a b c d e
14. a b c d e 34. a b c d e 54. a b c d e
15. a b c d e 35. a b c d e 55. a b c d e
16. a b c d e 36. a b c d e 56. a b c d e
17. a b c d e 37. a b c d e 57. a b c d e
18. a b c d e 38. a b c d e 58. a b c d e
19. a b c d e 39. a b c d e 59. a b c d e
20. a b c d e 40. a b c d e 60. a b c d e
Geometry

Problem 1. (Circles) If the diameter of a circle is $D$, what is its radius?

(a) $\pi D^2/2$  
(b) $2\pi D^2$  
*(c) $D/2$  
(d) $2D$  
(e) None of these

Solution: $D = 2R \Rightarrow R = D/2$

Problem 2. (Circles) If the area of a circle is $A$, what is its circumference?

(a) $2\pi \sqrt{A}$  
*(b) $2\sqrt{\pi A}$  
(c) $\sqrt{\frac{A}{\pi}}$  
(d) $\sqrt{\frac{2A}{\pi}}$  
(e) None of these

Solution: $A = \pi R^2$ and $C = 2\pi R$; so $R = \sqrt{\frac{A}{\pi}} \Rightarrow C = 2\pi \sqrt{\frac{A}{\pi}} = 2\sqrt{\pi A}$

Problem 3. (Rectangles) What is the area of a rectangular windowpane measuring 20 cm wide by 25 cm high?

(a) 50 cm$^2$  
(b) 80 cm$^2$  
(c) 125 cm$^2$  
*(d) 500 cm$^2$  
(e) None of these

Solution: $A = lw = (20 \text{ cm})(25 \text{ cm}) = 500 \text{ cm}^2$

Problem 4. (Rectangles) What is the volume of a rectangular room measuring 4 m long by 5 m wide by 3 m high?

(a) 12 m$^3$  
(b) 24 m$^3$  
(c) 30 m$^3$  
*(d) 60 m$^3$  
(e) None of these

Solution: $V = lwh = (4 \text{ m})(5 \text{ m})(3 \text{ m}) = 60 \text{ m}^3$

Problem 5. (Triangles) Find $r$ in the right triangle at right.

(a) 2  
(b) 3  
(c) 4  
*(d) 5  
(e) None of these

Solution: $r$ is a length, so it must be positive. Hence

$$r^2 = x^2 + y^2 \quad \Rightarrow \quad r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$$

This is the 3-4-5 triangle.
Problem 6. (Triangles) Find \( y \) in the right triangle at right.

(a) \( \sqrt{3} \)  
(b) 9 
*(c) \( \sqrt{21} \)  
(d) \( \sqrt{29} \)  
(e) None of these

Solution: \( y \) is a length, so it must be positive. Hence

\[
r^2 = x^2 + y^2
\]

\[
-x^2 \quad y^2 = r^2 - x^2
\]

\[
\sqrt{r^2 - x^2} = \sqrt{(5)^2 - (2)^2} = \sqrt{25 - 4} = \sqrt{21}
\]

Problem 7. (Triangles) In the right triangle at right, which of the following equations is true?

(a) \( b = \frac{1}{2} c + a \)  
(b) \( b = \frac{1}{2} c - a \)  
(c) \( b = \sqrt{c^2 + a^2} \)  
*(d) \( b = \sqrt{c^2 - a^2} \)  
(e) None of these

Solution: \( c^2 = a^2 + b^2 \implies b^2 = c^2 - a^2 \implies b = \sqrt{c^2 - a^2} \)

Problem 8. (Applications with triangles) An airplane flies a route involving three cities. From Rio Cordaro, it flies 100 miles straight east to Hackerville. From Hackerville, it flies 200 miles straight north to San Pitucco. How far does it fly from San Pitucco to Rio Cordaro?

(a) 225 miles  
(b) \( 100\sqrt{3} \) miles  
(c) 250 miles  
*(d) \( 100\sqrt{5} \) miles  
(e) None of these

Solution: Sketch a diagram, as at right. The route forms a right triangle, with the legs from Rio Cordaro to Hackerville and from Hackerville to San Pitucco on either side of the right angle. The leg from San Pitucco to Rio Cordaro is the hypotenuse. Use the theorem of Pythagoras:

\[
r^2 = x^2 + y^2
\]

\[
\sqrt{r^2} = \sqrt{x^2 + y^2} = \sqrt{(100 \text{ mi})^2 + (200 \text{ mi})^2} = \sqrt{50,000 \text{ mi}^2} = 100\sqrt{5} \text{ mi}
\]
Problem 9. (Applications with triangles) A blimp is attached to a cable whose other end is fastened to the ground. The wind is strong enough to pull the cable into a straight line. When you are standing directly below the blimp, you are 4 km from the place where the cable is anchored in the ground. The blimp is 3 km above the ground. How long is the cable?

(a) 1 km  
(b) $\sqrt{5}$ km  
(c) $\sqrt{7}$ km  
(d) 5 km  
(e) None of these

Solution: We have a sketch already; we can label the sides that we know and the side that we want to find. The system is a right triangle whose horizontal side is 4 km and whose vertical side is 3 km; we want to know the hypotenuse $r$. Use Pythagoras:

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(4 \text{ km})^2 + (3 \text{ km})^2} = \sqrt{25 \text{ km}} = 5 \text{ km}$$

Problem 10. (Applications with triangles) A physics instructor has locked himself out of his office, and tries to climb in through an upper window. He leans a 4-meter ladder against the side of the building so that the top of the ladder is 3 m above the ground. How far from the building is the bottom of the ladder?

(a) 1 m  
(b) $\sqrt{7}$ m  
(c) $\sqrt{12}$ m  
(d) 5 m  
(e) None of these

Solution: Sketch a diagram, as at right. The ground, the building, and the ladder form a right triangle whose vertical side is 3 m and whose hypotenuse is 4 m. We want to know the horizontal side $x$. Use the theorem of Pythagoras:

$$r^2 = x^2 + y^2$$

$$x^2 = r^2 - y^2$$

$$x = \sqrt{r^2 - y^2} = \sqrt{(4 \text{ m})^2 - (3 \text{ m})^2} = \sqrt{7} \text{ m}$$
Algebra

Problem 11. If \( f(x) = \frac{x^2 + 1}{3} \), find \( f(2) \).

(a) 5/9  
(b) 1  
* (c) 5/3  
(d) 3  
(e) None of these

**Solution:** \( f(2) = \frac{2^2 + 1}{3} = \frac{4 + 1}{3} = \frac{5}{3} \)

Problem 12. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(1) \).

(a) \(-2\)  
(b) 2  
(c) \(-\sqrt{6}\)  
* (d) No real value  
(e) None of these

**Solution:** \( f(1) = \sqrt{1^2 - 5} = \sqrt{1 - 5} = \sqrt{-4} \). Since the quantity under the square root sign is negative, there is no real value.

Problem 13. If \( f(x) = x^2 - 3 \), find \( f(a - 1) \).

* (a) \( a^2 - 2a - 2 \)  
(b) \( a^2 - 8a + 16 \)  
(c) \( a^2 - 6a + 8 \)  
(d) \( a^2 - 8a - 16 \)  
(e) None of these

**Solution:** \( f(a - 1) = (a - 1)^2 - 3 = a^2 - 2a + 1 - 3 = a^2 - 2a - 2 \)

Problem 14. If \( f(x) = (x - 2)^2 \), find \( f(a - 1) \).

(a) \( a^2 - 2a - 1 \)  
* (b) \( a^2 - 6a + 9 \)  
(c) \( a^2 - 4a + 3 \)  
(d) \( a^2 + 4a + 4 \)  
(e) None of these

**Solution:** \( f(a - 1) = ((a - 1) - 2)^2 = (a - 3)^2 = a^2 - 6a + 9 \)

Problem 15. If \( f(x) = 3x + 2 \), find \( f(a + 1) \).

(a) \( 3a + 1 \)  
(b) \( 3a + 2 \)  
(c) \( 3a + 3 \)  
* (d) \( 3a + 5 \)  
(e) None of these

**Solution:** \( f(a + 1) = 3(a + 1) + 2 = 3a + 3 + 2 = 3a + 5 \)

Problem 16. If \( f(x) = 3x + 2 \), find \( f(a - 3) \).

(a) \( 3a - 3 \)  
* (b) \( 3a - 7 \)  
(c) \( 3a - 13 \)  
(d) \( 3a - 15 \)  
(e) None of these

**Solution:** \( f(a - 3) = 3(a - 3) + 2 = 3a - 9 + 2 = 3a - 7 \)
Problem 17. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 20, and the gravitational acceleration is 10, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

* (a) 20 (b) 40
(c) $10\sqrt{2}$ (d) $20\sqrt{2}$
(e) None of these

Solution: $v = \sqrt{2gh} = \sqrt{2(10)(20)} = \sqrt{400} = 20$

Problem 18. The formula for the centripetal acceleration of an object moving in a circle is: $a = \frac{v^2}{r}$, where $v$ is the object’s speed and $r$ is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 6 at a speed of 3, what is its centripetal acceleration?

(a) $\frac{1}{4}$ (b) $\frac{3}{2}$
(c) 4 (d) 12
(e) None of these

Solution: $a = \frac{v^2}{r} = \frac{3^2}{6} = \frac{9}{6} = \frac{3}{2}$

Problem 19. The position of a moving object is given by the formula: $x = x_0 + vt$, where $x_0$ is the initial position, $v$ is the velocity, and $t$ is the time. What is the position of an object if $x_0 = 5$, $v = 2$, and $t = 3$?

* (a) 11 (b) 21
(c) 25 (d) 30
(e) None of these

Solution: $x = x_0 + vt = 5 + (2)(3) = 11$

Problem 20. The speed of an object is given by the formula: $v = v_0 + at$, where $v_0$ is the initial speed, $a$ is the acceleration, and $t$ is the time. What is the speed of an object if $v_0 = 5$, $a = 2$, and $t = 3$?

* (a) 11 (b) 13
(c) 25 (d) 30
(e) None of these

Solution: $v = v_0 + at = 5 + (2)(3) = 5 + 6 = 11$
Problem 21. If $3x + 11 = 6$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:** 
\[
3x + 11 = 6 
\rightarrow \quad 3x = -5
\quad \div 3
\rightarrow \quad x = -\frac{5}{3}
\]

Hence $x \leq -1$.

Problem 22. If $3x - 2 = -9$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:** 
\[
3x - 2 = -9 
\rightarrow \quad 3x = -7
\quad \div 3
\rightarrow \quad x = -\frac{7}{3}
\]

Hence $x < -1$.

Problem 23. If $-4x + 5 = 8$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:** 
\[
-4x + 5 = 8 
\rightarrow \quad -4x = 3
\quad \div (-4)
\rightarrow \quad x = -\frac{3}{4}
\]

Hence $-1 \leq x < 0$.

Problem 24. If $x + 7 = -4x + 9$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:** 
We need to collect all the terms with $x$ on one side of the equation, and all of the terms with no $x$ on the other.

\[
x + 7 = -4x + 9 
\rightarrow \quad 5x + 7 = 9
\quad \div 7
\rightarrow \quad 5x = 2
\quad \div 5
\rightarrow \quad x = \frac{2}{5}
\]

Hence $0 \leq x < 1$.
Problem 25. If $2x - 4 = 5x + 7$, find $x$. Which of the following statements is true?

*(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these

**Solution:** We need to collect all the terms with $x$ on one side of the equation, and all of the terms with no $x$ on the other.

\[
2x - 4 = 5x + 7 \quad \rightarrow \quad -3x - 4 = 7 \quad \rightarrow \quad -3x = 11 \quad \rightarrow \quad x = -\frac{11}{3}
\]

Hence $x < -1$

Problem 26. If $-3x - 5 = 2x - 11$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
*(d) $x \geq 1$
(e) None of these

**Solution:** We need to collect all the terms with $x$ on one side of the equation, and all of the terms with no $x$ on the other.

\[
-3x - 5 = 2x - 11 \quad \rightarrow \quad -5 = 5x - 11 \quad \rightarrow \quad 5x = 6 \quad \rightarrow \quad x = \frac{6}{5}
\]

Hence $x \geq 1$

Problem 27. If $PV = nRT$, find $P$.

(a) $P = \frac{V}{nRT}$  
*(b) $P = \frac{nRT}{V}$
(c) $P = V - nRT$  
(d) $P = nRT - V$
(e) None of these

**Solution:** $PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V}$

Problem 28. If $PV = nRT$, find $T$.

(a) $T = \frac{nR}{PV}$  
*(b) $T = \frac{PV}{nR}$
(c) $T = nR - PV$  
(d) $T = PV - nR$
(e) None of these

**Solution:** $PV = nRT \quad \rightarrow \quad nRT = PV \quad \rightarrow \quad T = \frac{PV}{nR}$
Problem 29. If \( v = v_0 + at \), find \( v_0 \).

(a) \( v_0 = \frac{v}{at} \)  
(b) \( v_0 = -\frac{v}{at} \)  
(c) \( v_0 = v + at \)  
*(d) \( v_0 = v - at \)  
(e) None of these

Solution: \( v = v_0 + at \) \text{ switch sides } \Rightarrow \( v_0 + at = v \) \text{ switch } \Rightarrow \( v_0 = v - at \)

Problem 30. If \( v = a(t - t_0) \), find \( t \).

(a) \( t = \frac{a - v}{t_0} \)  
* (b) \( t = \frac{v + at_0}{a} \)  
(c) \( t = v - at_0 \)  
(d) \( t = v + at_0 \)  
(e) None of these

Solution: Begin by expanding to get rid of the parentheses:

\[ v = a(t - t_0) = at - at_0 \] \text{ switch } \Rightarrow \( at = v + at_0 \) \text{ switch } \Rightarrow \( t = \frac{v + at_0}{a} \)

Problem 31. If \( x = a(t - t_0) \), find \( t_0 \).

* (a) \( t_0 = \frac{at - x}{a} \)  
(b) \( t_0 = \frac{x + at}{a} \)  
(c) \( t_0 = \frac{x - t}{a} \)  
(d) \( t_0 = \frac{x + t}{a} \)  
(e) None of these

Solution: It’s a good idea to begin by expanding the expression to get rid of the parentheses:

\[ x = a(t - t_0) = at - at_0 \] \text{ switch } \Rightarrow \( at_0 = at - x \) \text{ switch } \Rightarrow \( t_0 = \frac{at - x}{a} \)
Problem 32. For the following pair of equations, find $x$ and $y$:

\begin{align*}
6x - y &= -12 \\
-5x + 2y &= -4
\end{align*}

What is the product $xy$?

(a) $xy = -48$  
(b) $xy = -60$

*(c) $xy = 48$  
(d) $xy = 60$

(e) None of these

We’ll multiply the first equation by 2 and then add the two equations; that will let us eliminate $y$.

\begin{align*}
6x - y &= -12 & \times 2 & & 12x - 2y &= -24 \\
-5x + 2y &= -4 & \Rightarrow & & -5x + 2y &= -4 \\
\text{add} & & & & 7x + 0y &= -28 \Rightarrow x = -4
\end{align*}

Now we can substitute this value into either of the original two equations and solve for $y$. Since the coefficient of $y$ in the first equation is $-1$, it’s easiest to use that one:

\begin{align*}
6x - y &= -12 & +y+12 &\Rightarrow y &= 6x + 12 = 6(-4) + 12 = -12
\end{align*}

Then $xy = (-4)(-12) = 48$.

Problem 33. For the following pair of equations, find $x$ and $y$:

\begin{align*}
x + 3y &= -1 \\
-2x - 5y &= 5
\end{align*}

What is the product $xy$?

(a) $xy = 24$  
(b) $xy = 30$

(c) $xy = -24$  
*(d) $xy = -30$

(e) None of these

Solution: One way of solving this is to get rid of $x$ by multiplying the first equation by 2, then adding the two equations:

\begin{align*}
x + 3y &= -1 & \times 2 & & 2x + 6y &= -2 \\
-2x - 5y &= 5 & \Rightarrow & & -2x - 5y &= 5 \\
\text{add} & & & & 0x + y &= 3 \Rightarrow y = 3
\end{align*}

Now we can substitute this into either of the original two equations and solve for $x$. Since the coefficient of $x$ in the first equation is 1, that’s probably the easiest to use:

\begin{align*}
x + 3y &= -1 & -3y &\Rightarrow x = -3y - 1 = -3(3) - 1 = -10
\end{align*}

Hence $xy = (-10)(3) = -30$. 
Problem 34. Solve the equation: \(x^2 + 4x - 21 = 0\). There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) What is the difference \(x_1 - x_2\)?

(a) \(x_1 - x_2 = 0\)  
(b) \(x_1 - x_2 = 4\)  
(c) \(x_1 - x_2 = 10\)  
* (c) \(x_1 - x_2 = 10\)  
(d) \(x_1 - x_2 = 17\)  
(e) None of these

**Solution:** We can solve this by factoring, then by setting each factor equal to zero.

\[
x^2 + 4x - 21 = 0
\]

Factor: \((x - 3)(x + 7) = 0\)

Set factors equal to zero: \(x - 3 = 0\) or \(x + 7 = 0\)

Solve: \(x_1 = 3\) and \(x_2 = -7\)

Answer: \(x_1 - x_2 = 3 - (-7) = 10\)

Problem 35. Solve the equation: \(3x^2 - 3x = 1\)

(a) \(x = \frac{3 \pm \sqrt{3}}{2}\)  
(b) \(x = \frac{3 \pm \sqrt{21}}{6}\)  
(c) \(x = \frac{1}{3}\) or \(x = 1\)  
(d) \(x = 1\) or \(x = 2\)  
(e) No real solution

**Solution:** Begin by subtracting 1 from both sides of the equation, so that one side is zero: \(3x^2 - 3x - 1 = 0\). Now use the quadratic formula with \(a = 3\), \(b = -3\), and \(c = -1\).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{(-3)^2 - 4(3)(-1)}}{2(3)} = \frac{3 \pm \sqrt{21}}{6}
\]

Problem 36. Solve the equation: \(2x^2 - 5x = -1\)

(a) \(x = -1\) or \(x = 2\)  
(b) \(x = \frac{5 \pm \sqrt{17}}{4}\)  
(c) \(x = \frac{-5 \pm \sqrt{33}}{2}\)  
(d) \(x = \frac{-2 \pm \sqrt{23}}{2}\)  
(e) No real solution

**Solution:** First, we need an equation with zero on one side. We can then use the quadratic formula.

\[
2x^2 - 5x = -1 \quad \overset{+1}{\Rightarrow} \quad 2x^2 - 5x + 1 = 0
\]

Now we have an equation of the form \(ax^2 + bx + c = 0\), where \(a = 2\), \(b = -5\), and \(c = 1\). Use the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)} = \frac{5 \pm \sqrt{17}}{4}
\]
Problem 37. Solve the equation: $3x^2 - 4x = 7$. There are two solutions, $x_1$ and $x_2$, with $x_1 \geq x_2$. (It is possible that $x_1 = x_2$.) What is the difference $x_1 - x_2$?

(a) $x_1 - x_2 = 0$  (b) $x_1 - x_2 = \frac{1}{3}$

(c) $x_1 - x_2 = \frac{4}{3}$  * (d) $x_1 - x_2 = \frac{10}{3}$

(e) None of these

Solution: First, we need an equation with zero on one side. We can then solve by factoring, then by setting each factor equal to zero.

\[3x^2 - 4x = 7\]
\[-\rightarrow\]
\[3x^2 - 4x - 7 = 0\]

Factor: $(3x - 7)(x + 1) = 0$

Set factors equal to zero: $3x - 7 = 0$ or $x + 1 = 0$

Solve: $x_1 = \frac{7}{3}$ and $x_2 = -1$

Answer: $x_1 - x_2 = \frac{7}{3} - (-1) = \frac{7}{3} + \frac{3}{3} = \frac{10}{3}$

Problem 38. In the physics lab, you find two circular pieces of sheet metal. The radius of one of the circles is 5 centimeters greater than the radius of the other. The area of the larger circle is three times the area of the smaller one. Which of the following equations describes the radius of the smaller circle?

(a) $r^2 - 10r + 25 = 0$  *(b) $2r^2 - 10r - 25 = 0$

(c) $r^2 - 5r + 10 = 0$  (d) $3r^2 + 5r - 10 = 0$

(e) None of these

Solution: Let $r$ be the radius of the smaller circle. Then $r + 5$ is the radius of the larger one. The area of the small circle is $\pi r^2$; the area of the large circle is $\pi (r + 5)^2$. Since the area of the large circle is three times the area of the small one,

\[\pi (r + 5)^2 = 3\pi r^2\]
\[\Rightarrow (r + 5)^2 = 3r^2\]
\[\Rightarrow r^2 + 10r + 25 = 3r^2\]
\[\Rightarrow 2r^2 - 10r - 25 = 0\]
Problem 39. You go on a ten-mile bicycle ride. You ride the first nine miles at 15 miles per hour; but then you get a flat tire and have to walk your bike the remaining mile at 3 miles per hour. What is your average speed for the trip?

(a) 5 miles per hour  (b) 9 miles per hour

*(c) $10\frac{5}{7}$ miles per hour  (d) $13\frac{4}{5}$ miles per hour

(e) None of these

Solution: Your average speed is the total distance divided by the total time. The total distance is $x = 10$ miles. The total time is the time spent riding plus the time spent walking the bike:

$$t = \frac{9 \text{ miles}}{15 \text{ miles/hr}} + \frac{1 \text{ mile}}{3 \text{ miles/hr}} = \frac{9}{15} + \frac{1}{3} = \frac{9}{15} + \frac{5}{15} = \frac{14}{15} \text{ hr}$$

Hence the average speed is

$$v_{\text{ave}} = \frac{x}{t} = \frac{10 \text{ miles}}{\frac{14}{15} \text{ miles/hr}} = 10 \cdot \frac{15}{14} = \frac{150}{14} = \frac{75}{7} = 10\frac{5}{7} \text{ miles/hr}$$

Problem 40. A boat moves at 5 miles per hour in still water. It is launched in a river that flows at 3 miles per hour. From its launch point, it goes downstream for 4 miles, then turns around and comes back upstream to the launch point. How long does the round trip take?

(a) 4 hours 5 hours  (b) 8 hours

(c) 2 hours  *(d) 5 hours  

(e) None of these

Solution: The time for the round trip is the time that it takes to go downstream at $5 + 3 = 8$ miles per hour, plus the time that it takes to come back upstream at $5 - 3 = 2$ miles per hour. Thus

$$t = \frac{4 \text{ miles}}{8 \text{ miles/hour}} + \frac{4 \text{ miles}}{2 \text{ miles/hour}} = \frac{1}{2} + 2 = \frac{5}{2} \text{ hours}$$
Problem 41. An airplane flies at a speed of 80 miles per hour in still air. On a day when the wind is blowing from the north at 20 miles per hour, the airplane flies 200 miles straight north, then turns around and returns to its starting point. What is its average speed on the round trip?

(a) 64 miles per hour  
(b) 66 2/3 miles per hour  
*(c) 75 miles per hour  
(d) 80 miles per hour  
(e) None of these

Solution: The average speed for the round trip is the total distance divided by the total time. The total distance is $2 \times 200 = 400$ miles. The total time is

$$\frac{200}{80-20} + \frac{200}{80+20} = \frac{200}{60} + \frac{200}{100} = \frac{10}{3} + 2 = \frac{16}{3}$$

hours

Hence the average speed is

$$\frac{400}{\frac{16}{3}} \text{ hours} = 400 \cdot \frac{3}{16} = 75 \text{ miles/hour}$$

Problem 42. A runner and a bicyclist start from the same point at the same time, with the runner going straight north and the bicyclist going straight south. The bicyclist is 7 miles per hour faster than the runner. At the end of two hours, the two are 60 miles apart. What is the bicyclist’s speed?

(a) 11 1/2 miles per hour  
(b) 14 miles per hour  
*(c) 18 1/2 miles per hour  
(d) 23 miles per hour  
(e) None of these

Solution: Let $b$ be the bicyclist’s speed. Then the runner’s speed is $b - 7$. Since the two are going in opposite directions, after two hours the distance between them is

$$2b + 2(b - 7) = 4b - 14 = 60 \text{ miles} \Rightarrow b = \frac{60 + 14}{4} = \frac{37}{2} = 18\frac{1}{2} \text{ miles/hour}$$

Problem 43. You drive from Smithtown to Jonesville at a speed of $v$, making the trip in time $t$. On the return trip, you are able to drive 10 miles per hour faster, which shortens your travel time by one hour. Which of the following equations is true?

(a) $(v - 10)t = v(t + 1)$  
(b) $(v + 10)t = v(t - 1)$  
(c) $(v - 10)(t + 1) = vt$  
*(d) $(v + 10)(t - 1) = vt$  
(e) None of these

Solution: On the first leg of the trip, your speed is $v$ and your time is $t$. On the return leg, your speed is $v + 10$ and your time is $t - 1$. The distance is the same in each direction; so

$$vt = (v + 10)(t - 1)$$
Graphs

Problem 44. The graph at right shows four points labelled with letters. Which of the four points is $(2, -5)$?

(a) A  (b) B
(c) C  *(d) D

Solution: Of the four choices, only $D$ has $x > 0$ and $y < 0$.

Problem 45. The graph at right shows four points labelled with letters. The points are 

$(2, 2), (3, 9), (7, 2),$ and $(8, 8)$.

Which of the four points is $(2, 2)$?

*(a) A  (b) B
(c) D  (d) C

Solution: For point $D$, the $x$-coordinate is smaller than the $y$-coordinate. For point $B$, the $x$-coordinate is larger than the $y$-coordinate. Only $A$ and $C$ have coordinates that appear to be equal. For $C$, the values are larger than for $A$; so $C = (8, 8)$ and $A = (2, 2)$.

Problem 46. Which equation is shown on the graph at right? (The scale is the same for the $x$- and $y$-axes.)

(a) $y = 3x$
(b) $y = -3x$
(c) $y = \frac{x}{3}$
*(d) $y = -\frac{x}{3}$

Solution: If the equation of a line is written in the form: $y = mx + b$, then $m$ is the slope and $b$ is the $y$-intercept. In this case, all of the possible answers have $b = 0$, so we must focus on the slope $m$.

The line in the figure has a negative slope (as $x$ increases, $y$ decreases), so it must be $m = -3$ or $m = -1/3$. Since the value of $y$ decreases more slowly than the value of $x$ increases, $m > -1$. Hence $m = -1/3$; so $y = -x/3$. 
Problem 47. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
* (c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

Solution: The graph has a positive \( y \)-intercept: it crosses the \( y \)-axis above the \( x \)-axis. This means that when \( x = 0 \), \( y > 0 \). That allows us to rule out equations (b) and (d). In the graph, when \( x \) has large absolute values, \( y < 0 \). This is consistent with (c), where the coefficient of \( x^2 \) is negative, but not with (a), where the coefficient of \( x^2 \) is positive.

Problem 48. Which equation is shown on the graph at right?

* (a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

Solution: The graph has a positive \( y \)-intercept: it crosses the \( y \)-axis above the \( x \)-axis. This means that when \( x = 0 \), \( y > 0 \). That allows us to rule out equations (b) and (d). In the graph, when \( x \) has large absolute values, \( y > 0 \). This is consistent with (a), where the coefficient of \( x^2 \) is positive, but not with (c), where the coefficient of \( x^2 \) is negative.

Problem 49. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
* (d) \( y = -x^2 - 1 \)

Solution: The graph has a negative \( y \)-intercept: it crosses the \( y \)-axis below the \( x \)-axis. This means that when \( x = 0 \), \( y < 0 \). That allows us to rule out equations (a) and (c). In the graph, when \( x \) has large absolute values, \( y < 0 \). This is consistent with (d), where the coefficient of \( x^2 \) is negative, but not with (b), where the coefficient of \( x^2 \) is positive.
Problem 50. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[ y = 2x^2 \quad y = \frac{x^2}{2} \quad y = -2x^2 \quad y = -\frac{x^2}{2} \]

Which of the four graphs represents \( y = -2x^2 \)?

Solution: All four equations have \( y \)-intercepts of zero, so that won’t help us. In graphs (c) and (d), \( y \geq 0 \) for all values of \( x \). They must represent the two equations where the coefficient of \( x^2 \) is positive. That leaves us with graphs (a) and (b), which must represent the two equations with negative coefficients of \( x^2 \). Of these two equations, the curve of \( y = -2x^2 \) will fall faster than the curve of \( y = -\frac{x^2}{2} \): for example, the first of these includes the point \((1, -2)\), whereas the second includes the point \((1, -\frac{1}{2})\). Hence the graph of \( y = -2x^2 \) is (b).