Answer key for exam 2 (mathematical preliminaries)

1. a b ç d e 21. a b c d e 41. a b c ç e
d e
e d e
2. a b ç d e 22. a b c d e 42. a b c ç e
d e
e d e
3. a b ç d e 23. a b c d e 43. a b ç d e
d e
e d e
4. a b ç d e 24. a b c ç e 44. a b ç d e
d e
e d e
5. a b ç d e 25. a b ç d e 45. a b ç d e
d e
e d e
6. a b ç d e 26. a b ç d e 46. a b ç d e
d e
e d e
7. a b ç d e 27. a b ç d e 47. a b ç d e
d e
e d e
8. a b ç d e 28. a b ç d e 48. a b ç d e
d e
e d e
9. a b ç d e 29. a b ç d e 49. a b ç d e
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10. a b ç d e 30. a b ç d e 50. a b ç d e
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11. a b ç d e 31. a b ç d e 51. a b ç d e
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12. a b ç d e 32. a b ç d e 52. a b ç d e
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13. a b ç d e 33. a b ç d e 53. a b ç d e
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14. a b ç d e 34. a b ç d e 54. a b ç d e
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15. a b ç d e 35. a b ç d e 55. a b ç d e
d e
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16. a b ç d e 36. a b ç d e 56. a b ç d e
d e
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17. a b ç d e 37. a b ç d e 57. a b ç d e
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18. a b ç d e 38. a b ç d e 58. a b ç d e
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e d e
19. a b ç d e 39. a b ç d e 59. a b ç d e
d e
e d e
20. a b ç d e 40. a b ç d e 60. a b ç d e
Mathematical Preliminaries

Introduction to Measurement

Dimensional consistency

Problem 1. Consider the formulas (i) and (ii). Are the formulas dimensionally consistent? Here $V =$ volume, $v =$ velocity, $x =$ distance, $z =$ distance, and $t =$ time.

(i) $V = x^2 z$

(ii) $v = \frac{x - z}{t}$

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent

(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent

*(c) (i) and (ii) are both dimensionally consistent

(d) (i) and (ii) are both dimensionally inconsistent

Solution: In (i), $V$ has units of $(\text{length})^3$; $x$ has units of $(\text{length})$; and $z$ has units of $(\text{length})$. In units, the formula looks like

$$(\text{length})^3 = (\text{length})^2 \cdot (\text{length})$$

We get units of $(\text{length})^3$ on both sides of the equation, so it’s dimensionally consistent.

In (ii), $v$ has units $\frac{\text{length}}{\text{time}}$; $x$ and $z$ have units of $(\text{length})$; and $t$ has units of $(\text{time})$. In units, the equation looks like

$$\frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}} + \frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}}$$

Since the units are the same on both sides, the equation is dimensionally consistent.

Problem 2. Consider the formulas (i) and (ii). Are the formulas dimensionally consistent? Here $a =$ acceleration, $v =$ velocity, $x =$ distance, and $t =$ time.

(i) $a = \frac{x}{t - t_0}$

(ii) $v = a(t - t_0)$

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent

*(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent

(c) (i) and (ii) are both dimensionally consistent

(d) (i) and (ii) are both dimensionally inconsistent

Solution: In (i), $a$ has units $\frac{\text{length}}{(\text{time})^2}$; $x$ has units of $(\text{length})$; and $t$ and $t_0$ both have units of $(\text{length})$. In units, the equation looks like

$$\frac{\text{length}}{(\text{time})^2} = \frac{\text{length}}{(\text{time})} - \frac{\text{length}}{(\text{time})}$$
Since the units are not the same on both sides, the equation is dimensionally inconsistent.

In (ii), \(v\) has units of \(\frac{\text{length}}{\text{time}}\); so in units, the equation is
\[
\frac{\text{length}}{\text{time}} = \frac{\text{length}}{(\text{time})^2} \cdot (\text{time}) = \frac{\text{length}}{\text{time}}
\]

Since the units are the same on both sides, the equation is dimensionally consistent.

**Problem 3.** Consider the formulas (i) and (ii). Are the formulas dimensionally consistent? Here \(a = \text{acceleration}, V = \text{volume}, x = \text{distance}, \) and \(t = \text{time}.

(i) \(a = \frac{x^2}{t - t_0}\)  
(ii) \(V = xy^2\)

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent  
*(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent  
(c) (i) and (ii) are both dimensionally consistent  
(d) (i) and (ii) are both dimensionally inconsistent

**Solution:**

(i) is dimensionally inconsistent \((i.e., \text{can’t add velocity and time since they are dimensionally inconsistent})\); in terms of units, it’s
\[
\frac{(\text{distance})}{(\text{time})^2} = \frac{(\text{distance})^2}{(\text{time})}
\]

(ii) is dimensionally consistent:
\[
(\text{distance})^3 = (\text{distance})(\text{distance})^2 = (\text{distance})^3
\]

**Problem 4.** Consider the formulas (i) and (ii). Are the formulas dimensionally consistent? Here \(A = \text{area}, r = \text{distance}, x = \text{distance}, t = \text{time}, \) and \(v = \text{velocity}.

(i) \(A = 4\pi r^2\)  
(ii) \(v = \frac{x}{t}\)

(a) (i) is consistent; (ii) is not  
*(b) (ii) is consistent; (i) is not  
(c) Both (i) and (ii) are consistent  
(d) Neither (i) nor (ii) is consistent  
(e) None of these

**Solution:**

In (i), \(A\) has units of \((\text{length})^2\); 4 and \(\pi\) are dimensionless; and \(r\) has dimensions of \(\text{length}\). In units, the formula looks like
\[
(\text{length})^2 = (\text{length})^2
\]

Hence (i) is dimensionally consistent.

In (ii), \(v\) has units of \(\frac{\text{length}}{\text{time}}\); \(x\) has units of \(\text{length}\); and \(t\) has units of \(\text{time}\). In units, the formula looks like
\[
\frac{\text{length}}{\text{time}} = \frac{\text{length}}{\text{time}}
\]

Hence (ii) is also dimensionally consistent.
Problem 5. Determine whether equations (i) and (ii) below are dimensionally consistent.

\[(i) \quad x = at^2 + (1 + \frac{vt}{x})^2 \quad \quad \quad \quad (ii) \quad v_f^2 = v_i^2 + 2ax^2,\]

where \(x\) = distance, \(v\) = velocity, \(t\) = time, \(a\) = acceleration, \(r\) = distance, \(g\) = magnitude of gravity and \(\mu_s\) is the coefficient of static friction.

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent
(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent
(c) (i) and (ii) are both dimensionally consistent
*(d) (i) and (ii) are both dimensionally inconsistent

Solution: (i) Dimensionally inconsistent. \([x] = [a][t^2] + (1 + [(vt)/x])^2\) becomes \(L = \frac{L^2}{T^2} + (1 + \frac{([vt]=L)}{L})^2 = L + 1\), where we’ve used the fact that \([vt]\) = distance = \(L\). Since \(L\) and 1 are dimensional inconsistent, the RHS of the equation is dimensional inconsistent.

(ii) Dimensionally inconsistent. \([v_f^2] = [v_i^2] = L^2/T^2\), but \([2ax^2] = (L/T^2)(L^2) = L^3/T^2\)

Problem 6. (Using dimensional analysis to get statistical-partial credit)
Consider the following test question and multiple-choice answers. Without knowing how to solve the problem, can you identify the answer that cannot be a solution based solely on dimensional analysis?

You are towing a crate of physics books down the hallway, using a rope that makes an angle of \(\theta\) to the horizontal. The crate has a mass of \(m\); the coefficient of kinetic friction between the crate and the floor is \(\mu_k\). As you go down the hallway, you are giving the crate a forward acceleration of \(a\). What is the tension \(T\) in the rope, expressed in terms of the mass \(m\), the acceleration \(a\), the angle \(\theta\) (dimensionless), \(\mu_k\) (dimensionless), and the gravitational constant of acceleration \(g\)? Note: Tension has units of force.

*(a) \(mg\mu_k + a\) cos \(\theta\) \quad (b) \(\frac{m(g\mu_k + a)}{\cos \theta + \mu_k \sin \theta}\)
(c) \(\frac{m(g\mu_k + a)}{\cos \theta}\) \quad (d) \((mg\mu_k + a)(\cos \theta + \mu_k \sin \theta)\)

Solution: The answer is (a). Gravity \(g\) has units of acceleration, so each term in the factor \((g\mu_k + a)\) has units of acceleration and is therefore dimensionally consistent with acceleration. The factor \((\cos \theta + \mu_k \sin \theta)\) is composed of non-dimensional terms, so it’s dimensionless. Thus, the equation in (a) has units of acceleration. The equation in (b) is mass times acceleration (a.k.a force), which is consistent. Similarly, the equations in (c) and (d) have units of force.
Units

Problem 7. To get to your physics class in time, you must bicycle across town at an average speed of 13 miles per hour. What is this speed in meters per second? Round your answer to the nearest 0.1 m/s.

*(a) 5.8 m/s  (b) 6.4 m/s
(c) 7.0 m/s  (d) 7.7 m/s
(e) None of these

Solution: Use the conversion factors: 1610 m/mi and 3600 s/hr. To make the units work out, we need to multiply by the first and the reciprocal of the second:

\[
\frac{\text{mi}}{\text{hr}} \cdot \frac{\text{m}}{\text{mi}} \cdot \left(\frac{\text{s}}{\text{hr}}\right)^{-1} = \frac{\text{mi}}{\text{hr}} \cdot \frac{\text{m}}{\text{mi}} \cdot \frac{\text{hr}}{\text{s}} = \frac{\text{m}}{\text{s}}
\]

Your speed is

\[
12 \frac{\text{mi}}{\text{hr}} \cdot \left(1610 \frac{\text{m}}{\text{mi}}\right) \left(\frac{1}{3600} \frac{\text{hr}}{\text{s}}\right) = 5.8 \frac{\text{m}}{\text{s}}
\]

Problem 8. In a desperate bid for extra credit, you install a 42-foot statue of your physics instructor at the college entrance. What is the statue’s height in meters? Round your answer to the nearest meter.

*(a) 13 m  (b) 19 m
(c) 92 m  (d) 138 m
(e) None of these

Solution: Use the conversion factor 3.28 ft/m. To make the units come out, we have to divide by the conversion factor. In units,

\[
\text{ft} \div \frac{\text{ft}}{\text{m}} = \text{ft} \cdot \frac{\text{m}}{\text{ft}} = \text{m}
\]

Hence the height of the statue is

\[
\frac{42 \text{ ft}}{3.28 \text{ ft/m}} = 13 \text{ m}
\]

Problem 9. You are given a ticket for driving at 38 miles per hour in a school zone. What is your speed in meters per second? Round your answer to the nearest m/s.

(a) 15 m/s  *(b) 17 m/s
(c) 19 m/s  (d) 20 m/s
(e) None of these

Solution: Use the conversion factors: 1610 m/mi and 3600 s/hr. To make the units work out, we need to multiply by the first and the reciprocal of the second:

\[
\frac{\text{mi}}{\text{hr}} \cdot \frac{\text{m}}{\text{mi}} \cdot \left(\frac{\text{s}}{\text{hr}}\right)^{-1} = \frac{\text{mi}}{\text{hr}} \cdot \frac{\text{m}}{\text{mi}} \cdot \frac{\text{hr}}{\text{s}} = \frac{\text{m}}{\text{s}}
\]
The speed for which you got the ticket is

\[ 38 \, \text{mi/hr} \left( 1610 \, \text{m/mi} \right) \left( \frac{1}{3600} \, \text{hr/s} \right) = 17 \, \text{m/s} \]

**Scientific notation**

**Problem 10.** Convert the number 0.0070 to scientific notation. Your answer should have the appropriate number of significant figures.

(a) \(7.0 \times 10^{-2}\)  
(b) \(7.0 \times 10^{-3}\)  
(c) \(7.0000 \times 10^{-2}\)  
(d) \(7.0000 \times 10^{-3}\)  
(e) None of these

**Problem 11.** Convert the number 97,000 to scientific notation. Your answer should have three significant figures.

(a) \(9.70 \times 10^3\)  
(b) \(9.70 \times 10^4\)  
(c) \(9.7 \times 10^3\)  
(d) \(9.7 \times 10^4\)  
(e) None of these

**Solution:** Answers (b) and (d) are both equal to 97,000, but (d) only has two significant figures.

**Problem 12.** Convert the number \(6.2 \times 10^3\) from scientific notation to standard form.

(a) \(0.0062\)  
(b) \(6200\)  
(c) \(0.062\)  
(d) \(62\),000  
(e) None of these

**Significant figures**

**Problem 13.** How many significant digits are there in: \(v = 0.0420 \, \text{m/s}\)?

(a) 2  
(b) 3  
(c) 4  
(d) 5  
(e) None of these

**Solution:** The zeros to the left of the nonzero digits are placeholders and are not significant. The rightmost zero is significant.

**Problem 14.** How many significant digits are there in: \(y = 0.0504 \, \text{cm}\)?

(a) 2  
(b) 3  
(c) 4  
(d) 5  
(e) None of these

**Solution:** The first two zeros are placeholders and are not significant. The zero between the 5 and the 4 is significant.

**Problem 15.** How many significant digits are there in: \(r = 0.001000 \, \text{m}\)?

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) None of these
Solution: The zeros to the left of the first nonzero digit are not significant since they are placeholders. All of the zeros after the nonzero digit are significant figures. There are 4 significant figures.

Problem 16. Calculate \( \frac{3.21}{55.77} + 16 \). Round your answer to the correct number of significant figures.

*(a) 16  (b) 16.0
(c) 16.1  (d) 16.06
(e) None of these

Solution: Begin by calculating the quotient: \( \frac{3.21}{55.77} \approx 0.057558 \). The numerator has three significant figures and the denominator has four. We use the smaller of the two; so our result is only good to the nearest 0.0001. If this were the whole problem, we’d round it to 0.0576.

Now we do the addition. The first term (the quotient) is accurate to 0.0001; the second term (16) is only accurate to the ones place. Since we’re adding, we have to round the sum to the decimal place of the least accurate term, which is the ones place: so we round 16.057558 to 16.

Problem 17. Calculate \( \frac{122.1}{3.2} \). Round your answer to the correct number of significant figures.

*(a) 38  (b) 38.1
(c) 38.2  (d) 38.16
(e) None of these

Solution: The numerator has four significant figures; the denominator has two. Since we’re dividing, we use the smaller of these two numbers; so we round our result to two significant digits.

Problem 18. Calculate 120.51 + 3.2. Round your answer to the correct number of significant figures.

(a) 120  (b) 124
*(c) 123.7  (d) 123.71
(e) None of these

Solution: 120.51 + 3.2 = 123.71. The 3.2 is only accurate to the tenths place; so by rule 10, the sum is also only accurate to the tenths place. Hence we round it to 123.7.
Arc length

Problem 19. An angle measures 87°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 3.04 rad  
(b) 0.38 rad  
*(c) 1.52 rad  
(d) 6.07 rad  
(e) None of these

Solution: \( \frac{87\pi}{180} = 1.52 \text{ rad} \)

Problem 20. An angle measures 22°. What is its measurement in radians?

(a) \( \frac{22 \cdot 360}{\pi} \text{ rad} \)  
(b) \( \frac{22}{2\pi} \text{ rad} \)  
*(c) \( \frac{22\pi}{180} \text{ rad} \)  
(d) \( \frac{22\pi}{360} \text{ rad} \)  
(e) None of these

Problem 21. An angle measures 80°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

*(a) 1.40 rad  
(c) 1.69 rad  
(e) None of these

Solution: \( \frac{80\pi}{180} = 1.40 \text{ rad} \)

Problem 22. An angle measures 0.80 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

*(a) 46°  
(c) 55°  
(e) None of these

Solution: \( \frac{(0.80)(180^\circ)}{\pi} = 46^\circ \)

Problem 23. An angle measures 3 radians. What is its measurement in degrees?

*(a) \( \frac{(3)(180)}{\pi} \text{ degrees} \)  
(c) \( \frac{(3)(360)}{\pi} \text{ degrees} \)  
(e) None of these

(b) \( \frac{3}{2\pi} \text{ degrees} \)  
(d) \( \frac{3\pi}{360} \text{ degrees} \)
Problem 24. An angle measures 1.06 radians. What is its measurement in degrees?

(a) \( \frac{1.06\pi}{180} \) degrees  
(b) \( \frac{(1.06)(90)}{\pi} \) degrees  
(c) \( \frac{1.06}{2\pi} \) degrees  
*(d) \( \frac{(1.06)(180)}{\pi} \) degrees  
(e) None of these

Problem 25. In the figure at right, if \( \theta = \frac{1}{3} \) rad, what is the arc length \( s \)?

(a) \( \frac{\pi}{18} \) cm  
(b) \( \frac{\pi}{2} \) cm  
*(c) 2 cm  
(d) 18 cm  
(e) None of these

Solution: \( s = r\theta = (6 \text{ cm})(1/3) = 2 \text{ cm} \)

Problem 26. In the figure at right, what is the measure of the angle \( \theta \)?

(a) \( \sqrt{14} \) rad  
(b) \( \frac{5\pi}{7} \) rad  
(c) \( \frac{5}{7} \) rad  
*(d) \( \frac{7}{5} \) rad  
(e) None of these

Problem 27. In the figure at right, what is the measure of the angle \( \theta \)?

(a) \( \frac{4\pi}{5} \) rad  
*(b) \( \frac{5}{4} \) rad  
(c) \( 3 \) rad  
(d) \( \frac{3}{\pi} \) rad  
(e) None of these

Solution: \( \theta = \frac{s}{r} = \frac{5}{4} \)

Problem 28. A circle has a radius of 10 cm. A central angle \( \theta \) intercepts an arc whose length is \( 4\pi \) cm. What is the measure of \( \theta \)?

(a) \( \frac{5}{2} \) rad  
(b) \( \frac{5\pi}{2} \) rad  
(c) \( \frac{2}{5} \) rad  
*(d) \( \frac{2\pi}{5} \) rad  
(e) None of these

Solution: \( \theta = \frac{s}{r} = \frac{4\pi}{10} = \frac{2\pi}{5} \)
**Problem 29.** A circle has a radius of $3\pi$ cm. A central angle $\theta$ intercepts an arc whose length is 4 cm. What is the measure of $\theta$?

(a) $\frac{3}{4}$ rad  
(b) $\frac{3\pi^2}{4}$ rad  
*(c) $\frac{4}{3\pi}$ rad  
(d) $\frac{4}{3}$ rad  
(e) None of these

**Solution:** $s = r\theta \Rightarrow \theta = \frac{s}{r} = \frac{4 \text{ cm}}{3\pi \text{ cm}} = \frac{4}{3\pi} \text{ rad}$

**Problem 30.** A circle has a radius of 5 cm. A central angle has measure $\theta = \frac{\pi}{3}$ rad. What is the length of the arc intercepted by the angle?

(a) $\frac{2}{15}$ cm  
*(b) $\frac{5\pi}{3}$ cm  
(c) $\frac{5}{3}$ cm  
(d) $\frac{10\pi}{3}$ cm  
(e) None of these

**Solution:** $(5 \text{ cm})(\frac{\pi}{3}) = \frac{5\pi}{3} \text{ cm}$

**Problem 31.** A spider is clinging to the blade of a ceiling fan, 18 cm from the center. If the fan turns through an angle of 2.1 radians, how far does the spider travel? Round your answer to the nearest cm.

(a) 9 cm  
(c) 27 cm  
*(d) 38 cm  
(e) None of these

**Solution:** $\theta = \frac{s}{r} \Rightarrow s = r\theta = (18 \text{ cm})(2.1) = 38 \text{ cm}$

**Problem 32.** A physics instructor is riding on a merry-go-round, 2.3 m from the center. If the physics instructor travels a distance of 4.1 m, what angle has the merry-go-round turned through? Round your answer to two significant figures.

(a) 0.56 rad  
(c) 1.2 rad  
*(d) 1.8 rad  
(e) None of these

**Solution:** $s = r\theta \Rightarrow \theta = \frac{s}{r} = \frac{4.1 \text{ m}}{2.3 \text{ m}} = 1.8 \text{ rad}$

**Trigonometry**

**Problem 33.** Use a calculator or equivalent to find $\cos(0.70 \text{ rad})$. Round your answer to three decimal places.

(a) 0.558  
(c) 0.688  
*(d) 0.765  
(e) None of these
Problem 34. Use a calculator or equivalent to find \( \sin 77° \). Round your answer to three decimal places.

(a) 1.026  (b) 0.031  
(c) 0.331  * (d) 0.974  
(e) None of these

Problem 35. Use a calculator or equivalent to find \( \tan 35° \). Round your answer to three decimal places.

(a) 0.474  (b) 0.576  
* (c) 0.700  (d) 0.851  
(e) None of these

Problem 36. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \sin \theta = 0.31 \). Round your answer to the nearest degree.

(a) 13°  (b) 15°  
(c) 16°  * (d) 18°  
(e) None of these

Solution: \( \theta = \sin^{-1}(0.31) = 18° \)

Problem 37. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \tan \theta = 0.40 \). Round your answer to the nearest degree.

* (a) 22°  (b) 24°  
(c) 26°  (d) 29°  
(e) None of these

Solution: Since \( \theta \) is acute, \( \theta = \tan^{-1}(0.40) = 22° \)

Problem 38. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \cos \theta = 1.19 \). Round your answer to the nearest degree.

(a) 21°  (b) 50°  
(c) 53°  * (d) No such angle  
(e) None of these

Solution: No such angle; \( \cos \theta > 1 \) is impossible.

Problem 39. In the figure at right, \( \cos \theta = ? \)

(a) \( t/s \)  (b) \( s/t \)  
*(c) \( u/s \)  (d) \( s/u \)  
(e) None of these

Problem 40. In the figure at right, \( \cos \phi = ? \)

* (a) \( y/r \)  (b) \( r/x \)  
(c) \( x/r \)  (d) \( x/y \)  
(e) None of these
Problem 41. In the figure at right, $2/a = ?$

(a) $1/\tan \theta$
(b) $1/\cos \theta$
(c) $\sin \theta$
(d) $\cos \theta$
(e) None of these

Problem 42. In the figure at right, use a calculator or equivalent to find $a$. Round your answer to two significant figures. (The figure is not necessarily drawn to scale.)

(a) 4.5
(b) 6.0
(c) 6.7
(d) 7.8
(e) None of these

Solution: $\frac{5}{a} = \sin 40^\circ \Rightarrow a = \frac{5}{\sin 40^\circ} = 7.8$

Problem 43. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 24°
(b) 36°
(c) 49°
(d) 57°
(e) None of these

Solution: $\tan \phi = \frac{8}{7}$; so $\phi = \tan^{-1}(\frac{8}{7}) = 49^\circ$

Problem 44. A pole is supported by a diagonal guy wire. The wire attached to the pole 5 m above ground level, and is anchored in the ground 8 m from the base of the pole. At what angle $\theta$ does the wire meet the ground? Round your answer to the nearest degree.

(a) 32°
(b) 39°
(c) 51°
(d) 58°
(e) None of these

Solution: $\frac{5}{8} = \tan \theta \Rightarrow \theta = \tan^{-1}\left(\frac{5}{8}\right) = 32^\circ$
Problem 45. You want to know the width of a river. You begin by standing directly across from a tree on the opposite bank. You then walk 400 ft straight downstream. From this new point, the tree is at an angle of \( \theta = 60^\circ \) to the upstream direction. How wide is the river? Round your answer to two significant figures.

(a) 130 ft  (b) 230 ft
(c) 690 ft  (d) 1200 ft
(e) None of these

**Solution:** Let \( w \) be the width of the river. Then

\[
\frac{w}{400 \text{ ft}} = \tan 60^\circ \quad \Rightarrow \quad w = (400 \text{ ft}) \tan 60^\circ = 690 \text{ ft}
\]

Problem 46. If \( \theta = 190^\circ \), which quadrant is it in?

(a) Quadrant I  (b) Quadrant II
(c) Quadrant III  (d) Quadrant IV
(e) None of these

**Solution:** \( 180^\circ < 190^\circ < 270^\circ \); so \( 190^\circ \) is in quadrant III.

Problem 47. If \( \theta \) is in the third quadrant, which of the following is true?

(a) \( \sin \theta \leq 0 \) and \( \cos \theta \leq 0 \)  (b) \( \sin \theta \leq 0 \) and \( \cos \theta \geq 0 \)
(c) \( \sin \theta \geq 0 \) and \( \cos \theta \leq 0 \)  (d) \( \sin \theta \geq 0 \) and \( \cos \theta \geq 0 \)
(e) None of these

Vectors

Problem 48. Which of the vectors below corresponds to \( \vec{A} \) on the graph at right?

(a) \( \langle -1, 2 \rangle \)  (b) \( \langle -2, -1 \rangle \)
(c) \( \langle -1, -2 \rangle \)  (d) \( \langle -2, 1 \rangle \)
(e) None of these

**Solution:** \( A_x < 0 \) and \( A_y < 0 \); so (b) or (c). \( |A_x| > |A_y| \); so (b)
Problem 49. Which of the vectors in the graph corresponds to \(-10\hat{i} + 5\hat{j}\)?

(a) \(\vec{A}\)  
* (c) \(\vec{C}\)  
(b) \(\vec{B}\)  
(d) \(\vec{D}\)  
(e) None of these

Solution: Only \(\vec{B}\) and \(\vec{C}\) have a negative \(x\)-component and a positive \(y\)-component. In \(-10\hat{i} + 5\hat{j}\), the magnitude of the \(x\)-component (10) is greater than the magnitude of the \(y\)-component (5). This is true of \(\vec{C}\) but not of \(\vec{B}\).

Problem 50. Which of the vectors in the graph corresponds to \((6, -3)\)?

(a) \(\vec{A}\)  
(c) \(\vec{C}\)  
(b) \(\vec{B}\)  
* (d) \(\vec{D}\)  
(e) None of these

Solution: Only \(\vec{D}\) has a positive \(x\)-component and a negative \(y\)-component.

Problem 51. The vectors \(\vec{X}\), \(\vec{Y}\), and \(\vec{Z}\) are labelled on the figure at right. Which of the following equations is true?

* (a) \(\vec{Z} = \vec{X} - \vec{Y}\)  
(c) \(\vec{Z} = \vec{X} + \vec{Y}\)  
(b) \(\vec{Z} = \vec{Y} - \vec{X}\)  
(d) \(\vec{Z} = -\vec{X} - \vec{Y}\)  
(e) None of these

Solution: Sketch a copy of \(-\vec{Y}\), labelled \(-\vec{Y}'\), with its tail at the head of \(\vec{X}\). Then \(\vec{X} - \vec{Y}\) has its tail at the tail of \(\vec{X}\) and its head at the head of \(-\vec{Y}'\).
Problem 52. The vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are labelled on the figure at right. Which of the following equations is true?

(a) $\vec{C} = \vec{A} + \vec{B}$
(b) $\vec{C} = \vec{A} - \vec{B}$
(c) $\vec{C} = \vec{B} - \vec{A}$
(d) $\vec{C} = -\vec{A} - \vec{B}$
(e) None of these

Solution: Sketch a copy of $-\vec{A}$, labelled $-\vec{A}'$, with its tail at the tail of $\vec{A}$. Sketch a copy of $-\vec{B}$, labelled $-\vec{B}'$, with its tail at the head of $-\vec{A}'$. Then $-\vec{A} - \vec{B}$ has its tail at the tail of $-\vec{A}$ and its head at the head of $-\vec{B}'$.

Problem 53. $\vec{X} = (1, -3)$ and $\vec{Y} = (-4, 5)$. Find $\vec{X} + \vec{Y}$.

(a) $(4, -9)$
(b) $(5, -8)$
(c) $(-2, 1)$
(d) $(-3, 2)$
(e) None of these

Solution: $\vec{X} + \vec{Y} = (1, -3) + (-4, 5) = (1 + (-4), -3 + 5) = (1, 2)$

Problem 54. $\vec{X} = (1, -3)$ and $\vec{Y} = (-4, 5)$. Find $\vec{X} - \vec{Y}$.

(a) $(4, -9)$
(b) $(5, -8)$
(c) $(-2, 1)$
(d) $(-3, 2)$
(e) None of these

Solution: $\vec{X} - \vec{Y} = (1 - (-4), -3 - 5) = (5, -8)$

Problem 55. If $\vec{A} = 2.0\hat{i} + 3.0\hat{j}$, what is the direction of $\vec{A}$? Round your answer to the nearest degree.

(a) $34^\circ$
(b) $42^\circ$
(c) $48^\circ$
(d) $56^\circ$
(e) None of these

Solution: $A_x > 0$ and $A_y > 0$; so $\theta$ is in quadrant I. Hence:

$$\theta = \tan^{-1} \left| \frac{A_y}{A_x} \right| = \tan^{-1} \left( \frac{3.0}{2.0} \right) = 56^\circ$$
Problem 56. If \( \vec{A} = 2.2 \hat{i} + 3.4 \hat{j} \), what is the magnitude of \( \vec{A} \)? Round your answer to two significant figures.

\[ \| \vec{A} \| = \sqrt{2.2^2 + 3.4^2} = \sqrt{5.68} \approx 2.4 \]

\[ \text{Solution:} \quad \| \vec{A} \| = \sqrt{2.2^2 + 3.4^2} = \sqrt{5.68} = 2.4 \]

Problem 57. At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. Which of the vectors on the figure at right is in the direction of the spider’s displacement vector at 12:30?

(a) The zero vector \( \vec{0} \)  
(b) \( \vec{A} \)  
(c) \( \vec{B} \)  
(d) \( \vec{E} \)  
(e) None of these

Problem 58. Consider the statements (i) and (ii). Are the statements true or false?

(i) If \( k \) is a scalar, then \( k + k = 2k \)

(ii) Let \( \vec{v} = \langle a, b \rangle \). If \( \| \vec{v} \| > 0 \), then \( a > 0 \) and \( b > 0 \).

Choose the correct answer from below.

(a) (i) is true; (ii) is false  
(b) (i) is false; (ii) is true  
(c) Both statements are true  
(d) Both statements are false

\[ \text{Solution:} \quad (i) \text{ is true. (ii) is false: every nonzero vector has a positive magnitude, including those with negative components. For example, if } \vec{v} = \langle -3, 4 \rangle, \text{ then } \| \vec{v} \| = \sqrt{(-3)^2 + 4^2} = 5. \]

Problem 59. You are riding a motorcycle at a speed of 45 miles per hour, up a mountain road that slopes upward at an angle of 13° to the horizontal. What is the vertical component of your velocity? Round your answer to the nearest mile per hour.

(a) 8 mi/hr  
(b) 9 mi/hr  
(c) 10 mi/hr  
(d) 11 mi/hr  
(e) None of these

\[ \text{Solution:} \quad v_y = v \sin \theta = (45 \text{ mi/hr}) \sin 13^\circ = 10 \text{ mi/hr} \]
Problem 60. A gun is fired at an elevation of 22° above the horizontal. The shell emerges from the muzzle at 350 m/s. What is the vertical component of the shell’s velocity? Round your answer to the nearest 10 m/s.

*(a) 130 m/s (b) 180 m/s (c) 240 m/s (d) 320 m/s (e) None of these

Solution: \( v_y = v \sin \theta = (350 \text{ m/s}) \sin 22° = 130 \text{ m/s} \)