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I Mathematical Prerequisites

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0 About the mathematical prerequisites material

0.1 What is a College Physics Course?

Typically, in the United States a college physics course is a physics course that is taught without the use of calculus. The prerequisites are high school algebra (a.k.a. Algebra II from high school, or equivalently intermediate algebra from a community college) and trigonometry. It is also assumed that the student has had an introduction to dimensions, units, and significant figures.

0.2 What are the math-prerequisite topics for college physics?

College Physics Prerequisite Topics:

- Geometry section 1 and all subsections therein
- Algebra section 2 and all subsections therein
- Graphing section 3 and all subsections therein
- Basic Trigonometry section 4 and all subsections therein (except identities)
- An introduction to vectors section 6 and all subsections therein (except the dot and cross-product)
- Introduction to measurement and all subsections therein

0.3 What is a University Physics course?

Typically, in the United States a university physics course is a calculus-based physics course taken by engineering, physics, and math majors. The prerequisites are a high-school physics course and a minimum mathematical background of Calculus I from high school, or equivalently from a college, or university. Remember, this is an absolute minimum! A background of calculus II and a strong grasp of trigonometry will make the course more bearable.

University Physics Prerequisite Topics:

- Geometry section 1 and all subsections therein
- Algebra section 2 and all subsections therein
- Graphing section 3 and all subsections therein
- Basic Trigonometry section 4 and all subsections therein
• Calculus section ?? and all subsections therein
• An introduction to vectors section 6 and all subsections therein
• Introduction to measurement and all subsections therein

0.4 What is a college physics course in the state of Arizona?

Unfortunately, the University of Arizona and Pima community college only require high-school algebra as a prerequisite for college physics. As such, the physics 121 course taught at Pima presupposes no trigonometry background what-so-ever. However, a background in basic trigonometry will make the first few weeks of the course more bearable. College algebra is also a required co-requisite for this course.
Part I

Mathematical Prerequisites
1 Geometry

1.1 Circles

1.1.1 Letter-based problems

Problem 1. If the radius of a circle is \( R \), what is its diameter?
(a) \( R/2 \)  
(b) \( 2R \)  
(c) \( \pi R/2 \)  
(d) \( 2\pi R \)  
(e) None of these

Problem 2. If the radius of a circle is \( R \), what is its circumference?
(a) \( R/2 \)  
(b) \( 2R \)  
(c) \( \pi R/2 \)  
(d) \( 2\pi R \)  
(e) None of these

Problem 3. If the radius of a circle is \( R \), what is its area?
(a) \( \pi R \)  
(b) \( 2\pi R \)  
(c) \( \pi R^2 \)  
(d) \( 2\pi R^2 \)  
(e) None of these

Problem 4. If the diameter of a circle is \( D \), what is its radius?
(a) \( \pi D^2/2 \)  
(b) \( 2\pi D^2 \)  
(c) \( D/2 \)  
(d) \( 2D \)  
(e) None of these

Problem 5. If the diameter of a circle is \( D \), what is its circumference?
(a) \( \pi D \)  
(b) \( 2\pi D \)  
(c) \( \pi D/2 \)  
(d) \( \pi D/4 \)  
(e) None of these

Problem 6. If the diameter of a circle is \( D \), what is its area?
(a) \( \pi D \)  
(b) \( 2\pi D \)  
(c) \( \pi D^2/2 \)  
(d) \( \pi D^2/4 \)  
(e) None of these

Problem 7. If the area of a circle is \( A \), what is its radius?
(a) \( \frac{\sqrt{A}}{2\pi} \)  
(b) \( \sqrt{\frac{2A}{\pi}} \)  
(c) \( \frac{\sqrt{A}}{\pi} \)  
(d) \( \sqrt{\frac{A}{\pi}} \)  
(e) None of these
Problem 8. If the area of a circle is $A$, what is its diameter?

(a) $2\sqrt{\frac{A}{\pi}}$ (b) $\sqrt{\frac{2A}{\pi}}$

(c) $\frac{2\sqrt{A}}{\pi}$ (d) $\frac{\sqrt{2A}}{\pi}$

(e) None of these

Problem 9. If the area of a circle is $A$, what is its circumference?

(a) $2\pi \sqrt{A}$ (b) $2\sqrt{\pi A}$

(c) $\sqrt{\frac{A}{\pi}}$ (d) $\sqrt{\frac{2A}{\pi}}$

(e) None of these

Problem 10. If the circumference of a circle is $C$, what is its radius?

(a) $\frac{\sqrt{2C}}{\pi}$ (b) $\frac{2C}{\pi}$

(c) $\frac{\sqrt{C}}{2\pi}$ (d) $\frac{C}{2\pi}$

(e) None of these

Problem 11. If the circumference of a circle is $C$, what is its diameter?

(a) $\frac{\sqrt{C}}{2\pi}$ (b) $\sqrt{\frac{2C}{\pi}}$

(c) $\frac{C}{2\pi}$ (d) $\frac{C}{\pi}$

(e) None of these

Problem 12. If the circumference of a circle is $C$, what is its area?

(a) $\frac{\sqrt{2C}}{\pi}$ (b) $\frac{C^2}{4\pi}$

(c) $\sqrt{\frac{2C}{\pi}}$ (d) $\frac{\pi C^2}{4}$

(e) None of these
1.1.2 Non-calculator-based problems

Problem 13. What is the circumference of a circle with radius 6 cm?
(a) $6\pi$ cm  (b) $9\pi$ cm  
(c) $12\pi$ cm  (d) $36\pi$ cm  
(e) None of these

Problem 14. What is the circumference of a circle with diameter 3 m?
(a) $3\pi$ m  (b) $6\pi$ m  
(c) $9\pi$ m  (d) $36\pi$ m  
(e) None of these

Problem 15. What is the area of a circle with diameter 5 m?
(a) $\frac{5\pi}{2}$ m$^2$  (b) $\frac{25\pi}{4}$ m$^2$  
(c) $5\pi^2$ m$^2$  (d) $10\pi^2$ m$^2$  
(e) None of these

Problem 16. What is the area of a circle with radius 3 cm?
(a) $\frac{3\pi}{4}$ cm$^2$  (b) $\frac{9\pi}{4}$ cm$^2$  
(c) $\frac{9\pi}{2}$ cm$^2$  (d) $9\pi$ cm$^2$  
(e) None of these

Problem 17. What is the circumference of a circle with diameter 10 cm?
(a) $5\pi$ cm  (b) $10\pi$ cm  
(c) $25\pi$ cm  (d) $100\pi$ cm  
(e) None of these

Problem 18. What is the area of a circle with diameter 10 m?
(a) $5\pi$ m$^2$  (b) $10\pi$ m$^2$  
(c) $\frac{25\pi}{4}$ cm$^2$  (d) $25\pi$ cm$^2$  
(e) None of these

Problem 19. What is the circumference of a circle with radius 4 cm?
(a) $2\pi$ cm  (b) $4\pi$ cm  
(c) $8\pi$ cm  (d) $16\pi$ cm  
(e) None of these
Problem 20. What is the area of a circle with radius 6 cm?
(a) $3\pi \text{ cm}^2$  (b) $6\pi \text{ cm}^2$
(c) $9\pi \text{ cm}^2$  (d) $36\pi \text{ cm}^2$
(e) None of these

Problem 21. If the circumference of a circle is 5 cm, what is its area?
(a) $\frac{25}{4\pi} \text{ cm}^2$  (b) $50\pi \text{ cm}^2$
(c) $\frac{25}{2\pi} \text{ cm}^2$  (d) $\frac{25\pi}{2} \text{ cm}^2$
(e) None of these

Problem 22. If the area of a circle is 9 cm$^2$, what is its radius?
(a) $3\pi \text{ cm}^2$  (b) $3\sqrt{\pi} \text{ cm}$
(c) $\frac{3}{\pi} \text{ cm}$  (d) $\frac{3}{\sqrt{\pi}} \text{ cm}$
(e) None of these

Problem 23. If the circumference of a circle is 4 cm, what is its diameter?
(a) $\frac{4}{\pi} \text{ cm}$  (b) 8 cm
(c) $\frac{8}{\pi} \text{ cm}$  (d) $\frac{2}{\sqrt{\pi}} \text{ cm}^2$
(e) None of these

Problem 24. If the area of a circle is 4 m$^2$, what is its diameter?
(a) $\frac{2}{\pi} \text{ m}$  (b) $\frac{4}{\sqrt{\pi}} \text{ m}$
(c) $16\pi \text{ m}$  (d) $\frac{16}{\pi} \text{ m}$
(e) None of these

Problem 25. If the circumference of a circle is 10 m, what is its radius?
(a) $\frac{10}{\pi} \text{ m}$  (b) $\sqrt{\frac{10}{\pi}} \text{ m}$
(c) $\frac{5}{\pi} \text{ m}$  (d) $\sqrt{\frac{5}{\pi}} \text{ m}$
(e) None of these
Problem 26. If the area of a circle is $3 \text{ m}^2$, what is its circumference?

(a) $\frac{2\sqrt{3}}{\sqrt{\pi}} \text{ m}$  
(b) $2\sqrt{3\pi} \text{ m}$

(c) $\sqrt{\frac{3}{2\pi}} \text{ m}$  
(d) $\sqrt{\frac{6}{\pi}} \text{ m}$

(e) None of these

1.1.3 Calculator-based problems

Problem 27. What is the circumference of a circle with radius 11.9 cm? Round your answer to the nearest 0.1 cm.

(a) 74.8 cm  
(b) 82.2 cm

(c) 90.5 cm  
(d) 99.5 cm

(e) None of these

Problem 28. What is the circumference of a circle with radius 8.3 cm? Round your answer to the nearest centimeter.

(a) 38 cm  
(b) 42 cm

(c) 47 cm  
(d) 52 cm

(e) None of these

Problem 29. What is the circumference of a circle with diameter 5.6 cm? Round your answer to the nearest centimeter.

(a) 18 cm  
(b) 19 cm

(c) 21 cm  
(d) 23 cm

(e) None of these

Problem 30. What is the circumference of a circle with diameter 6.9 cm? Round your answer to the nearest centimeter.

(a) 16 cm  
(b) 18 cm

(c) 20 cm  
(d) 22 cm

(e) None of these

Problem 31. What is the area of a circle with radius 1.1 cm? Round your answer to the nearest 0.1 cm$^2$.

(a) 3.8 cm$^2$  
(b) 4.2 cm$^2$

(c) 4.6 cm$^2$  
(d) 5.1 cm$^2$

(e) None of these
Problem 32. What is the area of a circle with radius 6.1 cm? Round your answer to the nearest 10 cm$^2$.
(a) 90 cm$^2$    (b) 100 cm$^2$
(c) 110 cm$^2$    (d) 120 cm$^2$
(e) None of these

Problem 33. What is the area of a circle with diameter 4.4 cm? Round your answer to the nearest cm$^2$.
(a) 12 cm$^2$    (b) 14 cm$^2$
(c) 15 cm$^2$    (d) 17 cm$^2$
(e) None of these

Problem 34. What is the area of a circle with diameter 7.8 cm? Round your answer to the nearest cm$^2$.
(a) 39 cm$^2$    (b) 43 cm$^2$
(c) 48 cm$^2$    (d) 53 cm$^2$
(e) None of these

Problem 35. If the circumference of a circle is 4 cm, what is its area?
(a) $4\pi$ cm$^2$    (b) $16\pi$ cm$^2$
(c) $\frac{4}{\pi}$ cm$^2$    (d) $\frac{16}{\pi}$ cm$^2$
(e) None of these
1.2 Rectangles and rectangular solids

1.2.1 Letter-based problems

Problem 36. What is the area of a square with sides of length $s$?
(a) $4s$  
(b) $2\pi s$  
(c) $s^2$  
(d) $4\pi s^2$  
(e) None of these

Problem 37. What is the area of a rectangle with length $l$ and width $w$?
(a) $2l + 2w$  
(b) $lw$  
(c) $\sqrt{l^2 + w^2}$  
(d) $l^2 + w^2$  
(e) None of these

Problem 38. What is the volume of a cube with sides of length $s$?
(a) $8s$  
(b) $6s^2$  
(c) $s^3$  
(d) $\sqrt{3} s$  
(e) None of these

Problem 39. What is the volume of a rectangular solid with length $l$, width $w$, and height $h$?
(a) $2l + 2w + 2h$  
(b) $lwh$  
(c) $\sqrt{l^2 + w^2 + h^2}$  
(d) $l^3 + w^3 + h^3$  
(e) None of these

1.2.2 Non-calculator based problems

Problem 40. What is the area of a square tabletop measuring 3 feet on a side?
(a) $9 \text{ ft}^2$  
(b) $3\pi \text{ ft}^2$  
(c) $12 \text{ ft}^2$  
(d) $6\pi \text{ ft}^2$  
(e) None of these

Problem 41. What is the area of a rectangular sheet of paper measuring 10 inches long by 8 inches wide?
(a) $40 \text{ in}^2$  
(b) $80 \text{ in}^2$  
(c) $40\pi \text{ in}^2$  
(d) $80\pi \text{ in}^2$  
(e) None of these

Problem 42. What is the volume of a cube measuring 2 inches on a side?
(a) $4 \text{ in}^3$  
(b) $4\pi \text{ in}^3$  
(c) $8 \text{ in}^3$  
(d) $8\pi \text{ in}^3$  
(e) None of these
Problem 43. What is the volume of a rectangular box measuring 2 cm long by 3 cm wide by 5 cm high?
(a) 10 cm³  (b) 20 cm³
(c) 30 cm³  (d) 60 cm³
(e) None of these

Problem 44. What is the area of a square room measuring 5 m on a side?
(a) $5\pi$ m²  (b) $25\pi$ m²
(c) $\frac{25\pi}{2}$ m²  (d) 25 m²
(e) None of these

Problem 45. What is the volume of a rectangular room measuring 4 m long by 5 m wide by 3 m high?
(a) 12 m³  (b) 24 m³
(c) 30 m³  (d) 60 m³
(e) None of these

Problem 46. What is the volume of a cube measuring 10 cm on a side?
(a) $\frac{1000}{3}$ cm³  (b) 500 cm³
(c) 1000 cm³  (d) 2000 cm³
(e) None of these

Problem 47. What is the area of a rectangle measuring 6 m long by 2 m wide?
(a) 12 m²  (b) 24 m²
(c) 48 m²  (d) $6\pi$ m²
(e) None of these

Problem 48. What is the area of a square tabletop measuring 3 feet on a side?
(a) 9 ft²  (b) $3\pi$ ft²
(c) 12 ft²  (d) $6\pi$ ft²
(e) None of these
1.2.3 Calculator-based problems

Problem 49. What is the area of a square piece of ground whose sides are each 62 ft long? Round your answer to the nearest 100 ft$^2$.

(a) 3100 ft$^2$  
(b) 3500 ft$^2$  
(c) 3800 ft$^2$  
(d) 4200 ft$^2$  
(e) None of these

Problem 50. What is the area of a square tabletop whose sides are each 2.7 ft long? Round your answer to the nearest 0.1 ft$^2$.

(a) 7.3 ft$^2$  
(b) 8.0 ft$^2$  
(c) 8.8 ft$^2$  
(d) 9.7 ft$^2$  
(e) None of these

Problem 51. What is the area of a rectangular carpet measuring 2.4 m long by 1.7 m wide? Round your answer to the nearest 0.1 m$^2$.

(a) 3.7 m$^2$  
(b) 4.1 m$^2$  
(c) 4.5 m$^2$  
(d) 4.9 m$^2$  
(e) None of these

Problem 52. What is the area of a rectangular windowpane measuring 21 cm wide by 27 cm high? Round your answer to the nearest 10 cm$^2$.

(a) 510 cm$^2$  
(b) 570 cm$^2$  
(c) 620 cm$^2$  
(d) 690 cm$^2$  
(e) None of these

Problem 53. What is the volume of a cube measuring 13.4 cm on a side? Round your answer to the nearest 10 cm$^3$.

(a) 2170 cm$^3$  
(b) 2410 cm$^3$  
(c) 2650 cm$^3$  
(d) 2910 cm$^3$  
(e) None of these

Problem 54. What is the volume of a cube measuring 0.88 ft on a side? Round your answer to the nearest 0.01 ft$^3$.

(a) 0.50 ft$^3$  
(b) 0.55 ft$^3$  
(c) 0.61 ft$^3$  
(d) 0.68 ft$^3$  
(e) None of these

Problem 55. A rectangular fish-tank is 53 cm long, 22 cm wide, and 24 cm high. What is its volume? Round your answer to the nearest 1000 cm$^3$.

(a) 20,000 cm$^3$  
(b) 23,000 cm$^3$  
(c) 25,000 cm$^3$  
(d) 28,000 cm$^3$  
(e) None of these
Problem 56. A room is 5.7 m long, 4.8 m wide, and 2.4 m high. What is its volume? Round your answer to the nearest cubic meter.

(a) 66 m³  (b) 72 m³
(c) 79 m³  (d) 87 m³
(e) None of these
1.3 Right triangles

1.3.1 Letter-based problems

Problem 57. In the right triangle at right, which of the following equations is true?

(a) \( r = x + y \)  \quad (b) \( r = \frac{1}{2}(x + y) \)
(c) \( r^2 = x^2 + y^2 \)  \quad (d) \( r = \frac{1}{2}(x - y) \)
(e) None of these

Problem 58. In the right triangle at right, which of the following equations is true?

(a) \( a^2 = b^2 + c^2 \)  \quad (b) \( b^2 = a^2 + c^2 \)
(c) \( c^2 = a^2 + b^2 \)  \quad (d) \( a^2 + b^2 + c^2 = 1 \)
(e) None of these

Problem 59. In the right triangle at right, which of the following equations is true?

(a) \( x^2 = r^2 + y^2 \)  \quad (b) \( x^2 = r^2 - y^2 \)
(c) \( x^2 = \frac{1}{2}(r^2 + y^2) \)  \quad (d) \( x^2 = \frac{1}{2}(r^2 - y^2) \)
(e) None of these

Problem 60. In the right triangle at right, which of the following equations is true?

(a) \( a^2 = \frac{1}{2}(c - b)^2 \)  \quad (b) \( a^2 = \frac{1}{2}(c^2 - b^2) \)
(c) \( a^2 = (c - b)^2 \)  \quad (d) \( a^2 = c^2 - b^2 \)
(e) None of these

Problem 61. In the right triangle at right, which of the following equations is true?

(a) \( y^2 = r^2 - x^2 \)  \quad (b) \( y^2 = x^2 - r^2 \)
(c) \( y^2 = \frac{1}{2}(a^2 + r^2) \)  \quad (d) \( y^2 = \frac{1}{2}(a^2 - r^2) \)
(e) None of these

Problem 62. In the right triangle at right, which of the following equations is true?

(a) \( b^2 = \frac{1}{2}(c - a)^2 \)  \quad (b) \( b^2 = \frac{1}{2}(c + a)^2 \)
(c) \( b^2 = c^2 - a^2 \)  \quad (d) \( b^2 = c^2 + a^2 \)
(e) None of these
Problem 63. In the right triangle at right, which of the following equations is true?

(a) \( r = \sqrt{x^2 + y^2} \)  
(b) \( r = x^2 + y^2 \)
(c) \( r = \sqrt{(x+y)^2} \)  
(d) \( r = \frac{1}{2}(x+y) \)
(e) None of these

Problem 64. In the right triangle at right, which of the following equations is true?

(a) \( c = \frac{1}{2}(a^2 + b^2) \)  
(b) \( c = \frac{1}{2}\sqrt{a^2 + b^2} \)
(c) \( c = a^2 + b^2 \)  
(d) \( c = \sqrt{a^2 + b^2} \)
(e) None of these

Problem 65. In the right triangle at right, which of the following equations is true?

(a) \( x = \sqrt{(r-y)^2} \)  
(b) \( x = \sqrt{(r+y)^2} \)
(c) \( x = \sqrt{r^2 - y^2} \)  
(d) \( x = \sqrt{r^2 + y^2} \)
(e) None of these

Problem 66. In the right triangle at right, which of the following equations is true?

(a) \( a = \frac{bc}{2} \)  
(b) \( a = \frac{2c}{b} \)
(c) \( a = \frac{c^2 - b^2}{2} \)  
(d) \( a = \sqrt{c^2 - b^2} \)
(e) None of these

1.3.2 Non-calculator-based problems

Problem 67. Find \( r \) in the right triangle at right.

(a) \( \sqrt{7} \)  
(b) 5
(c) 7  
(d) 25
(e) None of these

Problem 68. Find \( r \) in the right triangle at right.

(a) 3  
(b) \( \sqrt{9} \)
(c) 9  
(d) \( \sqrt{41} \)
(e) None of these
Problem 69. Find $r$ in the right triangle at right.

(a) $\sqrt{8}$  
(b) $\sqrt{15}$  
(c) 4  
(d) $\sqrt{34}$  
(e) None of these

Problem 70. Find $x$ in the right triangle at right.

(a) 1  
(b) $\sqrt{2}$  
(c) $\sqrt{3}$  
(d) $\sqrt{5}$  
(e) None of these

Problem 71. Find $x$ in the right triangle at right.

(a) 2  
(b) $\sqrt{8}$  
(c) 4  
(d) $\sqrt{34}$  
(e) None of these

Problem 72. Find $y$ in the right triangle at right.

(a) 3  
(b) 9  
(c) $\sqrt{21}$  
(d) $\sqrt{34}$  
(e) None of these

Problem 73. Find $y$ in the right triangle at right.

(a) 1  
(b) $\sqrt{7}$  
(c) 5  
(d) 7  
(e) None of these

Problem 74. Find $y$ in the right triangle at right.

(a) 9  
(b) $\sqrt{17}$  
(c) $\sqrt{30}$  
(d) $\sqrt{51}$  
(e) None of these
1.3.3 Calculator-based problems

**Problem 75.** Find $r$ in the right triangle at right. Round your answer to one decimal place.

- (a) 6.0
- (b) 6.6
- (c) 7.4
- (d) 8.2
- (e) None of these

**Problem 76.** Find $r$ in the right triangle at right. Round your answer to the nearest integer.

- (a) 29
- (b) 32
- (c) 35
- (d) 39
- (e) None of these

**Problem 77.** Find $r$ in the right triangle at right. Round your answer to two decimal places.

- (a) 0.46
- (b) 0.51
- (c) 0.56
- (d) 0.62
- (e) None of these

**Problem 78.** Find $x$ in the right triangle at right. Round your answer to one decimal place.

- (a) 2.2
- (b) 2.4
- (c) 2.7
- (d) 3.0
- (e) None of these

**Problem 79.** Find $x$ in the right triangle at right. Round your answer to the nearest integer.

- (a) 33
- (b) 36
- (c) 40
- (d) 44
- (e) None of these

**Problem 80.** Find $x$ in the right triangle at right. Round your answer to two decimal places.

- (a) 0.69
- (b) 0.77
- (c) 0.85
- (d) 0.95
- (e) None of these

**Problem 81.** Find $y$ in the right triangle at right. Round your answer to two decimal places.

- (a) 0.10
- (b) 0.11
- (c) 0.13
- (d) 0.14
- (e) None of these
Problem 82. Find $y$ in the right triangle at right. Round your answer to one decimal place.

(a) 5.2  (b) 5.8  
(c) 6.4  (d) 7.0  
(e) None of these

Problem 83. Find $y$ in the right triangle at right. Round your answer to the nearest integer.

(a) 25  (b) 28  
(c) 31  (d) 34  
(e) None of these
1.4 Right triangle word problems

1.4.1 Non-calculator-based Problems

Problem 84. Pirates have buried their treasure in the desert, and you have obtained a copy of the directions to it. They tell you to begin at the entrance of the physics lab and go 2 km east, then to go north for another 5 km to the treasure. What is the straight-line distance from the physics lab to the treasure site?

(a) 4 km  
(b) 7 km  
(c) \(\sqrt{21}\) km  
(d) \(\sqrt{29}\) km  
(e) None of these

Problem 85. Pirates have buried their treasure in the desert, and you have obtained a copy of the directions to it. They tell you to begin at the entrance of the physics lab and go 3 km south, then to go west for another 4 km to the treasure. What is the straight-line distance from the physics lab to the treasure site?

(a) 5 km  
(b) \(\sqrt{7}\) km  
(c) \(\sqrt{12}\) km  
(d) 7 km  
(e) None of these

Problem 86. Your house is 5 km from the Pima Community College Missile Test Site. A missile is fired from the test site; it goes 3 km straight up, then explodes. How far is the missile from your house when it explodes?

(a) 4 km  
(b) \(\sqrt{8}\) km  
(c) \(\sqrt{15}\) km  
(d) \(\sqrt{34}\) km  
(e) None of these

Problem 87. Your house is 7 miles from the Pima Community College Missile Test Site. A missile is fired from the test site; it goes 1 mile straight up, then explodes. How far is the missile from your house when it explodes?

(a) 7.5 mi  
(b) 8 mi  
(c) \(\sqrt{50}\) mi  
(d) 10 mi  
(e) None of these

Problem 88. A vertical telephone pole is stabilized by a diagonal guy wire anchored in the ground, as shown at right. The wire is 10 m long; it is attached to the pole 7 m above ground level. How far from the base of the pole does the wire meet the ground?

(a) 9 m  
(b) \(\sqrt{51}\) m  
(c) \(\sqrt{93}\) m  
(d) \(\sqrt{149}\) m  
(e) None of these
Problem 89. A physics instructor has locked himself out of his office, and tries to climb in through an upper window. He leans a 5-meter ladder against the side of the building so that the top of the ladder is 3 m above the ground. How far from the building is the bottom of the ladder?

(a) 4 m  
(b) 8 m  
(c) $\sqrt{34}$ m  
(d) $\sqrt{44}$ m  
(e) None of these

Problem 90. A blimp is attached to a cable 3 km long. The other end of the cable is fastened to the ground. The wind is strong enough to pull the cable into a straight line. When you are standing directly below the blimp, you are 2 km from the place where the cable is anchored in the ground. How high above ground level is the blimp?

(a) 1 km  
(b) $\sqrt{5}$ km  
(c) 5 km  
(d) $\sqrt{13}$ km  
(e) None of these

1.4.2 Calculator-based Problems

Problem 91. An airplane flies a route involving three cities. From Rio Cordaro, it flies 210 miles straight east to Hackerville. From Hackerville, it flies 140 miles straight north to San Pitucco. How far does it fly from San Pitucco to Rio Cordaro? Round your answer to the nearest 10 miles.

(a) 250 miles  
(b) 280 miles  
(c) 310 miles  
(d) 340 miles  
(e) None of these

Problem 92. An airplane flies a route involving three cities. From Rio Cordaro, it flies 140 miles straight east to Hackerville. From Hackerville, it flies 320 miles straight north to San Pitucco. How far does it fly from San Pitucco to Rio Cordaro? Round your answer to the nearest 10 miles.

(a) 330 miles  
(b) 350 miles  
(c) 370 miles  
(d) 390 miles  
(e) None of these

Problem 93. A ship sails 58 miles straight south. It then sails another 44 miles straight west. At this point, what is its straight-line distance from its starting point? Round your answer to the nearest mile.

(a) 73 miles  
(b) 80 miles  
(c) 88 miles  
(d) 97 miles  
(e) None of these
Problem 94. A ship sails 79 miles straight south. It then sails another 34 miles straight west. At this point, what is its straight-line distance from its starting point? Round your answer to the nearest mile.

(a) 80 miles  (b) 83 miles
(c) 86 miles  (d) 89 miles
(e) None of these

Problem 95. A vertical telephone pole is stabilized by a diagonal guy wire anchored in the ground, as shown at right. The wire is 34 feet long; it is attached to the pole 21 feet above ground level. How far from the base of the pole does the wire meet the ground? Round your answer to the nearest foot.

(a) 24 ft  (b) 27 ft
(c) 29 ft  (d) 32 ft
(e) None of these

Problem 96. A vertical telephone pole is stabilized by a diagonal guy wire anchored in the ground, as shown at right. The wire is 47 feet long; it is attached to the pole 28 feet above ground level. How far from the base of the pole does the wire meet the ground? Round your answer to the nearest foot.

(a) 31 ft  (b) 34 ft
(c) 38 ft  (d) 42 ft
(e) None of these

Problem 97. A physics instructor has locked himself out of his office, and tries to climb in through an upper window. He leans a 21-foot ladder against the side of the building so that the top of the ladder is 18 feet above the ground. How far from the building is the bottom of the ladder? Round your answer to the nearest foot.

(a) 11 ft  (b) 12 ft
(c) 13 ft  (d) 14 ft
(e) None of these

Problem 98. A physics instructor has locked himself out of his office, and tries to climb in through an upper window. He leans a 13-foot ladder against the side of the building so that the top of the ladder is 9 feet above the ground. How far from the building is the bottom of the ladder? Round your answer to the nearest foot.

(a) 6 ft  (b) 7 ft
(c) 8 ft  (d) 9 ft
(e) None of these
Problem 99. A blimp is attached to a cable 890 meters long. The other end of the cable is fastened to the ground. The wind is strong enough to pull the cable into a straight line. When you are standing directly below the blimp, you are 660 meters from the place where the cable is anchored in the ground. How high above ground level is the blimp? Round your answer to the nearest 10 m.

(a) 540 m  
(b) 600 m  
(c) 660 m  
(d) 720 m  
(e) None of these

Problem 100. A blimp is attached to a cable 820 meters long. The other end of the cable is fastened to the ground. The wind is strong enough to pull the cable into a straight line. When you are standing directly below the blimp, you are 640 meters from the place where the cable is anchored in the ground. How high above ground level is the blimp? Round your answer to the nearest 10 m.

(a) 420 m  
(b) 460 m  
(c) 510 m  
(d) 560 m  
(e) None of these

Problem 101. A kite is attached to a string 310 ft long. The wind is strong enough to stretch the string into a straight line. When you are standing directly under the kite, you are 270 ft from the person holding the string. How high above the ground is the kite? Round your answer to the nearest 10 ft.

(a) 110 ft  
(b) 120 ft  
(c) 140 ft  
(d) 150 ft  
(e) None of these

Problem 102. A kite is attached to a string 560 ft long. The wind is strong enough to stretch the string into a straight line. When you are standing directly under the kite, you are 330 ft from the person holding the string. How high above the ground is the kite? Round your answer to the nearest 10 ft.

(a) 370 ft  
(b) 410 ft  
(c) 450 ft  
(d) 500 ft  
(e) None of these
2  Algebra

2.1  Evaluating functions with numerical arguments

2.1.1  Non-calculator based problems

Problem 103. If \( f(x) = x^2 - 5 \), find \( f(5) \).

(a) \(-5\)  (b) \(0\)
(c) \(9\)  (d) \(20\)
(e) None of these

Problem 104. If \( f(x) = \frac{x + 3}{7} \), find \( f(2) \).

(a) \(\frac{5}{7}\)  (b) \(\frac{10}{7}\)
(c) \(\frac{17}{7}\)  (d) \(\frac{23}{7}\)
(e) None of these

Problem 105. If \( f(x) = \frac{x + 2}{3} \), find \( f(4) \).

(a) \(\frac{5}{7}\)  (b) \(\frac{2}{3}\)
(c) \(2\)  (d) \(\frac{10}{3}\)
(e) None of these

Problem 106. If \( f(x) = \frac{x + 1}{2} \), find \( f(2) \).

(a) \(\frac{3}{4}\)  (b) \(1\)
(c) \(\frac{5}{4}\)  (d) \(\frac{3}{2}\)
(e) None of these

Problem 107. If \( f(x) = \frac{x^2 + 1}{3} \), find \( f(2) \).

(a) \(\frac{5}{9}\)  (b) \(1\)
(c) \(\frac{5}{3}\)  (d) \(3\)
(e) None of these
Problem 108. If \( f(x) = \frac{x^2 - 1}{3} \), find \( f(2) \).

(a) \( \frac{1}{9} \)  
(b) \( \frac{4}{9} \)  
(c) \( \frac{5}{3} \)  
(d) 1  
(e) None of these

Problem 109. If \( f(x) = \frac{x^2 - 3}{3} \), find \( f(3) \).

(a) 0  
(b) \( \frac{1}{9} \)  
(c) \( \frac{1}{3} \)  
(d) 2  
(e) None of these

Problem 110. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(1) \).

(a) \(-2\)  
(b) 2  
(c) \(-\sqrt{6}\)  
(d) No real value  
(e) None of these

Problem 111. If \( f(x) = \sqrt{x^2 + 2} \), find \( f(3) \).

(a) \( \sqrt{11} \)  
(b) 5  
(c) 25  
(d) No real value  
(e) None of these

Problem 112. If \( f(x) = \sqrt{x^2 - 3} \), find \( f(2) \).

(a) \(-1\)  
(b) 1  
(c) \( \sqrt{5} \)  
(d) No real value  
(e) None of these

Problem 113. If \( f(x) = 3x + 2 \), find \( f(4) \).

(a) 12  
(b) 14  
(c) 16  
(d) 18  
(e) None of these

Problem 114. If \( f(x) = 3x + 2 \), find \( f(7) \).

(a) 15  
(b) 21  
(c) 23  
(d) 27  
(e) None of these
Problem 115. If \( f(x) = x^2 - 5 \), find \( f(5) \).
(a) \(-5\)  
(b) \(0\)  
(c) \(9\)  
(d) \(20\)  
(e) None of these

Problem 116. If \( f(x) = x^2 - 5 \), find \( f(2) \).
(a) \(-1\)  
(b) \(1\)  
(c) \(9\)  
(d) \(49\)  
(e) None of these

2.1.2 Calculator based problems

Problem 117. If \( f(x) = 6.3x - 4.4 \), find \( f(3.9) \).
(a) \(14.70\)  
(b) \(16.34\)  
(c) \(18.15\)  
(d) \(20.17\)  
(e) None of these

Problem 118. If \( f(x) = 6.3x - 4.4 \), find \( f(2.1) \).
(a) \(7.15\)  
(b) \(7.95\)  
(c) \(8.83\)  
(d) \(9.71\)  
(e) None of these

Problem 119. If \( f(x) = -2.3x + 7.5 \), find \( f(1.1) \).
(a) \(4.97\)  
(b) \(5.47\)  
(c) \(6.01\)  
(d) \(6.62\)  
(e) None of these

Problem 120. If \( f(x) = -2.3x + 7.5 \), find \( f(4.6) \).
(a) \(-2.77\)  
(b) \(-3.08\)  
(c) \(-3.39\)  
(d) \(-3.73\)  
(e) None of these

Problem 121. If \( f(x) = \frac{x + 3}{7} \), find \( f(5) \). Round your answer to one decimal place.
(a) \(1.1\)  
(b) \(3.7\)  
(c) \(4.0\)  
(d) \(5.4\)  
(e) None of these
Problem 122. If \( f(x) = \frac{x + 3}{7} \), find \( f(9) \). Round your answer to one decimal place.

(a) 0.9  (b) 1.7  
(c) 4.3  (d) 9.4  
(e) None of these

Problem 123. If \( f(x) = \frac{x^2 - 5}{4} \), find \( f(5) \).

(a) 0  (b) 1.25  
(c) 5  (d) 23.75  
(e) None of these

Problem 124. If \( f(x) = \frac{x^2 - 5}{4} \), find \( f(9) \).

(a) 4  (b) 15.25  
(c) 19  (d) 79.75  
(e) None of these

Problem 125. If \( f(x) = \frac{(x - 5)^2}{4} \), find \( f(2) \).

(a) -5.75 (b) -4.25  
(c) -0.25 (d) 2.25  
(e) None of these

Problem 126. If \( f(x) = \frac{(x - 5)^2}{4} \), find \( f(7) \).

(a) -4.5 (b) 0.75  
(c) 1 (d) 11  
(e) None of these

Problem 127. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(7) \). Round your answer to one decimal place.

(a) 4.0 (b) 4.8  
(c) 6.6 (d) No real value  
(e) None of these

Problem 128. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(9) \). Round your answer to one decimal place.

(a) 4.0 (b) 6.8  
(c) 8.7 (d) No real value  
(e) None of these
Problem 129. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(-4) \). Round your answer to one decimal place.

(a) −6.2  
(b) 3.3  
(c) 9.0  
(d) No real value  
(e) None of these

Problem 130. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(2) \). Round your answer to one decimal place.

(a) −0.2  
(b) 3.0  
(c) 9.0  
(d) No real value  
(e) None of these
2.2 Evaluating functions with variable-expression arguments

Problem 131. If \( f(x) = 3x + 2 \), find \( f(a + 1) \).
(a) \( 3a + 1 \)  
(b) \( 3a + 2 \)  
(c) \( 3a + 3 \)  
(d) \( 3a + 5 \)  
(e) None of these

Problem 132. If \( f(x) = 3x + 2 \), find \( f(a - 5) \).
(a) \( 3a - 3 \)  
(b) \( 3a - 7 \)  
(c) \( 3a - 13 \)  
(d) \( 3a - 15 \)  
(e) None of these

Problem 133. If \( f(x) = x^2 \), find \( f(a + 4) \).
(a) \( a^2 + 4 \)  
(b) \( a^2 + 4a + 4 \)  
(c) \( a^2 + 16 \)  
(d) \( a^2 + 8a + 16 \)  
(e) None of these

Problem 134. If \( f(x) = x^2 \), find \( f(a - 3) \).
(a) \( a^2 + 3 \)  
(b) \( a^2 - 3 \)  
(c) \( a^2 + 9 \)  
(d) \( a^2 - 6a + 9 \)  
(e) None of these

Problem 135. If \( f(x) = x^2 \), find \( f(a - 2) \).
(a) \( a^2 - 2 \)  
(b) \( a^2 + 4 \)  
(c) \( a^2 - 4a + 4 \)  
(d) \( a^2 - 2a - 4 \)  
(e) None of these

Problem 136. If \( f(x) = x^2 - 2 \), find \( f(a + 3) \).
(a) \( a^2 + 6a + 7 \)  
(b) \( a^2 + 7 \)  
(c) \( a^2 + 2a + 1 \)  
(d) \( a^2 - 10a + 25 \)  
(e) None of these

Problem 137. If \( f(x) = x^2 - 2 \), find \( f(a - 1) \).
(a) \( a^2 - 2a - 1 \)  
(b) \( a^2 - 2a + 1 \)  
(c) \( a^2 - 6a - 9 \)  
(d) \( a^2 - 6a + 9 \)  
(e) None of these

Problem 138. If \( f(x) = x^2 - 2 \), find \( f(a + 2) \).
(a) \( a^2 \)  
(b) \( a^2 + 4 \)  
(c) \( a^2 + 4a + 2 \)  
(d) \( a^2 - 2a + 4 \)  
(e) None of these
Problem 139. If \( f(x) = \frac{x - 3}{4} \), find \( f(a + 3) \).

(a) \( \frac{a}{4} \)  
(b) \( \frac{4a + 9}{4} \)  
(c) \( \frac{a + 9}{4} \)  
(d) \( \frac{4a - 9}{4} \)  
(e) None of these

Problem 140. If \( f(x) = \frac{x - 3}{4} \), find \( f(a - 1) \).

(a) \( \frac{a - 1}{4} \)  
(b) \( \frac{a - 4}{4} \)  
(c) \( \frac{a + 1}{4} \)  
(d) \( \frac{a + 4}{4} \)  
(e) None of these

Problem 141. If \( f(x) = \frac{x - 3}{4} \), find \( f(a - 5) \).

(a) \( \frac{a - 23}{4} \)  
(b) \( \frac{a - 8}{4} \)  
(c) \( \frac{a + 23}{4} \)  
(d) \( \frac{a + 8}{4} \)  
(e) None of these

Problem 142. If \( f(x) = (x - 2)^2 \), find \( f(a + 2) \).

(a) \( a^2 \)  
(b) \( a^2 + 4 \)  
(c) \( a^2 + 4a + 2 \)  
(d) \( a^2 - 2a + 4 \)  
(e) None of these

Problem 143. If \( f(x) = (x - 2)^2 \), find \( f(a - 1) \).

(a) \( a^2 - 2a - 1 \)  
(b) \( a^2 - 6a + 9 \)  
(c) \( a^2 - 4a + 3 \)  
(d) \( a^2 + 4a + 4 \)  
(e) None of these

Problem 144. If \( f(x) = (x - 2)^2 \), find \( f(a - 2) \).

(a) \( a^2 \)  
(b) \( a^2 - 4a + 2 \)  
(c) \( a^2 - 4a - 6 \)  
(d) \( a^2 - 8a + 16 \)  
(e) None of these
2.3 Evaluating functions of multiple variables

2.3.1 Functions of two variables

Non-calculator based problems

Problem 145. The formula for the volume of a cylinder is: \( V = \pi r^2 l \), where \( r \) is the radius of the cylinder and \( l \) is its length. If a cylinder has radius 3 cm and length 2 cm, what is its volume?

(a) \( 9\pi \text{ cm}^3 \)  (b) \( 12\pi \text{ cm}^3 \)
(c) \( 18\pi \text{ cm}^3 \)  (d) \( 36\pi \text{ cm}^3 \)
(e) None of these

Problem 146. The formula for the volume of a cylinder is: \( V = \pi r^2 l \), where \( r \) is the radius of the cylinder and \( l \) is its length. If a cylinder has radius 2 cm and length 3 cm, what is its volume?

(a) \( 9\pi \text{ cm}^3 \)  (b) \( 12\pi \text{ cm}^3 \)
(c) \( 18\pi \text{ cm}^3 \)  (d) \( 36\pi \text{ cm}^3 \)
(e) None of these

Problem 147. The formula for the volume of a cylinder is: \( V = \pi r^2 l \), where \( r \) is the radius of the cylinder and \( l \) is its length. If a cylinder has radius 3 cm and length 4 cm, what is its volume?

(a) \( 9\pi \text{ cm}^3 \)  (b) \( 12\pi \text{ cm}^3 \)
(c) \( 18\pi \text{ cm}^3 \)  (d) \( 36\pi \text{ cm}^3 \)
(e) None of these

Problem 148. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 6 at a speed of 9, what is its centripetal acceleration?

(a) \( \frac{9}{4} \)  (b) 4
(c) \( \frac{27}{2} \)  (d) 54
(e) None of these
Problem 149. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 4 at a speed of 6, what is its centripetal acceleration?

(a) \( \frac{4}{9} \)  
(b) \( \frac{9}{4} \)  
(c) \( \frac{8}{3} \)  
(d) 9  
(e) None of these

Problem 150. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 6 at a speed of 4, what is its centripetal acceleration?

(a) \( \frac{4}{9} \)  
(b) \( \frac{9}{4} \)  
(c) \( \frac{8}{3} \)  
(d) 9  
(e) None of these

Problem 151. If an object is dropped from a height \( h \), the speed with which it hits the ground is: \( v = \sqrt{2gh} \), where \( g \) is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 5, and the gravitational acceleration is 10, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 5  
(b) \( \sqrt{50} \)  
(c) 10  
(d) \( \sqrt{200} \)  
(e) None of these

Problem 152. If an object is dropped from a height \( h \), the speed with which it hits the ground is: \( v = \sqrt{2gh} \), where \( g \) is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 6, and the gravitational acceleration is 2, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) \( \sqrt{12} \)  
(b) 3  
(c) \( \sqrt{24} \)  
(d) 9  
(e) None of these
Problem 153. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 3, and the gravitational acceleration is 5, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) $\sqrt{15}$  (b) 15  
(c) $\sqrt{30}$  (d) 30  
(e) None of these

Calculator-based problems

Problem 154. The formula for the volume of a cylinder is: $V = \pi r^2 l$, where $r$ is the radius of the cylinder and $l$ is its length. If a cylinder has radius 5.22 and length 7.80, what is its volume? Round your answer to the nearest integer.

(a) 487  (b) 541  
(c) 601  (d) 668  
(e) None of these

Problem 155. The formula for the volume of a cylinder is: $V = \pi r^2 l$, where $r$ is the radius of the cylinder and $l$ is its length. If a cylinder has radius 3.03 and length 5.51, what is its volume? Round your answer to the nearest integer.

(a) 116  (b) 129  
(c) 143  (d) 159  
(e) None of these

Problem 156. The formula for the volume of a cylinder is: $V = \pi r^2 l$, where $r$ is the radius of the cylinder and $l$ is its length. If a cylinder has radius 6.99 and length 2.43, what is its volume? Round your answer to the nearest integer.

(a) 373  (b) 410  
(c) 451  (d) 496  
(e) None of these

Problem 157. The formula for the centripetal acceleration of an object moving in a circle is: $a = \frac{v^2}{r}$, where $v$ is the object’s speed and $r$ is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 4.8 at a speed of 13, what is its centripetal acceleration? Round your answer to the nearest integer.

(a) 32  (b) 35  
(c) 39  (d) 43  
(e) None of these
Problem 158. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 4.4 at a speed of 19, what is its centripetal acceleration? Round your answer to the nearest integer.

(a) 60 (b) 66
(c) 74 (d) 82
(e) None of these

Problem 159. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you don’t know what centripetal acceleration is, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 8.2 at a speed of 27, what is its centripetal acceleration? Round your answer to the nearest integer.

(a) 70 (b) 82
(c) 89 (d) 98
(e) None of these

Problem 160. If an object is dropped from a height \( h \), the speed with which it hits the ground is: \( v = \sqrt{2gh} \), where \( g \) is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 23, and the gravitational acceleration is 32, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 31 (b) 35
(c) 38 (d) 42
(e) None of these

Problem 161. If an object is dropped from a height \( h \), the speed with which it hits the ground is: \( v = \sqrt{2gh} \), where \( g \) is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 39, and the gravitational acceleration is 9.8, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 20 (b) 22
(c) 25 (d) 28
(e) None of these
Problem 162. If an object is dropped from a height \( h \), the speed with which it hits the ground is: \( v = \sqrt{2gh} \), where \( g \) is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 120, and the gravitational acceleration is 32, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 71  
(b) 79  
(c) 88  
(d) 96  
(e) None of these

Problem 163. If an object is dropped from a height \( h \), the speed with which it hits the ground is: \( v = \sqrt{2gh} \), where \( g \) is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 56, and the gravitational acceleration is 9.8, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 33  
(b) 36  
(c) 40  
(d) 44  
(e) None of these

2.3.2 Functions of three variables

Non-calculator based problems

Problem 164. The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 3 \), \( a = 2 \), and \( t = 5 \)?

(a) 10  
(b) 13  
(c) 25  
(d) 30  
(e) None of these

Problem 165. The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 2 \), \( a = 3 \), and \( t = 5 \)?

(a) 17  
(b) 19  
(c) 25  
(d) 27  
(e) None of these

Problem 166. The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 5 \), \( a = 2 \), and \( t = 3 \)?

(a) 11  
(b) 21  
(c) 25  
(d) 30  
(e) None of these
Problem 167. The force exerted by a stretched spring is given by the formula: $F = k(x - x_0)$, where $k$ is the spring constant, $x$ is the stretched length of the spring, and $x_0$ is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force $F$ if $k = 10$, $x = 5$, and $x_0 = 4$?

(a) 10  (b) 45  
(c) 46  (d) 90  
(e) None of these

Problem 168. The force exerted by a stretched spring is given by the formula: $F = k(x - x_0)$, where $k$ is the spring constant, $x$ is the stretched length of the spring, and $x_0$ is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force $F$ if $k = 5$, $x = 10$, and $x_0 = 4$?

(a) 10  (b) 30  
(c) 46  (d) 60  
(e) None of these

Problem 169. The force exerted by a stretched spring is given by the formula: $F = k(x - x_0)$, where $k$ is the spring constant, $x$ is the stretched length of the spring, and $x_0$ is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force $F$ if $k = 4$, $x = 10$, and $x_0 = 5$?

(a) 10  (b) 20  
(c) 35  (d) 40  
(e) None of these

Problem 170. The mass of a cylindrical weight is given by the formula: $m = \pi r^2 l \rho$, where $r$ is the weight’s radius, $l$ is its length, and $\rho$ is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including $\rho$, which is “rho”.) What is the mass of such a weight if its radius is 2, its length is 3, and its density is 5?

(a) $10\pi$  (b) $30\pi$  
(c) $60\pi$  (d) $90\pi$  
(e) None of these

Problem 171. The mass of a cylindrical weight is given by the formula: $m = \pi r^2 l \rho$, where $r$ is the weight’s radius, $l$ is its length, and $\rho$ is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including $\rho$, which is “rho”.) What is the mass of such a weight if its radius is 3, its length is 2, and its density is 5?

(a) $10\pi$  (b) $30\pi$  
(c) $60\pi$  (d) $90\pi$  
(e) None of these
**Problem 172.** The mass of a cylindrical weight is given by the formula: \( m = \pi r^2 l \rho \), where \( r \) is the weight’s radius, \( l \) is its length, and \( \rho \) is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including \( \rho \), which is “rho”.) What is the mass of such a weight if its radius is 3, its length is 5, and its density is 2?

(a) 10\( \pi \)  
(b) 30\( \pi \)  
(c) 60\( \pi \)  
(d) 90\( \pi \)  
(e) None of these

**Calculator-based problems**

**Problem 173.** The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 12 \), \( a = 1.8 \), and \( t = 14 \)? Round your answer to the nearest integer.

(a) 30  
(b) 33  
(c) 37  
(d) 41  
(e) None of these

**Problem 174.** The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 45 \), \( a = 2.9 \), and \( t = 8.3 \)? Round your answer to the nearest integer.

(a) 62  
(b) 69  
(c) 76  
(d) 84  
(e) None of these

**Problem 175.** The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 28 \), \( a = -2.1 \), and \( t = 5.6 \)? Round your answer to the nearest integer.

(a) 15  
(b) 16  
(c) 18  
(d) 20  
(e) None of these

**Problem 176.** The force exerted by a stretched spring is given by the formula: \( F = k(x - x_0) \), where \( k \) is the spring constant, \( x \) is the stretched length of the spring, and \( x_0 \) is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force \( F \) if \( k = 28 \), \( x = 19 \), and \( x_0 = 12 \)? Round your answer to the nearest integer.

(a) 143  
(b) 159  
(c) 176  
(d) 196  
(e) None of these
Problem 177. The force exerted by a stretched spring is given by the formula: 
\[ F = k(x - x_0), \]
where \( k \) is the spring constant, \( x \) is the stretched length of the spring, and \( x_0 \) is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force \( F \) if \( k = 1700, x = 0.23, \) and \( x_0 = 0.19? \) Round your answer to the nearest integer.

(a) 68  (b) 75
(c) 82  (d) 91
(e) None of these

Problem 178. The force exerted by a stretched spring is given by the formula: 
\[ F = k(x - x_0), \]
where \( k \) is the spring constant, \( x \) is the stretched length of the spring, and \( x_0 \) is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force \( F \) if \( k = 2700, x = 0.41, \) and \( x_0 = 0.34? \) Round your answer to the nearest integer.

(a) 153  (b) 170
(c) 189  (d) 208
(e) None of these

Problem 179. The mass of a cylindrical weight is given by the formula: 
\[ m = \pi r^2 l \rho, \]
where \( r \) is the weight’s radius, \( l \) is its length, and \( \rho \) is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including \( \rho \), which is “rho”.) What is the mass of such a weight if its radius is 1.7, its length is 5.1, and its density is 7.9? Round your answer to the nearest integer.

(a) 296  (b) 329
(c) 366  (d) 402
(e) None of these

Problem 180. The mass of a cylindrical weight is given by the formula: 
\[ m = \pi r^2 l \rho, \]
where \( r \) is the weight’s radius, \( l \) is its length, and \( \rho \) is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including \( \rho \), which is “rho”.) What is the mass of such a weight if its radius is 0.32, its length is 1.1, and its density is 8.8? Round your answer to one decimal place.

(a) 3.1  (b) 3.5
(c) 3.8  (d) 4.2
(e) None of these

Problem 181. The mass of a cylindrical weight is given by the formula: 
\[ m = \pi r^2 l \rho, \]
where \( r \) is the weight’s radius, \( l \) is its length, and \( \rho \) is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including \( \rho \), which is “rho”.) What is the mass of such a weight if its radius is 0.55, its length is 1.8, and its density is 19.3? Round your answer to the nearest integer.

(a) 33  (b) 36
(c) 40  (d) 44
(e) None of these
2.4 Solving linear equations

2.4.1 Linear equations with numerical solutions

Problem 182. If $7x + 11 = 8$, find $x$. Which of the following statements is true?
(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these

Problem 183. If $9x - 11 = 7$, find $x$. Which of the following statements is true?
(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these

Problem 184. If $5x + 3 = 10$, find $x$. Which of the following statements is true?
(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these

Problem 185. If $3x - 2 = -9$, find $x$. Which of the following statements is true?
(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these

Problem 186. If $-4x + 5 = 8$, find $x$. Which of the following statements is true?
(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these

Problem 187. If $5x - 7 = 3x + 8$, find $x$. Which of the following statements is true?
(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these

Problem 188. If $2x - 4 = 5x + 7$, find $x$. Which of the following statements is true?
(a) $x < -1$  
(b) $-1 \leq x < 0$
(c) $0 \leq x < 1$  
(d) $x \geq 1$
(e) None of these
Problem 189. If \(-3x - 8 = x - 11\), find \(x\). Which of the following statements is true?

(a) \(x < -1\)  
(b) \(-1 \leq x < 0\)  
(c) \(0 \leq x < 1\)  
(d) \(x \geq 1\)  
(e) None of these

Problem 190. If \(-3x - 4 = 2x + 7\), find \(x\). Which of the following statements is true?

(a) \(x < -1\)  
(b) \(-1 \leq x < 0\)  
(c) \(0 \leq x < 1\)  
(d) \(x \geq 1\)  
(e) None of these

Problem 191. If \(x + 7 = -4x + 9\), find \(x\). Which of the following statements is true?

(a) \(x < -1\)  
(b) \(-1 \leq x < 0\)  
(c) \(0 \leq x < 1\)  
(d) \(x \geq 1\)  
(e) None of these

2.4.2 Linear equations with variable-expression solutions

Problem 192. If \(PV = nRT\), find \(P\).

(a) \(P = \frac{V}{nRT}\)  
(b) \(P = \frac{nRT}{V}\)  
(c) \(P = V - nRT\)  
(d) \(P = nRT - V\)  
(e) None of these

Problem 193. If \(PV = nRT\), find \(T\).

(a) \(T = \frac{PV}{nR}\)  
(b) \(T = \frac{nR}{PV}\)  
(c) \(T = PV - nR\)  
(d) \(T = nR - PV\)  
(e) None of these

Problem 194. If \(m = \pi r^2 l \rho\), find \(\rho\).

(a) \(\rho = m - \pi r^2 l\)  
(b) \(\rho = \pi r^2 l - m\)  
(c) \(\rho = \frac{m}{\pi r^2 l}\)  
(d) \(\rho = \frac{\pi r^2 l}{m}\)  
(e) None of these
Problem 195. If \( m = \pi r^2 l \rho \), find \( l \).
(a) \( l = m - \pi r^2 \rho \)  
(b) \( l = \pi r^2 \rho - m \)
(c) \( l = \frac{m}{\pi r^2 \rho} \)  
(d) \( l = \frac{\pi r^2 \rho}{m} \)
(e) None of these

Problem 196. If \( v = v_0 + at \), find \( v_0 \).
(a) \( v_0 = \frac{v}{at} \)  
(b) \( v_0 = -\frac{v}{at} \)
(c) \( v_0 = v + at \)  
(d) \( v_0 = v - at \)
(e) None of these

Problem 197. If \( v = v_0 + at \), find \( a \).
(a) \( a = \frac{v - v_0}{t} \)  
(b) \( a = \frac{v}{t} - v_0 \)
(c) \( a = \frac{vt - v_0}{t} \)  
(d) \( a = v - \frac{v_0}{t} \)
(e) None of these

Problem 198. If \( v = v_0 + at \), find \( t \).
(a) \( t = \frac{v}{a} - v_0 \)  
(b) \( t = \frac{va - v_0}{a} \)
(c) \( t = v - \frac{v_0}{a} \)  
(d) \( t = \frac{v - v_0}{a} \)
(e) None of these

Problem 199. If \( F = k(x - x_0) \), find \( k \).
(a) \( k = \frac{x_0 - x}{F} \)  
(b) \( k = \frac{x - x_0}{F} \)
(c) \( k = \frac{F}{x_0 - x} \)  
(d) \( k = \frac{F}{x - x_0} \)
(e) None of these

Problem 200. If \( F = k(x - x_0) \), find \( x \).
(a) \( x = \frac{k - F}{x_0} \)  
(b) \( x = \frac{F + kx_0}{k} \)
(c) \( x = F - kx_0 \)  
(d) \( x = F + kx_0 \)
(e) None of these
Problem 201. If $F = k(x - x_0)$, find $x_0$.

(a) $x_0 = \frac{kx - F}{k}$  
(b) $x_0 = \frac{F + kx}{k}$

(c) $x_0 = \frac{F - x}{k}$  
(d) $x_0 = \frac{F + x}{k}$

(e) None of these
2.5 Solving systems of linear equations

Problem 202. For the following pair of equations, find $x$ and $y$:

\[
\begin{align*}
5x - 2y &= 1 \\
-2x + y &= 2
\end{align*}
\]

What is the product $xy$?

(a) $xy = -48$  
(b) $xy = -60$  
(c) $xy = 48$  
(d) $xy = 60$  
(e) None of these

Problem 203. For the following pair of equations, find $x$ and $y$:

\[
\begin{align*}
2x + y &= 4 \\
3x + 2y &= 12
\end{align*}
\]

What is the product $xy$?

(a) $xy = -48$  
(b) $xy = -60$  
(c) $xy = 48$  
(d) $xy = 60$  
(e) None of these

Problem 204. For the following pair of equations, find $x$ and $y$:

\[
\begin{align*}
6x + y &= -2 \\
x - y &= 23
\end{align*}
\]

What is the product $xy$?

(a) $xy = -48$  
(b) $xy = -60$  
(c) $xy = 48$  
(d) $xy = 60$  
(e) None of these

Problem 205. For the following pair of equations, find $x$ and $y$:

\[
\begin{align*}
-x + y &= 2 \\
3x - 2y &= 2
\end{align*}
\]

What is the product $xy$?

(a) $xy = -48$  
(b) $xy = -60$  
(c) $xy = 48$  
(d) $xy = 60$  
(e) None of these
Problem 206. For the following pair of equations, find $x$ and $y$:

\[-x + y = 15\]
\[2x + 3y = 0\]

What is the product $xy$?

(a) $xy = -54$  (b) $xy = -56$
(c) $xy = 54$  (d) $xy = 56$
(e) None of these

Problem 207. For the following pair of equations, find $x$ and $y$:

\[x + 3y = 2\]
\[2x + 5y = 8\]

What is the product $xy$?

(a) $xy = -54$  (b) $xy = -56$
(c) $xy = 54$  (d) $xy = 56$
(e) None of these

Problem 208. For the following pair of equations, find $x$ and $y$:

\[5x - y = -3\]
\[-3x + y = 9\]

What is the product $xy$?

(a) $xy = -54$  (b) $xy = -56$
(c) $xy = 54$  (d) $xy = 56$
(e) None of these

Problem 209. For the following pair of equations, find $x$ and $y$:

\[x + y = 15\]
\[x - y = 1\]

What is the product $xy$?

(a) $xy = -54$  (b) $xy = -56$
(c) $xy = 54$  (d) $xy = 56$
(e) None of these
2.6 Solving quadratic equations

2.6.1 Factoring quadratic equations

Problem 210. Solve the equation: \( x^2 - 2bx + b^2 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  \hspace{1cm} (b) \( x_1 - x_2 = 2b \)
(c) \( x_1 - x_2 = -2b \)  \hspace{1cm} (d) \( x_1 - x_2 = 1 \)
(e) None of these

Problem 211. Solve the equation: \( x^2 - 11x + 28 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  \hspace{1cm} (b) \( x_1 - x_2 = 3 \)
(c) \( x_1 - x_2 = 8 \)  \hspace{1cm} (d) \( x_1 - x_2 = 11 \)
(e) None of these

Problem 212. Solve the equation: \( x^2 + 4x - 21 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  \hspace{1cm} (b) \( x_1 - x_2 = 4 \)
(c) \( x_1 - x_2 = 10 \)  \hspace{1cm} (d) \( x_1 - x_2 = 17 \)
(e) None of these

Problem 213. Solve the equation: \( x^2 - 3x - 18 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  \hspace{1cm} (b) \( x_1 - x_2 = 3 \)
(c) \( x_1 - x_2 = 6 \)  \hspace{1cm} (d) \( x_1 - x_2 = 9 \)
(e) None of these

Problem 214. Solve the equation: \( x^2 + 8x + 15 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  \hspace{1cm} (b) \( x_1 - x_2 = 2 \)
(c) \( x_1 - x_2 = 7 \)  \hspace{1cm} (d) \( x_1 - x_2 = 8 \)
(e) None of these

Problem 215. Solve the equation: \( 2x^2 - 5x + 3 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  \hspace{1cm} (b) \( x_1 - x_2 = \frac{1}{2} \)
(c) \( x_1 - x_2 = \frac{3}{2} \)  \hspace{1cm} (d) \( x_1 - x_2 = \frac{5}{2} \)
(e) None of these
Problem 216. Solve the equation: \(2x^2 + 11x + 5 = 0\). There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) What is the difference \(x_1 - x_2\)?

(a) \(x_1 - x_2 = 0\)  
(b) \(x_1 - x_2 = \frac{5}{2}\)  
(c) \(x_1 - x_2 = 3\)  
(d) \(x_1 - x_2 = \frac{9}{2}\)  
(e) None of these

Problem 217. Solve the equation: \(x^2 - 4x = 5\). There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) What is the difference \(x_1 - x_2\)?

(a) \(x_1 - x_2 = 0\)  
(b) \(x_1 - x_2 = 1\)  
(c) \(x_1 - x_2 = 4\)  
(d) \(x_1 - x_2 = 6\)  
(e) None of these

Problem 218. Solve the equation: \(x^2 + 7x = -10\). There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) What is the difference \(x_1 - x_2\)?

(a) \(x_1 - x_2 = 0\)  
(b) \(x_1 - x_2 = 3\)  
(c) \(x_1 - x_2 = 5\)  
(d) \(x_1 - x_2 = 7\)  
(e) None of these

2.6.2 Quadratic formula

Problem 219. Use a calculator or equivalent to solve the equation:

\[1.1x^2 + 3.4x - 5.5 = 0\]

There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) Which of the following statements is true of the larger solution \(x_1\)?

(a) \(x_1 < 1\)  
(b) \(1 \leq x_1 < 1.5\)  
(c) \(1.5 \leq x_1 < 2\)  
(d) \(2 \leq x_1\)  
(e) No real solution

Problem 220. Use a calculator or equivalent to solve the equation:

\[2.4x^2 + 3.7x - 0.61 = 0\]

There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) Which of the following statements is true of the larger solution \(x_1\)?

(a) \(x_1 < -1\)  
(b) \(-1 \leq x_1 < 0\)  
(c) \(0 \leq x_1 < 1\)  
(d) \(1 \leq x_1\)  
(e) No real solution
Problem 221. Use a calculator or equivalent to solve the equation:

\[ 0.82x^2 - 2.9x + 0.32 = 0 \]

There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) Which of the following statements is true of the larger solution \( x_1 \)?

(a) \( x_1 < -1 \)  
(b) \( -1 \leq x_1 < -0.5 \)  
(c) \( -0.5 \leq x_1 < 0 \)  
(d) \( 0 \leq x_1 \)  
(e) No real solution

Problem 222. Use a calculator or equivalent to solve the equation:

\[ 1.7x^2 + 8.9x - 0.77 = 0 \]

There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) Which of the following statements is true of the larger solution \( x_1 \)?

(a) \( x_1 < -2 \)  
(b) \( -2 \leq x_1 < 0 \)  
(c) \( 0 \leq x_1 < 2 \)  
(d) \( 2 \leq x_1 \)  
(e) No real solution

2.7 Algebra word problems

Problem 223. The length of a carpet is 3 feet greater than its width. The area of the carpet is 80 square feet. Which of the following equations describes the carpet’s width?

(a) \( w^2 + 3w - 80 = 0 \)  
(b) \( w^2 - 3w + 80 = 0 \)  
(c) \( w^2 - 20 = 0 \)  
(d) \( w^2 + 20 = 0 \)  
(e) None of these

Problem 224. A window is twice as long as it is wide. The area of the window is 40 square feet. Which of the following equations describes the window’s length?

(a) \( 3l^2 - 40 = 0 \)  
(b) \( l^2 - 20 = 0 \)  
(c) \( l^2 - 120 = 0 \)  
(d) \( l^2 - 80 = 0 \)  
(e) None of these

Problem 225. In the physics lab, you find two circular pieces of sheet metal. The radius of one of the circles is 3 centimeters greater than the radius of the other. The area of the larger circle is twice the area of the smaller one. Which of the following equations describes the radius of the smaller circle?

(a) \( r^2 - 6r - 9 = 0 \)  
(b) \( r^2 - 3r - 9 = 0 \)  
(c) \( r^2 + 6r + 9 = 0 \)  
(d) \( r^2 + 3r + 9 = 0 \)  
(e) None of these
Problem 226. A rectangular window is twice as long as it is wide. If its length were increased by 3 feet and its width were decreased by 1 foot, it would have the same area. Which of the following equations describes the window’s width?

(a) $w^2 - w + 3 = 0$  
(b) $w - 3 = 0$

(c) $w^2 + w - 3 = 0$  
(d) $w^2 - w - 3 = 0$

(e) None of these

Problem 227. The distance from Tucson to Phoenix is 120 miles. You want to drive there and back at an average speed of 60 miles per hour. Because of traffic congestion, your average speed from Tucson to Phoenix is 40 mph. How fast do you have to drive on the return trip?

(a) 60 miles per hour  
(b) 80 miles per hour

(c) 90 miles per hour  
(d) 120 miles per hour

(e) None of these

Problem 228. You drive 40 miles at 60 miles per hour, then bicycle an additional 10 miles at 12 miles per hour. What is your average speed for the entire trip?

(a) $\frac{33\frac{1}{3}}{}$ miles per hour  
(b) 36 miles per hour

(c) 50 miles per hour  
(d) 50.4 miles per hour

(e) None of these

Problem 229. A boat moves at 5 miles per hour in still water. It is launched in a river that flows at 3 miles per hour. From its launch point, it goes downstream for 4 miles, then turns around and comes back upstream to the launch point. How long does the round trip take?

(a) $\frac{4}{5}$ hours  
(b) $\frac{8}{5}$ hours

(c) 2 hours  
(d) $\frac{5}{2}$ hours

(e) None of these

Problem 230. An airplane flies at a speed of 80 miles per hour in still air. On a day when the wind is blowing from the north at 20 miles per hour, the airplane flies 200 miles straight north, then turns around and returns to its starting point. What is its average speed on the round trip?

(a) 64 miles per hour  
(b) $66\frac{2}{3}$ miles per hour

(c) 75 miles per hour  
(d) 80 miles per hour

(e) None of these
Problem 231. A runner and a bicyclist start from the same point at the same time, with the runner going straight north and the bicyclist going straight south. The bicyclist is 7 miles per hour faster than the runner. At the end of two hours, the two are 60 miles apart. What is the bicyclist’s speed?

(a) $11\frac{1}{2}$ miles per hour  
(b) 14 miles per hour  
(c) $18\frac{1}{2}$ miles per hour  
(d) 23 miles per hour

(e) None of these

Problem 232. You drive from Smithtown to Jonesville at a speed of $v$, making the trip in time $t$. On the return trip, you are able to drive 10 miles per hour faster, which shortens your travel time by one hour. Which of the following equations is true?

(a) $(v - 10)t = v(t + 1)$  
(b) $(v + 10)t = v(t - 1)$  
(c) $(v - 10)(t + 1) = vt$  
(d) $(v + 10)(t - 1) = vt$  
(e) None of these
3 Graphs

3.1 Single points on graphs

Problem 233. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\).

Which of the four points is \((3, 3)\)?

(a) A  (b) B  (c) C  (d) D

Problem 234. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\).

Which of the four points is \((2, -5)\)?

(a) A  (b) B  (c) C  (d) D

Problem 235. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\).

Which of the four points is \((-6, 5)\)?

(a) A  (b) B  (c) C  (d) D

Problem 236. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\).

Which of the four points is \((-4, -2)\)?

(a) A  (b) B  (c) C  (d) D
Problem 237. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5),\) and \((-4, -2)\).

Which of the four points is \(A\)?
(a) \((3, 3)\)  (b) \((2, -5)\)
(c) \((-6, 5)\)  (d) \((-4, -2)\)

Problem 238. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5),\) and \((-4, -2)\).

Which of the four points is \(B\)?
(a) \((3, 3)\)  (b) \((2, -5)\)
(c) \((-6, 5)\)  (d) \((-4, -2)\)

Problem 239. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5),\) and \((-4, -2)\).

Which of the four points is \(C\)?
(a) \((3, 3)\)  (b) \((2, -5)\)
(c) \((-6, 5)\)  (d) \((-4, -2)\)

Problem 240. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5),\) and \((-4, -2)\).

Which of the four points is \(D\)?
(a) \((3, 3)\)  (b) \((2, -5)\)
(c) \((-6, 5)\)  (d) \((-4, -2)\)
**Problem 241.** The graph at right shows four points labelled with letters. The points are 

(2, 2), (3, 9), (7, 2), and (8, 8).

Which of the four points is $A$?
(a) (2, 2)  
(b) (3, 9)  
(c) (7, 2)  
(d) (8, 8)

**Problem 242.** The graph at right shows four points labelled with letters. The points are 

(2, 2), (3, 9), (7, 2), and (8, 8).

Which of the four points is $B$?
(a) (2, 2)  
(b) (3, 9)  
(c) (7, 2)  
(d) (8, 8)

**Problem 243.** The graph at right shows four points labelled with letters. The points are 

(2, 2), (3, 9), (7, 2), and (8, 8).

Which of the four points is $C$?
(a) (2, 2)  
(b) (3, 9)  
(c) (7, 2)  
(d) (8, 8)

**Problem 244.** The graph at right shows four points labelled with letters. The points are 

(2, 2), (3, 9), (7, 2), and (8, 8).

Which of the four points is $D$?
(a) (2, 2)  
(b) (3, 9)  
(c) (7, 2)  
(d) (8, 8)
Problem 245. The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \((2, 2)\)?

(a) A  (b) B  
(c) C  (d) D

Problem 246. The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \((3, 9)\)?

(a) A  (b) B  
(c) C  (d) D

Problem 247. The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \((7, 2)\)?

(a) A  (b) B  
(c) C  (d) D

Problem 248. The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \((8, 8)\)?

(a) A  (b) B  
(c) C  (d) D
3.2 Matching graphs and equations

Problem 249. Which equation is shown on the graph at right?
(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = -2x$
(d) $y = -2x - 1$

Problem 250. Which equation is shown on the graph at right?
(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = -2x$
(d) $y = -2x - 1$

Problem 251. Which equation is shown on the graph at right?
(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = -2x$
(d) $y = -2x - 1$

Problem 252. Which equation is shown on the graph at right?
(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = -2x$
(d) $y = -2x - 1$
Problem 253. Which equation is shown on the graph at right? (The scale is the same for the $x$- and $y$-axes.)

(a) $y = 3x$
(b) $y = -3x$
(c) $y = \frac{x}{3}$
(d) $y = -\frac{x}{3}$

Problem 254. Which equation is shown on the graph at right? (The scale is the same for the $x$- and $y$-axes.)

(a) $y = 3x$
(b) $y = -3x$
(c) $y = \frac{x}{3}$
(d) $y = -\frac{x}{3}$

Problem 255. Which equation is shown on the graph at right? (The scale is the same for the $x$- and $y$-axes.)

(a) $y = 3x$
(b) $y = -3x$
(c) $y = \frac{x}{3}$
(d) $y = -\frac{x}{3}$

Problem 256. Which equation is shown on the graph at right? (The scale is the same for the $x$- and $y$-axes.)

(a) $y = 3x$
(b) $y = -3x$
(c) $y = \frac{x}{3}$
(d) $y = -\frac{x}{3}$
Problem 257. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = x + 1 \)?

Problem 258. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = x - 1 \)?

Problem 259. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = -x + 1 \)?
Problem 260. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = -x - 1 \)?

Problem 261. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

Problem 262. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

Problem 263. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)
Problem 264. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

Problem 265. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[
\begin{align*}
 y &= 2x^2 \\
 y &= \frac{x^2}{2} \\
 y &= -2x^2 \\
 y &= -\frac{x^2}{2}
\end{align*}
\]

Which of the four graphs represents \( y = 2x^2 \)?

Problem 266. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[
\begin{align*}
 y &= 2x^2 \\
 y &= \frac{x^2}{2} \\
 y &= -2x^2 \\
 y &= -\frac{x^2}{2}
\end{align*}
\]

Which of the four graphs represents \( y = \frac{x^2}{2} \)?
Problem 267. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[ y = 2x^2 \quad y = \frac{x^2}{2} \quad y = -2x^2 \quad y = -\frac{x^2}{2} \]

Which of the four graphs represents \( y = -2x^2 \)?

Problem 268. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[ y = 2x^2 \quad y = \frac{x^2}{2} \quad y = -2x^2 \quad y = -\frac{x^2}{2} \]

Which of the four graphs represents \( y = -\frac{x^2}{2} \)?
Part II

Mathematical Preliminaries
4 Basic Trigonometry

4.1 Arc-Length Problems using $s = \theta r$

4.1.1 Degrees to radians: formula

Problem 269. An angle measures $29^\circ$. What is its measurement in radians?

(a) $\frac{29\pi}{360}$ rad  (b) $\frac{29\pi}{180}$ rad

(c) $\frac{29 \cdot 360}{\pi}$ rad  (d) $\frac{29 \cdot 180}{\pi}$ rad

(e) None of these

Problem 270. An angle measures $47^\circ$. What is its measurement in radians?

(a) $\frac{47\pi}{180}$ rad  (b) $\frac{47}{360\pi}$ rad

(c) $\frac{47}{180\pi}$ rad  (d) $\frac{47}{2\pi}$ rad

(e) None of these

Problem 271. An angle measures $61^\circ$. What is its measurement in radians?

(a) $\frac{61\pi}{180}$ rad  (b) $\frac{61\pi}{90}$ rad

(c) $\frac{61}{\pi}$ rad  (d) $\frac{61 \cdot 360}{\pi}$ rad

(e) None of these

Problem 272. An angle measures $83^\circ$. What is its measurement in radians?

(a) $\frac{83}{\pi}$ rad  (b) $\frac{83\pi}{180}$ rad

(c) $\frac{83 \cdot 360}{\pi}$ rad  (d) $\frac{83}{2\pi}$ rad

(e) None of these

Problem 273. An angle measures $19^\circ$. What is its measurement in radians?

(a) $\frac{19}{\pi}$ rad  (b) $\frac{19\pi}{180}$ rad

(c) $\frac{19 \cdot 360}{\pi}$ rad  (d) $\frac{19\pi}{90}$ rad

(e) None of these
Problem 274. An angle measures $53^\circ$. What is its measurement in radians?

(a) $\frac{53 \cdot 360}{\pi}$ rad (b) $\frac{53}{2\pi}$ rad

(c) $\frac{53\pi}{180}$ rad (d) $\frac{53\pi}{360}$ rad

(e) None of these

Problem 275. An angle measures $7^\circ$. What is its measurement in radians?

(a) $\frac{7 \cdot 90}{\pi}$ rad (b) $\frac{7 \cdot 180}{\pi}$ rad

(c) $\frac{7\pi}{180}$ rad (d) $\frac{7}{2\pi}$ rad

(e) None of these

Problem 276. An angle measures $73^\circ$. What is its measurement in radians?

(a) $\frac{73 \cdot 180}{\pi}$ rad (b) $\frac{73\pi}{180}$ rad

(c) $\frac{73\cdot 360}{\pi}$ rad (d) $\frac{73\pi}{360}$ rad

(e) None of these

Problem 277. An angle measures $43^\circ$. What is its measurement in radians?

(a) $\frac{43\pi}{180}$ rad (b) $\frac{43 \cdot 90}{\pi}$ rad

(c) $\frac{43\pi}{360}$ rad (d) $\frac{43}{\pi}$ rad

(e) None of these

Problem 278. An angle measures $59^\circ$. What is its measurement in radians?

(a) $\frac{59 \cdot 180}{\pi}$ rad (b) $\frac{59}{\pi}$ rad

(c) $\frac{59\pi}{180}$ rad (d) $\frac{59\pi}{360}$ rad

(e) None of these
4.1.2 Radians to degrees: formula

Problem 279. An angle measures 0.40 radians. What is its measurement in degrees?

(a) \(\frac{0.40\pi}{360}\) degrees  
(b) \(\frac{0.40\pi}{180}\) degrees  
(c) \(\frac{(0.40)(360)}{\pi}\) degrees  
(d) \(\frac{(0.40)(180)}{\pi}\) degrees  
(e) None of these

Problem 280. An angle measures 1.06 radians. What is its measurement in degrees?

(a) \(\frac{1.06\pi}{180}\) degrees  
(b) \(\frac{(1.06)(90)}{\pi}\) degrees  
(c) \(\frac{1.06}{2\pi}\) degrees  
(d) \(\frac{(1.06)(180)}{\pi}\) degrees  
(e) None of these

Problem 281. An angle measures 0.88 radians. What is its measurement in degrees?

(a) \(\frac{(0.88)(180)}{\pi}\) degrees  
(b) \(\frac{(0.88)(360)}{\pi}\) degrees  
(c) \(\frac{(0.88)(90)}{\pi}\) degrees  
(d) \(\frac{(0.88)}{2\pi}\) degrees  
(e) None of these

Problem 282. An angle measures 0.04 radians. What is its measurement in degrees?

(a) \(\frac{(0.04)(360)}{\pi}\) degrees  
(b) \(\frac{0.04\pi}{360}\) degrees  
(c) \(\frac{0.04\pi}{90}\) degrees  
(d) \(\frac{(0.04)(180)}{\pi}\) degrees  
(e) None of these

Problem 283. An angle measures 0.15 radians. What is its measurement in degrees?

(a) \(\frac{0.15}{\pi}\) degrees  
(b) \(\frac{0.15\pi}{90}\) degrees  
(c) \(\frac{0.15}{2\pi}\) degrees  
(d) \(\frac{(0.15)(180)}{\pi}\) degrees  
(e) None of these

Problem 284. An angle measures 1.34 radians. What is its measurement in degrees?

(a) \(\frac{(1.34)(180)}{\pi}\) degrees  
(b) \(\frac{1.34\pi}{90}\) degrees  
(c) \(\frac{1.34\pi}{180}\) degrees  
(d) \(\frac{(1.34)(90)}{\pi}\) degrees  
(e) None of these
Problem 285. An angle measures 0.77 radians. What is its measurement in degrees?

(a) $\frac{0.77}{2\pi}$ degrees  
(b) $\frac{0.77\pi}{360}$ degrees  
(c) $\frac{0.77}{\pi}$ degrees  
(d) $\frac{(0.77)(180)}{\pi}$ degrees  
(e) None of these

Problem 286. An angle measures 0.12 radians. What is its measurement in degrees?

(a) $\frac{(0.12)(180)}{\pi}$ degrees  
(b) $\frac{0.12}{2\pi}$ degrees  
(c) $\frac{(0.12)(360)}{\pi}$ degrees  
(d) $\frac{0.12\pi}{360}$ degrees  
(e) None of these

Problem 287. An angle measures 0.85 radians. What is its measurement in degrees?

(a) $\frac{(0.85)(360)}{\pi}$ degrees  
(b) $\frac{0.85}{\pi}$ degrees  
(c) $\frac{(0.85)(180)}{\pi}$ degrees  
(d) $\frac{0.85\pi}{90}$ degrees  
(e) None of these

Problem 288. An angle measures 0.34 radians. What is its measurement in degrees?

(a) $\frac{0.34\pi}{360}$ degrees  
(b) $\frac{(0.34)(180)}{\pi}$ degrees  
(c) $\frac{(0.34)(360)}{\pi}$ degrees  
(d) $\frac{0.34\pi}{180}$ degrees  
(e) None of these

4.1.3 Degrees to radians: calculator

Problem 289. An angle measures 38°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 1.51 rad  
(b) 0.66 rad  
(c) 2.65 rad  
(d) 0.33 rad  
(e) None of these

Problem 290. An angle measures 87°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 3.04 rad  
(b) 0.38 rad  
(c) 1.52 rad  
(d) 6.07 rad  
(e) None of these
Problem 291. An angle measures 7°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 0.03 rad  
(b) 0.12 rad  
(c) 0.24 rad  
(d) 0.49 rad  
(e) None of these

Problem 292. An angle measures 72°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 0.38 rad  
(b) 5.03 rad  
(c) 1.26 rad  
(d) 0.80 rad  
(e) None of these

Problem 293. An angle measures 12°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 4.77 rad  
(b) 0.05 rad  
(c) 0.21 rad  
(d) 0.42 rad  
(e) None of these

Problem 294. An angle measures 84°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 2.93 rad  
(b) 1.47 rad  
(c) 0.73 rad  
(d) 0.37 rad  
(e) None of these

Problem 295. An angle measures 21°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 2.73 rad  
(b) 0.18 rad  
(c) 1.47 rad  
(d) 0.37 rad  
(e) None of these

Problem 296. An angle measures 75°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 1.31 rad  
(b) 2.63 rad  
(c) 0.65 rad  
(d) 0.76 rad  
(e) None of these

Problem 297. An angle measures 63°. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 0.55 rad  
(b) 1.10 rad  
(c) 0.91 rad  
(d) 3.09 rad  
(e) None of these
Problem 298. An angle measures $42^\circ$. Use a calculator or equivalent to find its measurement in radians. Round your answer to the nearest 0.01 rad.

(a) 0.73 rad  
(b) 1.36 rad  
(c) 1.47 rad  
(d) 3.10 rad  
(e) None of these

4.1.4 Radians to degrees: calculator

Problem 299. An angle measures 0.52 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 30°  
(b) 7°  
(c) 119°  
(d) 15°  
(e) None of these

Problem 300. An angle measures 1.14 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 131°  
(b) 16°  
(c) 7°  
(d) 65°  
(e) None of these

Problem 301. An angle measures 0.11 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 57°  
(b) 6°  
(c) 1°  
(d) 13°  
(e) None of these

Problem 302. An angle measures 1.52 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 348°  
(b) 4°  
(c) 87°  
(d) 10°  
(e) None of these

Problem 303. An angle measures 0.37 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 5°  
(b) 21°  
(c) 42°  
(d) 11°  
(e) None of these

Problem 304. An angle measures 0.89 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 51°  
(b) 7°  
(c) 102°  
(d) 13°  
(e) None of these
Problem 305. An angle measures 1.25 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 286°  (b) 143°
(c) 72°  (d) 5°
(e) None of these

Problem 306. An angle measures 0.66 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 151°  (b) 4°
(c) 38°  (d) 19°
(e) None of these

Problem 307. An angle measures 1.73 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 4°  (b) 99°
(c) 50°  (d) 36°
(e) None of these

Problem 308. An angle measures 0.44 rad. Use a calculator or equivalent to find its measurement in degrees. Round your answer to the nearest degree.

(a) 13°  (b) 6°
(c) 14°  (d) 25°
(e) None of these

4.1.5 Arc length to radians: pictures

Problem 309. In the figure at right, what is the measure of the angle \( \theta \)?

(a) \( \sqrt{41} \) rad  (b) 3\( \pi \) rad
(c) \( 5/4 \) rad  (d) 4\( \pi /5 \) rad
(e) None of these

Problem 310. In the figure at right, what is the measure of the angle \( \theta \)?

(a) 10\( \pi /9 \) rad  (b) \( \sqrt{19\pi} \) rad
(c) 9\( \pi /10 \) rad  (d) 10/9 rad
(e) None of these
Problem 311. In the figure at right, what is the measure of the angle $\theta$?

(a) $\frac{5}{3}$ rad  
(b) $\frac{3}{5}$ rad  
(c) 4 rad  
(d) $4\pi$ rad  
(e) None of these

Problem 312. In the figure at right, what is the measure of the angle $\theta$?

(a) 1 rad  
(b) $\sqrt{2}\pi$ rad  
(c) 3 rad  
(d) $\pi$ rad  
(e) None of these

Problem 313. In the figure at right, what is the measure of the angle $\theta$?

(a) $\sqrt{14}$ rad  
(b) $\frac{5\pi}{7}$ rad  
(c) $\frac{5}{7}$ rad  
(d) $\frac{7}{5}$ rad  
(e) None of these

Problem 314. In the figure at right, what is the measure of the angle $\theta$?

(a) $\frac{1}{3}$ rad  
(b) $\sqrt{10}\pi$ rad  
(c) 3 rad  
(d) $\pi/3$ rad  
(e) None of these

Problem 315. In the figure at right, what is the measure of the angle $\theta$?

(a) $\frac{4}{9}$ rad  
(b) $\frac{9}{4}$ rad  
(c) $\sqrt{65}\pi$ rad  
(d) $\sqrt{97}$ rad  
(e) None of these
Problem 316. In the figure at right, what is the measure of the angle $\theta$?

(a) $\frac{4}{7}$ rad  
(b) $\frac{7}{4}$ rad  
(c) $\frac{4\pi}{7}$ rad  
(d) $\sqrt{33}\pi$ rad  
(e) None of these

![Diagram](https://example.com/diagram.png)

4.1.6 Arc length to radians: descriptions

Problem 317. A circle has a radius of 5 cm. A central angle $\theta$ intercepts an arc whose length is 4 cm. What is the measure of $\theta$?

(a) 3 rad  
(b) $\frac{4\pi}{5}$ rad  
(c) $\sqrt{41}\pi$ rad  
(d) $\frac{4}{5}$ rad  
(e) None of these

Problem 318. A circle has a radius of 13 cm. A central angle $\theta$ intercepts an arc whose length is 5 cm. What is the measure of $\theta$?

(a) $\frac{5}{13}$ rad  
(b) $\frac{5\pi}{13}$ rad  
(c) $\sqrt{194}\pi$ rad  
(d) $\frac{13\pi}{5}$ rad  
(e) None of these

Problem 319. A circle has a radius of 3 cm. A central angle $\theta$ intercepts an arc whose length is 5 cm. What is the measure of $\theta$?

(a) $\sqrt{34}\pi$ rad  
(b) $\frac{5}{3}$ rad  
(c) $4\pi$ rad  
(d) $\sqrt{34}$ rad  
(e) None of these

Problem 320. A circle has a radius of 6 cm. A central angle $\theta$ intercepts an arc whose length is 7 cm. What is the measure of $\theta$?

(a) $\frac{7}{6}$ rad  
(b) $\sqrt{85}$ rad  
(c) $\sqrt{85}\pi$ rad  
(d) $\sqrt{13}$ rad  
(e) None of these

Problem 321. A circle has a radius of 6 cm. A central angle $\theta$ intercepts an arc whose length is 4 cm. What is the measure of $\theta$?

(a) $\frac{2}{3}$ rad  
(b) $\frac{3}{2}$ rad  
(c) $\frac{2\pi}{3}$ rad  
(d) $2\sqrt{13}\pi$ rad  
(e) None of these
Problem 322. A circle has a radius of 8 cm. A central angle $\theta$ intercepts an arc whose length is $4\pi$ cm. What is the measure of $\theta$?

(a) $1/2$ rad  
(b) $\pi/2$ rad
(c) $2$ rad  
(d) $2\pi$ rad
(e) None of these

Problem 323. A circle has a radius of 12 cm. A central angle $\theta$ intercepts an arc whose length is $4\pi$ cm. What is the measure of $\theta$?

(a) $3\pi$ rad  
(b) $\pi/3$ rad
(c) $3$ rad  
(d) $1/3$ rad
(e) None of these

Problem 324. A circle has a radius of $15\pi$ cm. A central angle $\theta$ intercepts an arc whose length is $3\pi$ cm. What is the measure of $\theta$?

(a) $\pi/5$ rad  
(b) $5\pi$ rad
(c) $1/5$ rad  
(d) $5$ rad
(e) None of these

Problem 325. A circle has a radius of 10 cm. A central angle $\theta$ intercepts an arc whose length is $15\pi$ cm. What is the measure of $\theta$?

(a) $2/3$ rad  
(b) $2\pi/3$ rad
(c) $3/2$ rad  
(d) $3\pi/2$ rad
(e) None of these

Problem 326. A circle has a radius of $6\pi$ cm. A central angle $\theta$ intercepts an arc whose length is 2 cm. What is the measure of $\theta$?

(a) $2\pi/3$ rad  
(b) $3\pi$ rad
(c) $1/3\pi$ rad  
(d) $3/2\pi$ rad
(e) None of these

4.1.7 Radians to arc length: pictures

Problem 327. In the figure at right, if $\theta = 1.4$ rad, what is the arc length $s$?

(a) 9.8 cm  
(b) $5\pi$ cm
(c) $\pi/5$ cm  
(d) $10\pi$ cm
(e) None of these
Problem 328. In the figure at right, if $\theta = 1.2$ rad, what is the arc length $s$?

(a) 5 cm  
(b) 7.2 cm  
(c) $\pi/5$ cm  
(d) $14.4\pi$ cm  
(e) None of these

Problem 329. In the figure at right, if $\theta = 4/9$ rad, what is the arc length $s$?

(a) $27\pi/2$ cm  
(b) $8/3$ cm  
(c) $27\pi$ cm  
(d) $2/27$ cm  
(e) None of these

Problem 330. In the figure at right, if $\theta = 2/3$ rad, what is the arc length $s$?

(a) $15\pi$ cm  
(b) $10/3$ cm  
(c) $15\pi/2$ cm  
(d) $2/15$ cm  
(e) None of these

Problem 331. In the figure at right, if $\theta = 9/5$ rad, what is the arc length $s$?

(a) $9\pi/10$ cm  
(b) $18\pi/5$ cm  
(c) $18/5$ cm  
(d) $20\pi/9$ cm  
(e) None of these

Problem 332. In the figure at right, if $\theta = 5/4$ rad, what is the arc length $s$?

(a) $5\pi/8$ cm  
(b) $15\pi/4$ cm  
(c) $12\pi/5$ cm  
(d) $15/4$ cm  
(e) None of these
4.1.8 Radians to arc length: descriptions

Problem 333. A circle has a radius of 10 cm. A central angle has measure $\theta = 3/5$ rad. What is the length of the arc intercepted by the angle?

(a) $6\pi/50$ cm (b) 6 cm
(c) $12\pi$ cm (d) $6\pi$ cm
(e) None of these

Problem 334. A circle has a radius of 4 cm. A central angle has measure $\theta = 1/2$ rad. What is the length of the arc intercepted by the angle?

(a) $\pi/4$ cm (b) 2 cm
(c) $2\pi$ cm (d) $4\pi$ cm
(e) None of these

Problem 335. A circle has a radius of 5 cm. A central angle has measure $\theta = \pi/3$ rad. What is the length of the arc intercepted by the angle?

(a) $2/15$ cm (b) $5\pi/3$ cm
(c) $5/3$ cm (d) $10\pi/3$ cm
(e) None of these

Problem 336. A circle has a radius of 6 cm. A central angle has measure $\theta = \pi/4$ rad. What is the length of the arc intercepted by the angle?

(a) $3\pi/2$ cm (b) $3/2$ cm
(c) $\pi/24$ cm (d) $\pi/12$ cm
(e) None of these

Problem 337. A circle has a radius of 4 cm. A central angle has measure $\theta = 4/3$ rad. What is the length of the arc intercepted by the angle?

(a) $6\pi$ cm (b) 3 cm
(c) $2\pi/3$ cm (d) $16/3$ cm
(e) None of these

Problem 338. A circle has a radius of 10 cm. A central angle has measure $\theta = \pi/5$ rad. What is the length of the arc intercepted by the angle?

(a) $2\pi$ cm (b) $\pi^2/25$ cm
(c) $50\pi$ cm (d) 2 cm
(e) None of these

Problem 339. A circle has a radius of 12 cm. A central angle has measure $\theta = 3/2$ rad. What is the length of the arc intercepted by the angle?

(a) $\pi/8$ cm (b) 18 cm
(c) $8\pi$ cm (d) $1/8$ cm
(e) None of these
Problem 340. A circle has a radius of 10 cm. A central angle has measure $\theta = \frac{2\pi}{5}$ rad. What is the length of the arc intercepted by the angle?

(a) $\frac{2\pi}{50}$ cm  
(b) $\frac{2}{25}$ cm  
(c) $\frac{25}{\pi}$ cm  
(d) $4\pi$ cm  
(e) None of these

4.1.9 Word problems: arc length and radians

Problem 341. A wheel has a radius of 28 inches. It turns through an angle of 1.2 rad. How far does a point on the rim travel during the turn? Round your answer to the nearest inch.

(a) 11 in  
(b) 34 in  
(c) 53 in  
(d) 106 in  
(e) None of these

Problem 342. A wheel has a radius of 71 cm. It turns through an angle of 0.6 rad. How far does a point on the rim travel during the turn? Round your answer to the nearest cm.

(a) 7 cm  
(b) 14 cm  
(c) 27 cm  
(d) 43 cm  
(e) None of these

Problem 343. A pizza has a radius of 8.2 inches. A slice cut from the pizza has a central angle of 0.39 rad. What is the length of the crust on the slice? Round your answer to the nearest 0.1 in.

(a) 0.5 in  
(b) 1.0 in  
(c) 2.0 in  
(d) 3.2 in  
(e) None of these

Problem 344. A pie has a radius of 13 cm. A slice cut from the pie has a central angle of 0.29 rad. What is the length of the crust on the slice? Round your answer to the nearest 0.1 cm.

(a) 1.2 cm  
(b) 2.7 cm  
(c) 3.8 cm  
(d) 4.1 cm  
(e) None of these

Problem 345. A door measures 34 in from the hinge to the outer edge. It swings through an angle of 1.4 rad. How far does the outer edge travel? Round your answer to the nearest inch.

(a) 45 in  
(b) 48 in  
(c) 51 in  
(d) 54 in  
(e) None of these
Problem 346. A wheel has a radius of 63 cm. Two adjacent spokes meet at the center at an angle of 0.51 rad. How long is the rim between the two spokes? Round your answer to the nearest cm.
(a) 32 cm (b) 34 cm
(c) 36 cm (d) 38 cm
(e) None of these

Problem 347. A bicycle wheel has a radius of 27 inches. It turns so that a point on the rim travels 11 inches. What angle has the wheel turned through? Round your answer to the nearest 0.01 rad.
(a) 0.31 rad (b) 0.35 rad
(c) 0.38 rad (d) 0.41 rad
(e) None of these

Problem 348. A pizza has a radius of 13 cm. A slice cut from the pizza has 10 cm of crust on the outside. What is the angle of the slice? Round your answer to the nearest 0.01 rad.
(a) 0.70 rad (b) 0.74 rad
(c) 0.77 rad (d) 0.81 rad
(e) None of these
4.2 Basic Right-Triangle Trigonometry

4.2.1 Basic trig functions: finding

Problem 349. In the figure at right, $\sin \theta = ?$
(a) $x/y$  (b) $y/x$
(c) $y/r$  (d) $x/r$
(e) None of these

Problem 350. In the figure at right, $\cos \theta = ?$
(a) $r/y$  (b) $x/r$
(c) $r/x$  (d) $y/x$
(e) None of these

Problem 351. In the figure at right, $\tan \theta = ?$
(a) $y/x$  (b) $y/r$
(c) $r/x$  (d) $r/y$
(e) None of these

Problem 352. In the figure at right, $\sin \phi = ?$
(a) $x/r$  (b) $r/x$
(c) $y/r$  (d) $y/x$
(e) None of these

Problem 353. In the figure at right, $\cos \phi = ?$
(a) $y/r$  (b) $r/x$
(c) $x/r$  (d) $x/y$
(e) None of these

Problem 354. In the figure at right, $\tan \phi = ?$
(a) $y/r$  (b) $r/x$
(c) $x/y$  (d) $y/x$
(e) None of these
Problem 355. In the figure at right, $\tan \theta = ?$
(a) $4/3$  
(b) $3/5$
(c) $4/5$  
(d) $3/4$
(e) None of these

Problem 356. In the figure at right, $\sin \phi = ?$
(a) $5/4$  
(b) $4/3$
(c) $3/5$  
(d) $4/5$
(e) None of these

Problem 357. In the figure at right, $\cos \theta = ?$
(a) $4/5$  
(b) $5/3$
(c) $3/4$  
(d) $4/3$
(e) None of these

Problem 358. In the figure at right, $\sin \theta = ?$
(a) $5/4$  
(b) $4/5$
(c) $4/3$  
(d) $3/5$
(e) None of these

Problem 359. In the figure at right, $\tan \phi = ?$
(a) $5/3$  
(b) $3/5$
(c) $4/3$  
(d) $5/4$
(e) None of these

Problem 360. In the figure at right, $\cos \phi = ?$
(a) $5/3$  
(b) $5/4$
(c) $4/3$  
(d) $3/5$
(e) None of these

Problem 361. In the figure at right, $\tan B = ?$
(a) $c/b$  
(b) $b/c$
(c) $b/a$  
(d) $a/c$
(e) None of these
Problem 362. In the figure at right, \( \sin B = ? \)
(a) \( c/b \)  
(b) \( b/a \)  
(c) \( b/c \)  
(d) \( a/c \)  
(e) None of these

Problem 363. In the figure at right, \( \cos B = ? \)
(a) \( b/c \)  
(b) \( c/a \)  
(c) \( a/b \)  
(d) \( b/a \)  
(e) None of these

Problem 364. In the figure at right, \( \sin C = ? \)
(a) \( a/b \)  
(b) \( c/b \)  
(c) \( c/a \)  
(d) \( a/c \)  
(e) None of these

Problem 365. In the figure at right, \( \cos C = ? \)
(a) \( a/c \)  
(b) \( a/b \)  
(c) \( b/a \)  
(d) \( c/b \)  
(e) None of these

Problem 366. In the figure at right, \( \tan C = ? \)
(a) \( a/b \)  
(b) \( b/c \)  
(c) \( c/a \)  
(d) \( c/b \)  
(e) None of these

Problem 367. In the figure at right, \( \cos \beta = ? \)
(a) \( u/v \)  
(b) \( w/u \)  
(c) \( w/v \)  
(d) \( v/u \)  
(e) None of these

Problem 368. In the figure at right, \( \tan \beta = ? \)
(a) \( v/w \)  
(b) \( u/w \)  
(c) \( u/v \)  
(d) \( w/v \)  
(e) None of these
Problem 369. In the figure at right, \( \sin \alpha = ? \)

(a) \( u/w \)  
(b) \( v/w \)  
(c) \( w/v \)  
(d) \( w/u \)  
(e) None of these

Problem 370. In the figure at right, \( \sin \beta = ? \)

(a) \( w/u \)  
(b) \( w/v \)  
(c) \( u/v \)  
(d) \( v/w \)  
(e) None of these

Problem 371. In the figure at right, \( \cos \alpha = ? \)

(a) \( v/w \)  
(b) \( u/v \)  
(c) \( v/u \)  
(d) \( w/v \)  
(e) None of these

Problem 372. In the figure at right, \( \tan \alpha = ? \)

(a) \( w/u \)  
(b) \( v/w \)  
(c) \( u/v \)  
(d) \( v/u \)  
(e) None of these

4.2.2 Basic trig functions: identifying

Problem 373. In the figure at right, \( y/x = ? \)

(a) \( \cos \theta \)  
(b) \( 1/\cos \theta \)  
(c) \( \sin \theta \)  
(d) \( \tan \theta \)  
(e) None of these

Problem 374. In the figure at right, \( y/r = ? \)

(a) \( 1/\sin \theta \)  
(b) \( \sin \theta \)  
(c) \( 1/\tan \theta \)  
(d) \( \cos \theta \)  
(e) None of these

Problem 375. In the figure at right, \( x/r = ? \)

(a) \( 1/\tan \theta \)  
(b) \( 1/\cos \theta \)  
(c) \( \sin \theta \)  
(d) \( \cos \theta \)  
(e) None of these
Problem 376. In the figure at right, $r/x = ?$
(a) $1/\cos \theta$  
(b) $1/\sin \theta$  
(c) $\sin \theta$  
(d) $1/\tan \theta$  
(e) None of these

Problem 377. In the figure at right, $r/y = ?$
(a) $\sin \theta$  
(b) $1/\sin \theta$  
(c) $\tan \theta$  
(d) $1/\tan \theta$  
(e) None of these

Problem 378. In the figure at right, $x/y = ?$
(a) $1/\tan \theta$  
(b) $1/\sin \theta$  
(c) $1/\cos \theta$  
(d) $\sin \theta$  
(e) None of these

Problem 379. In the figure at right, $y/r = ?$
(a) $1/\sin \phi$  
(b) $1/\cos \phi$  
(c) $\tan \phi$  
(d) $\cos \phi$  
(e) None of these

Problem 380. In the figure at right, $r/x = ?$
(a) $1/\cos \phi$  
(b) $1/\sin \phi$  
(c) $\tan \phi$  
(d) $\sin \phi$  
(e) None of these

Problem 381. In the figure at right, $x/y = ?$
(a) $1/\tan \phi$  
(b) $1/\sin \phi$  
(c) $\tan \phi$  
(d) $\sin \phi$  
(e) None of these

Problem 382. In the figure at right, $x/r = ?$
(a) $1/\tan \phi$  
(b) $\sin \phi$  
(c) $\cos \phi$  
(d) $1/\cos \phi$  
(e) None of these
Problem 383. In the figure at right, \( r/y = ? \)
(a) \( \tan \phi \)  
(b) \( \sin \phi \)  
(c) \( 1/\tan \phi \)  
(d) \( 1/\cos \phi \)  
(e) None of these

Problem 384. In the figure at right, \( y/x = ? \)
(a) \( 1/\cos \phi \)  
(b) \( \cos \phi \)  
(c) \( 1/\sin \phi \)  
(d) \( 1/\tan \phi \)  
(e) None of these

Problem 385. In the figure at right, \( 3/4 = ? \)
(a) \( 1/\cos \theta \)  
(b) \( \tan \theta \)  
(c) \( 1/\sin \theta \)  
(d) \( \sin \theta \)  
(e) None of these

Problem 386. In the figure at right, \( 4/3 = ? \)
(a) \( \sin \theta \)  
(b) \( 1/\tan \theta \)  
(c) \( 1/\cos \theta \)  
(d) \( \cos \theta \)  
(e) None of these

Problem 387. In the figure at right, \( 5/4 = ? \)
(a) \( 1/\cos \theta \)  
(b) \( \cos \theta \)  
(c) \( 1/\tan \theta \)  
(d) \( \sin \theta \)  
(e) None of these

Problem 388. In the figure at right, \( 3/5 = ? \)
(a) \( \tan \theta \)  
(b) \( \cos \theta \)  
(c) \( 1/\sin \theta \)  
(d) \( \sin \theta \)  
(e) None of these

Problem 389. In the figure at right, \( 5/3 = ? \)
(a) \( \sin \theta \)  
(b) \( 1/\tan \theta \)  
(c) \( 1/\cos \theta \)  
(d) \( 1/\sin \theta \)  
(e) None of these
Problem 390. In the figure at right, $4/5 = ?$

(a) $\cos \theta$  
(b) $1/\sin \theta$
(c) $1/\tan \theta$  
(d) $1/\cos \theta$
(e) None of these

Problem 391. In the figure at right, $4/3 = ?$

(a) $1/\cos \phi$  
(b) $\tan \phi$
(c) $\cos \phi$  
(d) $\sin \phi$
(e) None of these

Problem 392. In the figure at right, $3/5 = ?$

(a) $\sin \phi$  
(b) $1/\cos \phi$
(c) $\cos \phi$  
(d) $\tan \phi$
(e) None of these

Problem 393. In the figure at right, $5/3 = ?$

(a) $\cos \phi$  
(b) $\tan \phi$
(c) $1/\tan \phi$  
(d) $1/\cos \phi$
(e) None of these

Problem 394. In the figure at right, $5/4 = ?$

(a) $1/\tan \phi$  
(b) $1/\cos \phi$
(c) $1/\sin \phi$  
(d) $\tan \phi$
(e) None of these

Problem 395. In the figure at right, $4/5 = ?$

(a) $\tan \phi$  
(b) $1/\sin \phi$
(c) $\cos \phi$  
(d) $\sin \phi$
(e) None of these

Problem 396. In the figure at right, $3/4 = ?$

(a) $1/\tan \phi$  
(b) $\tan \phi$
(c) $1/\cos \phi$  
(d) $1/\sin \phi$
(e) None of these
Problem 397. In the figure at right, \( a/c = ? \)

(a) \( \sin A \)  
(b) \( \tan A \)  
(c) \( \cos A \)  
(d) \( 1/ \cos A \)  
(e) None of these

Problem 398. In the figure at right, \( c/b = ? \)

(a) \( 1/ \sin B \)  
(b) \( 1/ \tan B \)  
(c) \( \cos B \)  
(d) \( \sin B \)  
(e) None of these

Problem 399. In the figure at right, \( a/b = ? \)

(a) \( \tan A \)  
(b) \( \cos A \)  
(c) \( 1/ \cos A \)  
(d) \( 1/ \sin A \)  
(e) None of these

Problem 400. In the figure at right, \( c/a = ? \)

(a) \( 1/ \tan B \)  
(b) \( \sin B \)  
(c) \( 1/ \sin B \)  
(d) \( 1/ \cos B \)  
(e) None of these

Problem 401. In the figure at right, \( b/c = ? \)

(a) \( \tan B \)  
(b) \( \sin B \)  
(c) \( 1/ \cos B \)  
(d) \( 1/ \sin B \)  
(e) None of these

Problem 402. In the figure at right, \( b/a = ? \)

(a) \( 1/ \tan B \)  
(b) \( \tan B \)  
(c) \( 1/ \sin B \)  
(d) \( \cos B \)  
(e) None of these

Problem 403. In the figure at right, \( u/v = ? \)

(a) \( 1/ \tan \alpha \)  
(b) \( \sin \alpha \)  
(c) \( 1/ \cos \alpha \)  
(d) \( 1/ \sin \alpha \)  
(e) None of these
Problem 404. In the figure at right, \( w/v = ? \)
(a) \( \cos \alpha \)  
(b) \( 1/ \tan \alpha \)  
(c) \( 1/ \cos \alpha \)  
(d) \( 1/ \sin \alpha \)  
(e) None of these

Problem 405. In the figure at right, \( v/w = ? \)
(a) \( \sin \beta \)  
(b) \( 1/ \tan \beta \)  
(c) \( 1/ \sin \beta \)  
(d) \( \cos \beta \)  
(e) None of these

Problem 406. In the figure at right, \( w/u = ? \)
(a) \( \cos \beta \)  
(b) \( \sin \beta \)  
(c) \( \tan \beta \)  
(d) \( 1/ \cos \beta \)  
(e) None of these

Problem 407. In the figure at right, \( u/w = ? \)
(a) \( 1/ \sin \beta \)  
(b) \( 1/ \tan \beta \)  
(c) \( \cos \beta \)  
(d) \( \tan \beta \)  
(e) None of these

Problem 408. In the figure at right, \( v/u = ? \)
(a) \( 1/ \sin \alpha \)  
(b) \( \tan \alpha \)  
(c) \( 1/ \tan \alpha \)  
(d) \( \sin \alpha \)  
(e) None of these

4.2.3 Basic trig functions: calculator; radians

Problem 409. Use a calculator or equivalent to find \( \sin(0.35 \text{ rad}) \). Round your answer to three decimal places.
(a) 0.343  
(b) 1.071  
(c) 2.603  
(d) 0.359  
(e) None of these

Problem 410. Use a calculator or equivalent to find \( \cos(1.08 \text{ rad}) \). Round your answer to three decimal places.
(a) 1.871  
(b) 0.534  
(c) 0.815  
(d) 0.471  
(e) None of these
Problem 411. Use a calculator or equivalent to find \( \tan(0.89 \text{ rad}) \). Round your answer to three decimal places.

(a) 1.589 \hspace{1cm} (b) 0.629
(c) 1.339 \hspace{1cm} (d) 1.235
(e) None of these

Problem 412. Use a calculator or equivalent to find \( \sin(1.21 \text{ rad}) \). Round your answer to three decimal places.

(a) 0.353 \hspace{1cm} (b) 2.833
(c) 0.211 \hspace{1cm} (d) 0.936
(e) None of these

Problem 413. Use a calculator or equivalent to find \( \cos(0.57 \text{ rad}) \). Round your answer to three decimal places.

(a) 0.842 \hspace{1cm} (b) 1.853
(c) 0.322 \hspace{1cm} (d) 1.188
(e) None of these

Problem 414. Use a calculator or equivalent to find \( \tan(1.39 \text{ rad}) \). Round your answer to three decimal places.

(a) 0.892 \hspace{1cm} (b) 5.471
(c) 0.507 \hspace{1cm} (d) 1.971
(e) None of these

Problem 415. Use a calculator or equivalent to find \( \sin(0.74 \text{ rad}) \). Round your answer to three decimal places.

(a) 0.674 \hspace{1cm} (b) 0.013
(c) 1.354 \hspace{1cm} (d) 0.913
(e) None of these

Problem 416. Use a calculator or equivalent to find \( \cos(0.88 \text{ rad}) \). Round your answer to three decimal places.

(a) 1.569 \hspace{1cm} (b) 0.988
(c) 0.637 \hspace{1cm} (d) 0.771
(e) None of these

Problem 417. Use a calculator or equivalent to find \( \tan(0.97 \text{ rad}) \). Round your answer to three decimal places.

(a) 0.565 \hspace{1cm} (b) 0.685
(c) 1.459 \hspace{1cm} (d) 1.211
(e) None of these
Problem 418. Use a calculator or equivalent to find $\sin(0.18 \text{ rad})$. Round your answer to three decimal places.

(a) 0.179  
(b) 5.495  
(c) 1.288  
(d) 0.984  
(e) None of these

Problem 419. Use a calculator or equivalent to find $\cos(0.33 \text{ rad})$. Round your answer to three decimal places.

(a) 0.058  
(b) 0.946  
(c) 0.998  
(d) 3.086  
(e) None of these

Problem 420. Use a calculator or equivalent to find $\tan(0.41 \text{ rad})$. Round your answer to three decimal places.

(a) 0.399  
(b) 0.071  
(c) 0.435  
(d) 0.998  
(e) None of these

4.2.4 Basic trig functions: calculator; degrees

Problem 421. Use a calculator or equivalent to find $\sin 37^\circ$. Round your answer to three decimal places.

(a) 0.754  
(b) 0.644  
(c) 0.602  
(d) 1.327  
(e) None of these

Problem 422. Use a calculator or equivalent to find $\cos 81^\circ$. Round your answer to three decimal places.

(a) 1.233  
(b) 0.630  
(c) 0.156  
(d) 0.158  
(e) None of these

Problem 423. Use a calculator or equivalent to find $\tan 52^\circ$. Round your answer to three decimal places.

(a) 0.163  
(b) 1.280  
(c) 0.781  
(d) 0.616  
(e) None of these

Problem 424. Use a calculator or equivalent to find $\sin 77^\circ$. Round your answer to three decimal places.

(a) 1.026  
(b) 0.031  
(c) 0.331  
(d) 0.974  
(e) None of these
Problem 425. Use a calculator or equivalent to find $\cos 29^\circ$. Round your answer to three decimal places.

(a) 0.875
(b) 1.127
(c) 0.263
(d) 1.143
(e) None of these

Problem 426. Use a calculator or equivalent to find $\tan 23^\circ$. Round your answer to three decimal places.

(a) 0.846
(b) 0.391
(c) 0.424
(d) 1.877
(e) None of these

Problem 427. Use a calculator or equivalent to find $\sin 41^\circ$. Round your answer to three decimal places.

(a) 0.656
(b) 0.869
(c) 1.524
(d) 0.159
(e) None of these

Problem 428. Use a calculator or equivalent to find $\cos 52^\circ$. Round your answer to three decimal places.

(a) 0.788
(b) 0.616
(c) 0.624
(d) 1.269
(e) None of these

Problem 429. Use a calculator or equivalent to find $\tan 69^\circ$. Round your answer to three decimal places.

(a) 2.790
(b) 0.654
(c) 1.071
(d) 2.605
(e) None of these

Problem 430. Use a calculator or equivalent to find $\sin 16^\circ$. Round your answer to three decimal places.

(a) 0.958
(b) 0.288
(c) 0.276
(d) 0.301
(e) None of these

Problem 431. Use a calculator or equivalent to find $\cos 43^\circ$. Round your answer to three decimal places.

(a) 0.367
(b) 0.555
(c) 0.498
(d) 0.731
(e) None of these
Problem 432. Use a calculator or equivalent to find $\tan 41^\circ$. Round your answer to three decimal places.

(a) 6.304  (b) 0.656
(c) 0.987   (d) 0.869
(e) None of these

4.2.5 Using basic trig functions: formulas

Problem 433. In the figure at right, $y =$ ?

(a) $r \cos \theta$  (b) $r \sin \theta$
(c) $r / \sin \theta$  (d) $r / \tan \theta$
(e) None of these

Problem 434. In the figure at right, $x =$ ?

(a) $r \sin \theta$  (b) $r \cos \theta$
(c) $r / \cos \theta$  (d) $r \tan \theta$
(e) None of these

Problem 435. In the figure at right, $x =$ ?

(a) $y \cos \theta$  (b) $y \tan \theta$
(c) $y \sin \theta$  (d) $y / \tan \theta$
(e) None of these

Problem 436. In the figure at right, $r =$ ?

(a) $x \tan \theta$  (b) $x / \sin \theta$
(c) $x \sin \theta$  (d) $x / \cos \theta$
(e) None of these

Problem 437. In the figure at right, $r =$ ?

(a) $y / \sin \theta$  (b) $y / \cos \theta$
(c) $y \tan \theta$  (d) $y / \tan \theta$
(e) None of these

Problem 438. In the figure at right, $y =$ ?

(a) $x \cos \theta$  (b) $x \tan \theta$
(c) $x \sin \theta$  (d) $x / \sin \theta$
(e) None of these
Problem 439. In the figure at right, \( s = ? \)
(a) \( \frac{5}{\cos \theta} \)  
(b) \( \frac{5}{\tan \theta} \)  
(c) \( 5 \sin \theta \)  
(d) \( \frac{5}{\sin \theta} \)  
(e) None of these

Problem 440. In the figure at right, \( s = ? \)
(a) \( \frac{3}{\sin \theta} \)  
(b) \( \frac{3}{\tan \theta} \)  
(c) \( 3 \tan \theta \)  
(d) \( 3 \cos \theta \)  
(e) None of these

Problem 441. In the figure at right, \( t = ? \)
(a) \( 13 \tan \alpha \)  
(b) \( \frac{13}{\tan \alpha} \)  
(c) \( \frac{\cos \alpha}{13} \)  
(d) \( \frac{13}{\cos \alpha} \)  
(e) None of these

Problem 442. In the figure at right, \( t = ? \)
(a) \( 8 \sin \alpha \)  
(b) \( \frac{8}{\cos \alpha} \)  
(c) \( \frac{8}{\tan \alpha} \)  
(d) \( 8 \cos \alpha \)  
(e) None of these

Problem 443. In the figure at right, \( u = ? \)
(a) \( \frac{\cos \phi}{9} \)  
(b) \( \frac{9}{\tan \phi} \)  
(c) \( 9 \tan \phi \)  
(d) \( \frac{\tan \phi}{9} \)  
(e) None of these

Problem 444. In the figure at right, \( u = ? \)
(a) \( \frac{\cos \phi}{5} \)  
(b) \( \frac{5}{\sin \phi} \)  
(c) \( 5 \tan \phi \)  
(d) \( \frac{5}{\cos \phi} \)  
(e) None of these

Problem 445. In the figure at right, \( u = ? \)
(a) \( 5 \tan \phi \)  
(b) \( \frac{(\sin \phi)}{5} \)  
(c) \( \frac{5}{\tan \phi} \)  
(d) \( \frac{5}{\cos \phi} \)  
(e) None of these
Problem 446. In the figure at right, \( s = ? \)

(a) \( 9 \cos \phi \)  
(b) \( \frac{9}{\cos \phi} \)  
(c) \( \frac{\sin \phi}{9} \)  
(d) \( \frac{9}{\sin \phi} \)  
(e) None of these

Problem 447. In the figure at right, \( t = ? \)

(a) \( \frac{\sin \theta}{3} \)  
(b) \( \frac{3}{\tan \theta} \)  
(c) \( 3 \cos \theta \)  
(d) \( \frac{\tan \theta}{3} \)  
(e) None of these

Problem 448. In the figure at right, \( u = ? \)

(a) \( 8 \sin \alpha \)  
(b) \( \frac{8}{\cos \alpha} \)  
(c) \( \frac{8}{\sin \alpha} \)  
(d) \( \frac{\tan \alpha}{8} \)  
(e) None of these

Problem 449. In the figure at right, \( s = ? \)

(a) \( \frac{\cos \theta}{3} \)  
(b) \( 3 \tan \theta \)  
(c) \( \frac{\sin \theta}{3} \)  
(d) \( 3 \cos \theta \)  
(e) None of these

Problem 450. In the figure at right, \( u = ? \)

(a) \( \frac{\cos \theta}{13} \)  
(b) \( \frac{\tan \theta}{13} \)  
(c) \( \frac{13}{\sin \theta} \)  
(d) \( \frac{13}{\cos \theta} \)  
(e) None of these

4.2.6 Using basic trig functions: calculator

Problem 451. In the figure at right, use a calculator or equivalent to find \( s \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 11.02  
(b) 2.62  
(c) 3.27  
(d) 5.40  
(e) None of these
Problem 452. In the figure at right, use a calculator or equivalent to find \( t \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 4.23  
(b) 11.78  
(c) 4.60  
(d) 12.80  
(e) None of these

Problem 453. In the figure at right, use a calculator or equivalent to find \( w \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 6.79  
(b) 6.63  
(c) 4.23  
(d) 5.40  
(e) None of these

Problem 454. In the figure at right, use a calculator or equivalent to find \( u \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 4.47  
(b) 14.31  
(c) 7.79  
(d) 4.23  
(e) None of these

Problem 455. In the figure at right, use a calculator or equivalent to find \( w \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 4.23  
(b) 10.24  
(c) 4.60  
(d) 12.80  
(e) None of these

Problem 456. In the figure at right, use a calculator or equivalent to find \( s \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 1.87  
(b) 3.62  
(c) 2.49  
(d) 4.02  
(e) None of these
**Problem 457.** In the figure at right, use a calculator or equivalent to find \( s \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 8.83  
(b) 7.25  
(c) 17.16  
(d) 3.38  
(e) None of these

**Problem 458.** In the figure at right, use a calculator or equivalent to find \( a \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 20.02  
(b) 7.08  
(c) 8.44  
(d) 10.90  
(e) None of these

**Problem 459.** In the figure at right, use a calculator or equivalent to find \( a \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 2.67  
(b) 6.22  
(c) 3.06  
(d) 2.98  
(e) None of these

**Problem 460.** In the figure at right, use a calculator or equivalent to find \( t \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 11.12  
(b) 15.31  
(c) 6.54  
(d) 5.29  
(e) None of these

**Problem 461.** In the figure at right, use a calculator or equivalent to find \( w \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 5.75  
(b) 4.77  
(c) 3.77  
(d) 3.28  
(e) None of these
Problem 462. In the figure at right, use a calculator or equivalent to find \( a \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 11.26  
(b) 4.16  
(c) 7.51  
(d) 15.01  
(e) None of these

Problem 463. In the figure at right, use a calculator or equivalent to find \( t \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 14.95  
(b) 11.25  
(c) 17.23  
(d) 9.81  
(e) None of these

Problem 464. In the figure at right, use a calculator or equivalent to find \( a \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 5.60  
(b) 13.95  
(c) 12.59  
(d) 9.77  
(e) None of these

Problem 465. In the figure at right, use a calculator or equivalent to find \( t \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 7.64  
(b) 4.18  
(c) 1.92  
(d) 2.36  
(e) None of these

Problem 466. In the figure at right, use a calculator or equivalent to find \( s \). Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 1.75  
(b) 1.83  
(c) 4.42  
(d) 5.74  
(e) None of these
Problem 467. In the figure at right, use a calculator or equivalent to find $a$. Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 3.76  
(b) 4.25  
(c) 7.04  
(d) 5.05  
(e) None of these

Problem 468. In the figure at right, use a calculator or equivalent to find $a$. Round your answer to two decimal places. (The figure is not necessarily drawn to scale.)

(a) 4.60  
(b) 3.86  
(c) 4.00  
(d) 7.15  
(e) None of these

4.2.7 Reference angles

Problem 469. If $\theta = 320^\circ$, which quadrant is it in?

(a) Quadrant I  
(b) Quadrant II  
(c) Quadrant III  
(d) Quadrant IV  
(e) None of these

Problem 470. If $\theta = 109^\circ$, which quadrant is it in?

(a) Quadrant I  
(b) Quadrant II  
(c) Quadrant III  
(d) Quadrant IV  
(e) None of these

Problem 471. If $\theta = 194^\circ$, which quadrant is it in?

(a) Quadrant I  
(b) Quadrant II  
(c) Quadrant III  
(d) Quadrant IV  
(e) None of these

Problem 472. If $\theta = 32^\circ$, which quadrant is it in?

(a) Quadrant I  
(b) Quadrant II  
(c) Quadrant III  
(d) Quadrant IV  
(e) None of these

Problem 473. If $\theta = 5.91$ rad, which quadrant is it in?

(a) Quadrant I  
(b) Quadrant II  
(c) Quadrant III  
(d) Quadrant IV  
(e) None of these
Problem 474. If \( \theta = 0.32 \text{ rad}, \) which quadrant is it in?

(a) Quadrant I  (b) Quadrant II  
(c) Quadrant III (d) Quadrant IV  
(e) None of these

Problem 475. If \( \theta = 2.15 \text{ rad}, \) which quadrant is it in?

(a) Quadrant I  (b) Quadrant II  
(c) Quadrant III (d) Quadrant IV  
(e) None of these

Problem 476. If \( \theta = 3.40 \text{ rad}, \) which quadrant is it in?

(a) Quadrant I  (b) Quadrant II  
(c) Quadrant III (d) Quadrant IV  
(e) None of these

Problem 477. An angle with measure \( \theta \) degrees is in quadrant II. What is the reference angle \( \theta_{\text{ref}}? \)

(a) \( \theta - 180^\circ \)  (b) \( 180^\circ - \theta \)  
(c) \( \theta - 360^\circ \) (d) \( 360^\circ - \theta \)  
(e) None of these

Problem 478. An angle with measure \( \theta \) degrees is in quadrant IV. What is the reference angle \( \theta_{\text{ref}}? \)

(a) \( \theta - 180^\circ \)  (b) \( 180^\circ - \theta \)  
(c) \( \theta - 360^\circ \) (d) \( 360^\circ - \theta \)  
(e) None of these

Problem 479. An angle with measure \( \theta \) degrees is in quadrant III. What is the reference angle \( \theta_{\text{ref}}? \)

(a) \( \theta - 180^\circ \)  (b) \( 180^\circ - \theta \)  
(c) \( \theta - 360^\circ \) (d) \( 360^\circ - \theta \)  
(e) None of these

Problem 480. An angle with measure \( \theta \) radians is in quadrant III. What is the reference angle \( \theta_{\text{ref}}? \)

(a) \( 2\pi - \theta \)  (b) \( \theta - 2\pi \)  
(c) \( \pi - \theta \) (d) \( \theta - \pi \)  
(e) None of these

Problem 481. An angle with measure \( \theta \) radians is in quadrant II. What is the reference angle \( \theta_{\text{ref}}? \)

(a) \( 2\pi - \theta \)  (b) \( \theta - 2\pi \)  
(c) \( \pi - \theta \) (d) \( \theta - \pi \)  
(e) None of these
Problem 482. An angle with measure $\theta$ radians is in quadrant IV. What is the reference angle $\theta_{\text{ref}}$?

(a) $2\pi - \theta$  
(b) $\theta - 2\pi$  
(c) $\pi - \theta$  
(d) $\theta - \pi$  
(e) None of these

Problem 483. If $\theta = 311^\circ$, what is $\theta_{\text{ref}}$?

(a) $31^\circ$  
(b) $39^\circ$  
(c) $41^\circ$  
(d) $49^\circ$  
(e) None of these

Problem 484. If $\theta = 122^\circ$, what is $\theta_{\text{ref}}$?

(a) $32^\circ$  
(b) $42^\circ$  
(c) $58^\circ$  
(d) $68^\circ$  
(e) None of these

Problem 485. If $\theta = 195^\circ$, what is $\theta_{\text{ref}}$?

(a) $15^\circ$  
(b) $25^\circ$  
(c) $65^\circ$  
(d) $75^\circ$  
(e) None of these

Problem 486. If $\theta = 272^\circ$, what is $\theta_{\text{ref}}$?

(a) $2^\circ$  
(b) $28^\circ$  
(c) $72^\circ$  
(d) $88^\circ$  
(e) None of these

Problem 487. If $\theta = 163^\circ$, what is $\theta_{\text{ref}}$?

(a) $17^\circ$  
(b) $27^\circ$  
(c) $63^\circ$  
(d) $73^\circ$  
(e) None of these

Problem 488. If $\theta = 210^\circ$, what is $\theta_{\text{ref}}$?

(a) $10^\circ$  
(b) $30^\circ$  
(c) $40^\circ$  
(d) $60^\circ$  
(e) None of these

Problem 489. If $\theta$ is in the first quadrant, which of the following is true?

(a) $\sin \theta \leq 0$ and $\cos \theta \leq 0$  
(b) $\sin \theta \leq 0$ and $\cos \theta \geq 0$  
(c) $\sin \theta \geq 0$ and $\cos \theta \leq 0$  
(d) $\sin \theta \geq 0$ and $\cos \theta \geq 0$  
(e) None of these
Problem 490. If $\theta$ is in the second quadrant, which of the following is true?

(a) $\sin \theta \leq 0$ and $\cos \theta \leq 0$
(b) $\sin \theta \leq 0$ and $\cos \theta \geq 0$
(c) $\sin \theta \geq 0$ and $\cos \theta \leq 0$
(d) $\sin \theta \geq 0$ and $\cos \theta \geq 0$
(e) None of these

Problem 491. If $\theta$ is in the third quadrant, which of the following is true?

(a) $\sin \theta \leq 0$ and $\cos \theta \leq 0$
(b) $\sin \theta \leq 0$ and $\cos \theta \geq 0$
(c) $\sin \theta \geq 0$ and $\cos \theta \leq 0$
(d) $\sin \theta \geq 0$ and $\cos \theta \geq 0$
(e) None of these

Problem 492. If $\theta$ is in the fourth quadrant, which of the following is true?

(a) $\sin \theta \leq 0$ and $\cos \theta \leq 0$
(b) $\sin \theta \leq 0$ and $\cos \theta \geq 0$
(c) $\sin \theta \geq 0$ and $\cos \theta \leq 0$
(d) $\sin \theta \geq 0$ and $\cos \theta \geq 0$
(e) None of these

Problem 493. An angle with measure $\theta$ degrees is in the third quadrant. What is $\sin \theta$?

(a) $\sin(\theta - 180^\circ)$
(b) $\sin(360^\circ - \theta)$
(c) $-\sin(\theta - 180^\circ)$
(d) $-\sin(\theta - 360^\circ)$
(e) None of these

Problem 494. An angle with measure $\theta$ degrees is in the fourth quadrant. What is $\sin \theta$?

(a) $\sin(\theta - 180^\circ)$
(b) $\sin(360^\circ - \theta)$
(c) $\sin(\theta + 180^\circ)$
(d) $-\sin(360^\circ - \theta)$
(e) None of these

Problem 495. An angle with measure $\theta$ degrees is in the second quadrant. What is $\sin \theta$?

(a) $\sin(180^\circ - \theta)$
(b) $\sin(180^\circ + \theta)$
(c) $\sin(360^\circ - \theta)$
(d) $-\sin(360^\circ + \theta)$
(e) None of these

Problem 496. An angle with measure $\theta$ degrees is in the second quadrant. What is $\cos \theta$?

(a) $\cos(180^\circ - \theta)$
(b) $-\cos(180^\circ - \theta)$
(c) $\cos(\theta - 180^\circ)$
(d) $-\cos(360^\circ + \theta)$
(e) None of these

Problem 497. An angle with measure $\theta$ degrees is in the third quadrant. What is $\cos \theta$?

(a) $\cos(\theta - 180^\circ)$
(b) $\cos(180^\circ - \theta)$
(c) $-\cos(\theta - 180^\circ)$
(d) $\cos(180^\circ + \theta)$
(e) None of these
Problem 498. An angle with measure $\theta$ degrees is in the fourth quadrant. What is $\cos \theta$?

(a) $\cos(\theta - 180^\circ)$
(b) $\cos(180^\circ - \theta)$
(c) $\cos(\theta + 180^\circ)$
(d) $\cos(360^\circ - \theta)$
(e) None of these

4.2.8 Arc functions: definition

Problem 499. Complete the statement: If $\theta$ is an acute angle and $\sin \theta = 0.3$, then

(a) $\theta = \sin 0.3$
(b) $\theta = \cos 0.3$
(c) $\theta = \arcsin 0.3$
(d) $\theta = 0.3/\sin$
(e) None of these

Problem 500. Complete the statement: If $\theta$ is an acute angle and $\cos \theta = 0.4$, then

(a) $\theta = 1/\cos 0.4$
(b) $\theta = \cos 0.4$
(c) $\theta = \cos^{-1} 0.4$
(d) $\theta = \sin 0.4$
(e) None of these

Problem 501. Complete the statement: If $\theta$ is an acute angle and $\tan \theta = 3.1$, then

(a) $\theta = \tan^{-1} 3.1$
(b) $\theta = 3.1/\tan$
(c) $\theta = 1/\tan 3.1$
(d) $\theta = \tan(1/3.1)$
(e) None of these

Problem 502. Complete the statement: If $\theta$ is an acute angle and $\sin \theta = 0.9$, then

(a) $\theta = \sin^{-1} 0.9$
(b) $\theta = \sin(1/0.9)$
(c) $\theta = \cos 0.9$
(d) $\theta = 1/\sin 0.9$
(e) None of these

Problem 503. Complete the statement: If $\theta$ is an acute angle and $\cos \theta = 0.2$, then

(a) $\theta = \arccos 0.2$
(b) $\theta = \cos 0.2$
(c) $\theta = \sin 0.2$
(d) $\theta = \cos(5)$
(e) None of these

Problem 504. Complete the statement: If $\theta$ is an acute angle and $\tan \theta = 0.5$, then

(a) $\theta = \arctan 0.5$
(b) $\theta = \tan 2$
(c) $\theta = 1/\tan 0.5$
(d) $\theta = 0.5/\tan$
(e) None of these

Problem 505. Complete the statement: If $\theta = \arcsin 0.5$, then

(a) $\sin \theta = 2$
(b) $\sin \theta = 0.5$
(c) $\theta = \sin 2$
(d) $\cos \theta = 2$
(e) None of these
Problem 506. Complete the statement: If $\theta = \arccos 0.2$, then
(a) $\cos \theta = 5$       (b) $\theta = \cos 5$
(c) $\cos \theta = 0.2$     (d) $\theta = \cos 0.2$
(e) None of these

Problem 507. Complete the statement: If $\theta = \arctan 2$, then
(a) $\tan \theta = 0.5$     (b) $\theta = \tan 0.5$
(c) $\theta = 2/\tan$       (d) $\tan \theta = 2$
(e) None of these

Problem 508. Complete the statement: If $\theta = \sin^{-1} 0.4$, then
(a) $\sin \theta = 2.5$     (b) $1/\sin \theta = 0.4$
(c) $\sin \theta = 0.4$     (d) $\theta = 2.5$
(e) None of these

Problem 509. Complete the statement: If $\theta = \cos^{-1}(1/3)$, then
(a) $\cos \theta = 3$       (b) $3\theta = \cos$
(c) $\cos \theta = 1/3$     (d) $\sin \theta = 3$
(e) None of these

Problem 510. Complete the statement: If $\theta = \tan^{-1}(2/3)$, then
(a) $\tan \theta = 3/2$     (b) $\theta = 1/\tan(2/3)$
(c) $\tan \theta = 2/3$     (d) $\theta = \tan(3/2)$
(e) None of these

4.2.9  Arc functions: definition; diagrams

Problem 511. Which statement is true of the figure at right?
(a) $\theta = \sin(5/3)$     (b) $\theta = \sin(3/5)$
(c) $\theta = \arcsin(5/3)$  (d) $\theta = \arcsin(3/5)$
(e) None of these

Problem 512. Which statement is true of the figure at right?
(a) $\theta = \cos(5/4)$     (b) $\theta = \arccos(5/4)$
(c) $\theta = \cos(4/5)$     (d) $\theta = \arccos(4/5)$
(e) None of these
Problem 513. Which statement is true of the figure at right?

(a) $\theta = \tan(3/4)$  
(b) $\theta = \tan(4/3)$  
(c) $\theta = \arctan(3/4)$  
(d) $\theta = \arctan(4/3)$  
(e) None of these

Problem 514. Which statement is true of the figure at right?

(a) $\phi = \sin^{-1}(4/5)$  
(b) $\phi = \sin(4/5)$  
(c) $\phi = \sin^{-1}(5/4)$  
(d) $\phi = \sin(5/4)$  
(e) None of these

Problem 515. Which statement is true of the figure at right?

(a) $\phi = \cos(4/5)$  
(b) $\phi = \cos^{-1}(4/5)$  
(c) $\phi = \cos(3/5)$  
(d) $\phi = \cos^{-1}(3/5)$  
(e) None of these

Problem 516. Which statement is true of the figure at right?

(a) $\phi = \tan^{-1}(3/4)$  
(b) $\phi = \tan^{-1}(4/3)$  
(c) $\phi = \tan(3/4)$  
(d) $\phi = \tan(4/3)$  
(e) None of these

4.2.10 Arc functions: calculator; radians

Problem 517. Use a calculator or equivalent to determine the value of $\theta$ if $\theta$ is an acute angle and $\cos \theta = 0.41$. Round your answer to the nearest 0.01 radian.

(a) 0.39 rad  
(b) 36.83 rad  
(c) 1.15 rad  
(d) No such angle  
(e) None of these

Problem 518. Use a calculator or equivalent to determine the value of $\theta$ if $\theta$ is an acute angle and $\sin \theta = 0.83$. Round your answer to the nearest 0.01 radian.

(a) 1.48 rad  
(b) 0.59 rad  
(c) 0.98 rad  
(d) No such angle  
(e) None of these

Problem 519. Use a calculator or equivalent to determine the value of $\theta$ if $\theta$ is an acute angle and $\tan \theta = 1.03$. Round your answer to the nearest 0.01 radian.

(a) 0.86 rad  
(b) 0.80 rad  
(c) 0.51 rad  
(d) No such angle  
(e) None of these
Problem 520. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \cos \theta = 0.77 \). Round your answer to the nearest 0.01 radian.

(a) 0.65 rad  (b) 0.69 rad
(c) 0.35 rad  (d) No such angle
(e) None of these

Problem 521. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \sin \theta = 1.21 \). Round your answer to the nearest 0.01 radian.

(a) 0.39 rad  (b) 36.83 rad
(c) 1.15 rad  (d) No such angle
(e) None of these

Problem 522. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \tan \theta = 0.22 \). Round your answer to the nearest 0.01 radian.

(a) 0.22 rad  (b) 0.29 rad
(c) 18.45 rad  (d) No such angle
(e) None of these

Problem 523. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \cos \theta = 1.03 \). Round your answer to the nearest 0.01 radian.

(a) 1.17 rad  (b) 1.67 rad
(c) 0.86 rad  (d) No such angle
(e) None of these

Problem 524. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \sin \theta = 0.55 \). Round your answer to the nearest 0.01 radian.

(a) 1.91 rad  (b) 0.58 rad
(c) 0.52 rad  (d) No such angle
(e) None of these

Problem 525. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \tan \theta = 1.71 \). Round your answer to the nearest 0.01 radian.

(a) 1.04 rad  (b) 0.14 rad
(c) 89.48 rad  (d) No such angle
(e) None of these
4.2.11 Arc function: calculator; degrees

Problem 526. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \sin \theta = 0.63 \). Round your answer to the nearest degree.

(a) 46°  (b) 39°
(c) 32°  (d) No such angle
(e) None of these

Problem 527. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \cos \theta = 1.19 \). Round your answer to the nearest degree.

(a) 21°  (b) 50°
(c) 53°  (d) No such angle
(e) None of these

Problem 528. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \tan \theta = 1.30 \). Round your answer to the nearest degree.

(a) 52°  (b) 34°
(c) 15°  (d) No such angle
(e) None of these

Problem 529. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \cos \theta = 0.81 \). Round your answer to the nearest degree.

(a) 36°  (b) 39°
(c) 77°  (d) No such angle
(e) None of these

Problem 530. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \tan \theta = 0.46 \). Round your answer to the nearest degree.

(a) 39°  (b) 25°
(c) 1°  (d) No such angle
(e) None of these

Problem 531. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \sin \theta = 0.32 \). Round your answer to the nearest degree.

(a) 3°  (b) 19°
(c) 27°  (d) No such angle
(e) None of these

Problem 532. Use a calculator or equivalent to determine the value of \( \theta \) if \( \theta \) is an acute angle and \( \sin \theta = 0.67 \). Round your answer to the nearest degree.

(a) 42°  (b) 48°
(c) 45°  (d) No such angle
(e) None of these
Problem 533. Use a calculator or equivalent to determine the value of $\theta$ if $\theta$ is an acute angle and $\cos \theta = 0.96$. Round your answer to the nearest degree.

(a) $1^\circ$  
(b) $2^\circ$  
(c) $16^\circ$  
(d) No such angle  
(e) None of these

Problem 534. Use a calculator or equivalent to determine the value of $\theta$ if $\theta$ is an acute angle and $\tan \theta = 0.02$. Round your answer to the nearest degree.

(a) $57^\circ$  
(b) $1^\circ$  
(c) $66^\circ$  
(d) No such angle  
(e) None of these

4.2.12 Arc functions: calculator; diagram; radians

Problem 535. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

- $7$  
- $5$

(a) $0.78\ \text{rad}$  
(b) $1.32\ \text{rad}$  
(c) $1.53\ \text{rad}$  
(d) $1.81\ \text{rad}$  
(e) None of these

Problem 536. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

- $10$  
- $4$

(a) $0.41\ \text{rad}$  
(b) $1.16\ \text{rad}$  
(c) $1.57\ \text{rad}$  
(d) $0.31\ \text{rad}$  
(e) None of these

Problem 537. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

- $8$  
- $11$

(a) $0.32\ \text{rad}$  
(b) $0.34\ \text{rad}$  
(c) $0.63\ \text{rad}$  
(d) $1.34\ \text{rad}$  
(e) None of these

Problem 538. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

- $5$  
- $8$

(a) $0.90\ \text{rad}$  
(b) $0.32\ \text{rad}$  
(c) $0.56\ \text{rad}$  
(d) $0.68\ \text{rad}$  
(e) None of these
Problem 539. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

(a) 0.91 rad  (b) 0.24 rad  
(c) 0.68 rad  (d) 0.49 rad  
(e) None of these

Problem 540. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

(a) 0.46 rad  (b) 0.69 rad  
(c) 0.66 rad  (d) 0.48 rad  
(e) None of these

Problem 541. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

(a) 1.03 rad  (b) 1.07 rad  
(c) 1.09 rad  (d) 1.18 rad  
(e) None of these

Problem 542. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

(a) 1.04 rad  (b) 0.84 rad  
(c) 0.87 rad  (d) 1.13 rad  
(e) None of these

Problem 543. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

(a) 0.91 rad  (b) 1.06 rad  
(c) 1.40 rad  (d) 0.85 rad  
(e) None of these

Problem 544. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to 0.01 rad. (The figure is not necessarily drawn to scale.)

(a) 1.42 rad  (b) 1.29 rad  
(c) 1.63 rad  (d) 1.15 rad  
(e) None of these
4.2.13 Arc functions: calculator; diagram; degrees

Problem 545. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 59° (b) 53°
(c) 40° (d) 47°
(e) None of these

Problem 546. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 24° (b) 36°
(c) 49° (d) 57°
(e) None of these

Problem 547. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 52° (b) 65°
(c) 54° (d) 67°
(e) None of these

Problem 548. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 50° (b) 52°
(c) 54° (d) 56°
(e) None of these

Problem 549. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 60° (b) 63°
(c) 66° (d) 69°
(e) None of these
Problem 550. In the figure at right, use a calculator or equivalent to find $\phi$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 76°  
(b) 78°  
(c) 80°  
(d) 82°  
(e) None of these

Problem 551. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 47°  
(b) 50°  
(c) 53°  
(d) 56°  
(e) None of these

Problem 552. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 21°  
(b) 23°  
(c) 25°  
(d) 27°  
(e) None of these

Problem 553. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 40°  
(b) 42°  
(c) 44°  
(d) 46°  
(e) None of these

Problem 554. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) 23°  
(b) 25°  
(c) 27°  
(d) 29°  
(e) None of these
Problem 555. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) $6^\circ$  
(b) $8^\circ$  
(c) $10^\circ$  
(d) $12^\circ$  
(e) None of these

Problem 556. In the figure at right, use a calculator or equivalent to find $\theta$. Round your answer to the nearest degree. (The figure is not necessarily drawn to scale.)

(a) $15^\circ$  
(b) $17^\circ$  
(c) $19^\circ$  
(d) $21^\circ$  
(e) None of these

4.2.14 Word problems: basic trig functions

Problem 557. A tree casts a shadow 53 ft long. The sun is $\theta = 49^\circ$ above the horizon. How tall is the tree? Round your answer to the nearest foot.

(a) 46 ft  
(b) 51 ft  
(c) 56 ft  
(d) 61 ft  
(e) None of these

Problem 558. You want to know the width of a river. You begin by standing directly across from a tree on the opposite bank. You then walk 400 ft straight downstream. From this new point, the tree is at an angle of $\theta = 67^\circ$ to the upstream direction. How wide is the river? Round your answer to the nearest foot.

(a) 170 ft  
(b) 942 ft  
(c) 1070 ft  
(d) 1413 ft  
(e) None of these
Problem 559. A prisoner is trying to escape by digging a tunnel under a wall 147 ft away. Unfortunately, his compass is inaccurate, and his tunnel is off course by $\theta = 19^\circ$. How long does the tunnel have to be to reach the wall? Round your answer to the nearest foot.

(a) 149 ft  (b) 151 ft
(c) 153 ft  (d) 155 ft
(e) None of these

Problem 560. A highway runs directly east and west. An airplane flies across the highway in a direction $\theta = 34^\circ$ north of east. What is the total distance that the airplane will fly before it is 21 miles north of the highway? Round your answer to the nearest tenth of a mile.

(a) 25.3 miles  (b) 29.4 miles
(c) 33.5 miles  (d) 37.6 miles
(e) None of these

Problem 561. A car is driving up a mountain road with a slope of $\theta = 4.5^\circ$. It goes 9000 ft along the road by the odometer. At the end of this time, how high is it above its starting point? Round your answer to the nearest 10 ft.

(a) 680 ft  (b) 710 ft
(c) 740 ft  (d) 770 ft
(e) None of these

Problem 562. From where you stand, a mountain peak has an elevation of $\theta = 12.5^\circ$. On a map, the horizontal distance between you and the peak is 4.11 miles. How high above you is the peak? Round your answer to the nearest 0.01 miles.

(a) 0.83 miles  (b) 0.87 miles
(c) 0.91 miles  (d) 0.95 miles
(e) None of these
Problem 563. You and a friend are trying to find out how high airplanes fly over his house. He phones you when a plane is directly overhead. At that moment, the airplane is at an elevation of \( \theta = 36.2^\circ \), as seen from your house. If your house is 4100 ft from your friend’s house, how high is the airplane? Round your answer to the nearest 100 ft.

(a) 3000 ft  
(b) 3300 ft  
(c) 3600 ft  
(d) 3900 ft  
(e) None of these

Problem 564. A pole is supported by a diagonal guy wire. The wire is anchored in the ground 13.8 m from the base of the pole, and meets the ground at an angle of \( \theta = 59^\circ \). How long is the wire? Round your answer to the nearest 0.1 m.

(a) 25.8 m  
(b) 26.3 m  
(c) 26.8 m  
(d) 27.3 m  
(e) None of these

Problem 565. You are flying a kite in a wind strong enough to stretch the string into a straight line. The string is 71 m long, and makes an angle of \( \theta = 51^\circ \) with the horizontal. How high above the ground is the kite? Round your answer to the nearest meter.

(a) 55 m  
(b) 57 m  
(c) 59 m  
(d) 61 m  
(e) None of these

4.2.15 Word problems: arc functions

Problem 566. A tree is 73 ft tall. It casts a shadow 93 ft long. What is the sun’s angle of elevation \( \theta \)? Round your answer to the nearest degree.

(a) 38°  
(b) 40°  
(c) 42°  
(d) 44°  
(e) None of these
Problem 567. A prisoner is trying to escape by digging a tunnel under a wall 113 ft away. Unfortunately, his compass is inaccurate, and the tunnel is off course by an angle of $\theta$. He does not reach the wall until the tunnel is 128 ft long. What is the value of $\theta$? Round your answer to the nearest degree.

(a) 26°  
(b) 28°  
(c) 30°  
(d) 32°  
(e) None of these

Problem 568. A highway runs directly east and west. An airplane flies across the highway in a direction $\theta$ north of east. When the airplane has flown a total distance of 19 miles, it is 5 miles north of the highway. What is $\theta$? Round your answer to the nearest degree.

(a) 13°  
(b) 15°  
(c) 17°  
(d) 19°  
(e) None of these

Problem 569. A car is driving up a mountain road with a slope of $\theta$. It goes 12,000 ft along the road by the odometer. At the end of this time, it is 1800 ft above its starting point. What is $\theta$? Round your answer to the nearest 0.1°.

(a) 8.0°  
(b) 8.3°  
(c) 8.6°  
(d) 8.9°  
(e) None of these

Problem 570. A mountain peak is 7200 ft higher than your house. On the map, there is a horizontal distance of 53,000 ft from the house to the peak. What is the peak’s elevation $\theta$ as seen from the house? Round your answer to the nearest 0.1°.

(a) 7.1°  
(b) 7.4°  
(c) 7.7°  
(d) 8.0°  
(e) None of these
**Problem 571.** Your house is 3800 ft from your friend’s house. An airplane flies over the friend’s house at a height of 960 ft. When the plane is directly above your friend’s house, what is its elevation $\theta$ as seen from your house? Round your answer to the nearest degree.

- (a) $14^\circ$
- (b) $15^\circ$
- (c) $16^\circ$
- (d) $17^\circ$
- (e) None of these

**Problem 572.** A pole is supported by a diagonal guy wire. The wire is 19.9 m long, and is anchored in the ground 15.2 m from the base of the pole. At what angle $\theta$ does the wire meet the ground? Round your answer to the nearest degree.

- (a) $36^\circ$
- (b) $38^\circ$
- (c) $40^\circ$
- (d) $42^\circ$
- (e) None of these

**Problem 573.** You are flying a kite in a wind strong enough to stretch the string into a straight line. The string is 94 m long, and the kite is 75 m above the ground. What angle $\theta$ does the string make with the ground? Round your answer to the nearest degree.

- (a) $49^\circ$
- (b) $51^\circ$
- (c) $53^\circ$
- (d) $55^\circ$
- (e) None of these
5 Introduction to measurement

The following topics should be known by every well-prepared student entering an entry-level physics course (i.e., both college and university-level physics courses). If you do not know these subjects, then you are already behind! You should be ready to put in a lot of work in this course.

5.1 Dimensions

5.1.1 Dimension or unit?

Problem 574. What are the three fundamental dimensions?

Problem 575. What’s the difference between fundamental dimensions \{length, mass, time\} and the base units \{meter, kilogram, second\}? 

Problem 576. For the given physical quantities, identify the three fundamental dimensions:
(a) mass (c) time (e) force
(b) volume (d) density (f) length
5.1.2 Dimensional consistency

In the following problems, determine which of the formulas could not be correct because it violates the consistency-of-dimensions principle discussed in class. Do not worry about the origin or application of the formulas: just focus on the issue of consistency of units. That is, based solely on consistency of units in an equation, could the given formula be correct?

**Definition:** The Dimensionator \([\quad]\) is the “take the fundamental dimensions of” operator.

**Warning:** Do not let the subscripts confuse you. The subscripts are only used to label variables, they are dimensionless numbers and letters and do not affect the outcome of dimensionator one bit.

The dimensions of the fundamental quantities: \{Length L, Mass M, Time T\} are

- \(d = \text{distance}; \quad [d] = L\)
- \(x = \text{distance}; \quad [x] = L\)
- \(R = \text{radius}; \quad [R] = L\)
- \(h = \text{height}; \quad [h] = L\)
- \(m = \text{mass}; \quad [m] = M\)
- \(t = \text{time}; \quad [t] = T\)

The dimensions of the derived quantities (i.e., the quantities derived from the fundamental quantities) are

- \(r = \text{rate (speed)}; \quad [r] = L/T\)
- \(v = \text{velocity}; \quad [v] = L/T\)
- \(a = \text{acceleration}; \quad [a] = L/T^2\)
- \(A = \text{area}; \quad [A] = L^2\)
- \(V = \text{volume}; \quad [V] = L^3\)

**Problem 577.** \(d = rt\)

**Problem 578.** \(v = x + t\)

**Problem 579.** \(V = \frac{4}{3} \pi R^2\)

**Problem 580.** \(V = \pi R^2 h\)

**Problem 581.** \(V = Ad\)
Problem 582. \( V = Ah \)
Problem 583. \( V = Ax^2 \)
Problem 584. \( v = \frac{x}{t} \)
Problem 585. \( v_2^2 = v_1^2 + x^2 \)
Problem 586. \( v_2^2 = v_1^2 + \frac{x^2}{t^2} \)

Problem 587. Determine whether equations (i) and (ii) below are dimensionally consistent.

(i) \( v = \frac{x - x_0}{t} \)  \hspace{1cm} (ii) \( x = a(t + v)^2 \),
where \( x = \) distance, \( v = \) velocity, \( t = \) time, and \( a = \) acceleration.

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent
(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent
(c) (i) and (ii) are both dimensionally consistent
(d) (i) and (ii) are both dimensionally inconsistent

Problem 588. Consider the formulas (i) and (ii). Are the formulas dimensionally consistent? Here \( x \) = distance, \( v \) = velocity, \( V \) = volume, and \( A \) = area.

(i) \( V = Ax \)  \hspace{1cm} (ii) \( v_2^2 = v_1^2 + x^2 \)

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent
(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent
(c) (i) and (ii) are both dimensionally consistent
(d) (i) and (ii) are both dimensionally inconsistent

Problem 589. Consider the formulas (i) and (ii). Are the formulas dimensionally consistent? Here \( V \) = volume, \( r \) = distance, \( x \) = distance, and \( t \) = time.

(i) \( V = \pi xr^2 \)  \hspace{1cm} (ii) \( t = \frac{x - t_0}{2} \)

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent
(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent
(c) (i) and (ii) are both dimensionally consistent
(d) (i) and (ii) are both dimensionally inconsistent
Problem 590. Determine whether equations (i) and (ii) below are dimensionally consistent.

\[ \text{(i)} \quad a = \frac{x - x_0}{t^2} \quad \text{and} \quad \text{(ii)} \quad x^2 = at^2 + (x + vt)^2, \]

where \( x \) = distance, \( v \) = velocity, \( t \) = time, and \( a \) = acceleration.

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent

(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent

(c) (i) and (ii) are both dimensionally consistent

(d) (i) and (ii) are both dimensionally inconsistent

Problem 591. Determine whether equations (i) and (ii) below are dimensionally consistent.

\[ \text{(i)} \quad x = at^2 + (1 + (vt)/x)^2 \quad \text{and} \quad \text{(ii)} \quad v_f^2 = v_i^2 + 2ax^2, \]

where \( x \) = distance, \( v \) = velocity, \( t \) = time, \( a \) = acceleration, \( r \) = distance, \( g \) = magnitude of gravity and \( \mu_s \) is the coefficient of static friction.

(a) (i) is dimensionally consistent; (ii) is dimensionally inconsistent

(b) (i) is dimensionally inconsistent; (ii) is dimensionally consistent

(c) (i) and (ii) are both dimensionally consistent

(d) (i) and (ii) are both dimensionally inconsistent

Problem 592. (Using dimensional analysis to get statistical-partial credit)
Consider the following test question and multiple-choice answers. Without knowing how to solve the problem, can you identify the answer that cannot be a solution based solely on dimensional analysis?

You are towing a crate of physics books down the hallway, using a rope that makes an angle of \( \theta \) to the horizontal. The crate has a mass of \( m \); the coefficient of kinetic friction between the crate and the floor is \( \mu_k \). As you go down the hallway, you are giving the crate a forward acceleration of \( a \). What is the tension \( T \) in the rope, expressed in terms of the mass \( m \), the acceleration \( a \), the angle \( \theta \) (dimensionless), \( \mu_k \) (dimensionless), and the gravitational constant of acceleration \( g \)?

(a) \( (mg\mu_k + a) \cos \theta \)

(b) \( \frac{m(g\mu_k + a)}{\cos \theta + \mu_k \sin \theta} \)

(c) \( \frac{m(g\mu_k + a)}{\cos \theta} \)

(d) \( (mg\mu_k + a)(\cos \theta + \mu_k \sin \theta) \)

5.1.3 Practical questions involving dimensions

Problem 593. In the construction business, concrete is often sold by the yard. What is the precise meaning of a yard of concrete?

Problem 594. In the carpet business, carpet is sold by the yard. What do they really mean by a yard of carpet?
Problem 595. Name some physical phenomena that could be used as crude timing devices.

Problem 596. Suppose your lab partner tells you that the volume of a cylinder is $V = \pi r^3 h$. Could this formula be correct?
5.2 Units

Problem 597. What are the units of the number $\pi$?

Problem 598. What are the units of the number $e$?

Problem 599. What are the units of the number $-5$?

Problem 600. Give an example from everyday life of something that’s about 1 m long.

Problem 601. Give an example from everyday life of something that has a mass of about 1 kg.

Problem 602. Give an example from everyday life of something that weighs about 1 N.

Problem 603. Give an example from everyday life of something with a volume of about 1 l.

Problem 604. Give an example from everyday life of a distance that’s about 1 km.

Problem 605. How much do you weigh in pounds? Round your answer to the nearest pound. Guess if you don’t have a scale handy. If that’s your weight in pounds, what is your mass in kg? Round to the nearest kg.

Problem 606. What is your height in inches? Round your answer to the nearest inch. Guess if you don’t have a tape measure handy. If that’s your height in inches, what is your height in cm? Round to the nearest cm.

Problem 607. How far is it in miles from where you live to your physics classroom? Round your answer to the nearest 0.1 mile. Guess if you haven’t measured it. If that’s the distance in miles, what is the distance in km? Round to the nearest 0.1 km.

5.2.1 Converting between different sets of units

Problem 608. A box is 13 inches tall. What is its height in centimeters? Round to the nearest cm.

Problem 609. A bookshelf is 2.3 m tall. How tall is it in feet? Round to the nearest 0.1 ft.

Problem 610. Two towns are 37 miles apart. How far apart are they in kilometers? Round your answer to the nearest km.

Problem 611. Two other towns are 113 km apart. How far apart are they in miles? Round your answer to the nearest mile.

Problem 612. A house sits on a lot of 7260 square feet. What is the size of the lot in square meters? Round your answer to the nearest m$^2$.

Problem 613. A pond has a surface area of 3900 m$^2$. What is its surface area in square feet? Round to the nearest 100 ft$^2$. 
Problem 614. You are given a ticket for driving at 38 miles per hour in a school zone. What is your speed in meters per second? Round your answer to the nearest m/s.

(a) 15 m/s  (b) 17 m/s  
(c) 19 m/s  (d) 20 m/s  
(e) None of these

Problem 615. You are given a ticket for driving at 37 miles per hour in a school zone. What was your speed in m/s? Round your answer to the nearest m/s.

Problem 616. A water-balloon dropped from a tenth-floor window hits the ground at 24.5 m/s. What is its speed in miles per hour? Round your answer to the nearest 0.1 mile/hour. (Hint: 1.61 km = 1 mi)

Problem 617. A water-balloon dropped from a 38th-floor window hits the ground at 86 m/s. What is its speed in ft/s? Round your answer to the nearest ft/s.

(a) 26 ft/s  (b) 29 ft/s  
(c) 257 ft/s  (d) 282 ft/s  
(e) None of these

Problem 618. An evil scientist grows a giant carrot with a mass of 60 kg. What is the carrot’s weight in pounds? Round your answer to the nearest pound.

(a) 17 lb  (b) 27 lb  
(c) 132 lb  (d) 207 lb  
(e) None of these

Problem 619. A can of beer has a volume of 355 cm³. What is its volume in cubic inches? Round your answer to the nearest 0.1 in³.

Problem 620. A pond has a volume of 325,000 ft³. What is its volume in m³? Round your answer to the nearest 100 cubic meters.

Problem 621. A river is flowing at 46,500 cubic feet per second. What is the river’s flow rate in cubic meters per second? Round your answer to the nearest 10 m³/s.

Problem 622. A piano is pushed with a force of 450 N. What is the equivalent force in pounds? Round your answer to the nearest 10 lb.

Problem 623. A box weighs 46 lb. What is the equivalent weight in newtons? Round your answer to the nearest N.

Problem 624. A sack of potatoes has a mass of 12 kg. What is the weight of the potatoes in pounds? Round your answer to the nearest lb.

Problem 625. A sack of potatoes has a mass of 20 kg. What is the weight of the potatoes in pounds? Round your answer to the nearest lb.
**Problem 626.** A German shepherd weighs 72 lb. What is the dog’s mass in kg? Round your answer to the nearest kg.

**Problem 627.** A cable weighs 5.2 lb/ft. What is the cable’s mass in kg/m? Round your answer to the nearest 0.1 kg/m.

**Problem 628.** The pressure at the bottom of a swimming pool is 18 lb/in$^2$. What is the pressure in N/cm$^2$? Round your answer to the nearest N/cm$^2$.

**Problem 629.** An engine burns fuel at a rate of 11.2 g/s. What is the consumption rate in kg/hr? Round your answer to the nearest 0.1 kg/hr.

**Problem 630.** You have just discovered the new element explodium. It has a density of 9.70 g/cm$^3$. The *USA Today* reporter says that their readers don’t understand all that centi-whatsis stuff, and wants the density in pounds per cubic foot. Round your answer to the nearest lb/ft$^3$.

**Problem 631.** Your male friend is trying to impress a pretty German girl at your local pub. He says to her “I can bench press 330lbs.” She then asks “How much is that in kilograms?” He is dumbfounded. Can you save him by converting 330 lbs to kgs, so that she can be properly impressed?

**Problem 632.** You have a measuring stick that has centimeters and inches on it. You notice that 1.00 in = 2.54 cm. Using only this information together with the relationship 1 mile = 5,280 ft., find a relationship between miles and kilometers. That is, 1 mile = how many kilometers?

**Problem 633.** Use the relationship 1.00 in = 2.54 cm to find a relationship between a yard and a meter. Your answer should show that a yard is close in length to a meter.

### 5.3 Scientific notation

For the following problems express the following numbers in scientific notation without using a calculator. Assume 3 significant figures in each number with more than 3 trailing zeros.

**Problem 634.** 1

**Problem 635.** 12

**Problem 636.** 123

**Problem 637.** 1234

**Problem 638.** 12345

**Problem 639.** 0.1

**Problem 640.** 0.12

**Problem 641.** 0.012
Problem 642. 0.0012
Problem 643. 0.00012
Problem 644. 395
Problem 645. 97,000
Problem 646. .0000345
Problem 647. 10,000,000
Problem 648. .0000000500

For the following problems convert the expressions that are given in scientific notation into standard form without using a calculator.

Problem 649. $1 \times 10^0$
Problem 650. $1.2 \times 10^1$
Problem 651. $1.23 \times 10^2$
Problem 652. $1.234 \times 10^3$
Problem 653. $1.2345 \times 10^4$
Problem 654. $1 \times 10^{-1}$
Problem 655. $1.2 \times 10^{-1}$
Problem 656. $1.2 \times 10^{-2}$
Problem 657. $1.2 \times 10^{-3}$
Problem 658. $1.2 \times 10^{-4}$
Problem 659. $3.95 \times 10^2$
Problem 660. $9.70 \times 10^4$
Problem 661. $3.45 \times 10^{-5}$
Problem 662. $1.00 \times 10^7$
Problem 663. $5.00 \times 10^{-9}$

Problem 664. Convert the number $6.2 \times 10^{-3}$ from scientific notation to standard form.
   (a) 0.00062 (b) −6200
   (c) 0.0062 (d) −62,000
   (e) None of these

Problem 665. Convert the number $9.70 \times 10^{-4}$ to standard form.
   (a) 0.000097 (b) 0.00097
   (c) 0.0000970 (d) 0.000970
   (e) None of these
Problem 666. Convert the number 97,000 to scientific notation. Your answer should have three significant figures.

(a) $9.70 \times 10^3$
(b) $9.70 \times 10^4$
(c) $9.7 \times 10^3$
(d) $9.7 \times 10^4$
(e) None of these

For the following problems perform the following arithmetic without using a calculator.

Problem 667. $\frac{(1.0 \times 10^3) \cdot (9.0 \times 10^8)}{3.0 \times 10^7}$

Problem 668. $\frac{(4.67 \times 10^7) \cdot (1.01 \times 10^2)}{6.24 \times 10^3}$

Problem 669. $\frac{(7.3 \times 10^2) \cdot (2.23 \times 10^5)}{2.45 \times 10^4}$

Problem 670. $(3.4 \times 10^7) - (3.0 \times 10^6)$

Problem 671. $(3.4 \times 10^3) + (3.0 \times 10^4)$

5.4 Significant figures

Problem 672. A useful approximation for the number of seconds in a year is $\pi \times 10^7$ s. Determine the number of significant figures in this approximation given that there are 365.25 days in a year.

Problem 673. A common approximation for $\pi = 3.141592659 \ldots$ is $\frac{22}{7}$. That is, $\pi \approx \frac{22}{7}$. To how many significant figures is this approximation accurate?

Problem 674. A common approximation for $\pi = 3.141592659 \ldots$ is $\frac{355}{113}$. That is, $\pi \approx \frac{355}{113}$. To how many significant figures is this approximation accurate?

For the following problems, perform the indicated operations. Assume that all of the numbers come from real data with the correct number of significant figures. Be sure to express your answer in the correct number of significant figures. Please refer to rules 1-11 in the lecture notes.

Problem 675. $566.3 + 32.50 + 2.197 = ?$

Problem 676. $261.72 - 90.715 = ?$

Problem 677. $523.498 - 32.907527 + 0.52 = ?$

Problem 678. $225 + 1.087 - 45.3975 = ?$

Problem 679. $596.9/3.986 = ?$

Problem 680. $505 \times 0.0065$
Problem 681. $2.00 \times 10^7 + 3.5 \times 332 = ?$

Problem 682. $(3.7)^3 = ?$

Problem 683. What is the average of 2.5212, 2.5214, 2.5213, 2.5212, and 2.5220?

Problem 684. Calculate $2.00 \times 1.07 + 3.5 \times 332$. Your answer should have the correct number of significant figures.

(a) 1200  
(b) 1160  
(c) 1164  
(d) 1164.14  
(e) None of these

Problem 685. Evaluate the following expression. Round your answer to the correct number of significant figures: $1.24 + 24.038 + 6.925$

(a) 32  
(b) 32.2  
(c) 32.20  
(d) 32.203  
(e) None of these

Problem 686. $\frac{(7.3 \times 10^2) \cdot (2.23 \times 10^5)}{2.45 \times 10^4}$

Problem 687. How many significant digits are there in: $m = 0.0507 \text{ kg}$?

Problem 688. How many significant digits are there in: $m = 28.350 \text{ g}$?

Problem 689. How many significant digits are there in: $v = 1.022 \text{ m/s}$?

Problem 690. How many significant digits are there in: $r = 0.028 \text{ cm}$?

Problem 691. How many significant digits are there in: $h = 0.08 \text{ cm}$?

Problem 692. How many significant digits are there in: $r = 1.028 \text{ cm}$?

Problem 693. How many significant digits are there in: $r = 10.028 \text{ cm}$?

Problem 694. How many significant digits are there in: $V = 44,000 \text{ ft}^3$?

Problem 695. How many significant digits are there in: $r = 0.03070 \text{ m}$?

(a) 2  
(b) 3  
(c) 4  
(d) 5  
(e) None of these

Problem 696. How many significant digits are there in: $v = 1.022 \text{ m/s}$?

(a) 2  
(b) 3  
(c) 4  
(d) 5  
(e) None of these

Problem 697. How many significant digits are there in: $r = 0.02070 \text{ m}$?

(a) 2  
(b) 3  
(c) 4  
(d) 5  
(e) None of these
Problem 698. How many significant digits are there in: \( r = 0.001000 \text{ m} \)?

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) None of these

Problem 699. Use a calculator to determine the decimal value of 1/7. Round your answer to 3 significant digits.

(a) 0.14  
(b) 0.15  
(c) 0.142  
(d) 0.143  
(e) None of these

Problem 700. Use a calculator to determine the decimal value of 22/7. Round your answer to 3 significant digits.

Problem 701. Use a calculator to determine the decimal value of \( \pi \). Round your answer to 3 significant digits. How does this answer compare to the previous answer for 22/7?

Problem 702. Use a calculator to determine the decimal value of 1/237. Round your answer to 3 significant digits.

Problem 703. You have calculated the volume of a pond as 83,512.338 ft\(^3\). Round this answer to 2 significant digits.

Problem 704. You have taken several measurements of the size of a very small particle. The mean of your measurements is 0.00013722 cm. Round this answer to 2 significant digits.

Problem 705. (Significant Figures and Conversion) If you were to convert the measured length 7.923 ft into yards by multiplying by the conversion factor (1 yd/3 ft), how many significant figures should the answer contain? Justify your answer.

Problem 706. Evaluate the following expression. Round your answer to the correct number of significant figures: \( \frac{\left(7.3 \times 10^2\right) \cdot \left(2.23 \times 10^5\right)}{2.45 \times 10^4} \)

(a) 6.64 \times 10^3  
(b) 6.6 \times 10^3  
(c) 6.6 \times 10^2  
(d) 6.64 \times 10  
(e) None of these

Problem 707. Evaluate the following expression. Round your answer to the correct number of significant figures.

\( \frac{(3.021 \times 10^{-2})(8.446 \times 10^{11})}{1.9 \times 10^3} \)
5.5 Determining derived units from definitions and equations

In the following problems, use the given physical governing equation to determine the dimensions of the given derived quantity in terms of the fundamental dimensions of length, time, and mass, or in terms of other derived quantities as specified in the problem. Use the SI units when expressing the dimensions.

Problem 708. Determine the expression for the unit of work in terms of the fundamental units from the definition of work (in one spacial dimension by a constant force) as force times distance $W = Fd$, where $W$ is the symbol for work (a method of transferring energy), $F$ is the force exerted on the object and $d$ is the distance that the force acts on the object.

Problem 709. Determine the expression for the unit of kinetic energy (K.E.) in terms of the fundamental units from the definition of K.E. as $K = \frac{1}{2}mv^2$ where $K$ is the symbol for kinetic energy, $m$ is the mass of the object, $v$ is the speed of the object.

Problem 710. Determine the expression for the unit of gravitational potential energy (P.E.) in terms of the fundamental units from the definition of P.E. as $U = mgh$, where $U$ is the symbol for potential energy, $m$ is the mass of the object, $g$ is the magnitude of acceleration due to gravity, and $h$ is the height of an object above some reference level.

Problem 711. Determine the expression for the unit of elastic potential energy (P.E.) stored in a spring in terms of the fundamental units from the definition of P.E. as $U = \frac{1}{2}kx^2$, where $U$ is the symbol for potential energy, $k$ is the spring constant, and $x$ is the displacement from the equilibrium position.

Problem 712. Compare your results from the four previous expressions for work, kinetic, and potential energy. What do you conclude about the relation between the various units?

Problem 713. Determine the fundamental dimensions for density = mass/volume, denoted by $\rho$.

Problem 714. Express pressure, denoted by $P$, in terms of the fundamental dimensions. For now, just take pressure to be force/area. **Note:** In a later chapter we will use the mathematical expression for Newton’s second law to show that the dimensions of force are $[F] = MLT^{-2}$. For now, just take this as a fact.

Problem 715. Determine the expression for the unit of pressure (the Pascal Pa) in terms of the SI units for pressure and area from the equation defining pressure $P = \frac{F}{A}$, where $F$ is force and $A$ is area.

Problem 716. Determine the units of the universal gas constant $R$ from the equation for the ideal gas law: $PV = nRT$, where $P$ is pressure, $V$ is volume, $n$ is moles (denoted mol), and $T$ is temperature (with SI units of absolute temperature the kelvin).

Problem 717. In the study of thermodynamics one of the common expressions that you encounter is pressure $P$ times volume $V$. It turns out that this term has the dimensions of work = force $\times$ distance. To see this we write volume = area $\times$ length, then using the
definitions of pressure and work we have

\[ \text{pressure} \times \text{volume} = \frac{\text{force}}{\text{area}} \times (\text{area} \times \text{length}) = \text{force} \times \text{length} = \text{work}. \]

Verify our word equation using the fundamental dimensions for the product of pressure and volume: pressure × volume = \( PV \) and work as work = \( Fd \), where \( d \) is the distance that the force acts over.
6 Introduction to Vectors

Unless stated otherwise, the direction of a vector in $xy$-space is the counterclockwise angle $\theta$ that it makes with the $x$-axis; $0 \leq \theta < 360^\circ$ or $0 \leq \theta < 2\pi$ rad., and angle measurements in your answers should be in the same units as in the statement of the problem.

6.1 Identifying Vectors

6.1.1 Given vector, identify on figure

Problem 718. Which of the vectors in the graph corresponds to $-10\hat{i} - 5\hat{j}$?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 719. Which of the vectors in the graph corresponds to $-6\hat{i} + 2\hat{j}$?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these
Problem 720. Which of the vectors in the graph corresponds to $4\hat{i} + 4\hat{j}$?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 721. Which of the vectors in the graph corresponds to $-\hat{i} + 2\hat{j}$?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 722. Which of the vectors in the graph corresponds to $-3\hat{i} + \hat{j}$?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these
Problem 723. Which of the vectors in the graph corresponds to $2\mathbf{i} + \mathbf{j}$?
(a) $\vec{A}$  (b) $\vec{B}$  
(c) $\vec{C}$  (d) $\vec{D}$  
(e) None of these

Problem 724. Which of the vectors in the graph corresponds to $\mathbf{i} + 2\mathbf{j}$?
(a) $\vec{A}$  (b) $\vec{B}$  
(c) $\vec{C}$  (d) $\vec{D}$  
(e) None of these

Problem 725. Which of the vectors in the graph corresponds to $3\mathbf{i} - 4\mathbf{j}$?
(a) $\vec{A}$  (b) $\vec{B}$  
(c) $\vec{C}$  (d) $\vec{D}$  
(e) None of these
Problem 726. Which of the vectors in the graph corresponds to \((-3, 1)\)?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 727. Which of the vectors in the graph corresponds to \((2, 2)\)?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 728. Which of the vectors in the graph corresponds to \((-1, 2)\)?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these
Problem 729. Which of the vectors in the graph corresponds to $\langle -2, -1 \rangle$?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 730. Which of the vectors in the graph corresponds to $\langle 1, 3 \rangle$?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 731. Which of the vectors in the graph corresponds to $\langle -4, -2 \rangle$?
(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these
Problem 732. Which of the vectors in the graph corresponds to $(10, -5)$?

(a) $\vec{A}$
(b) $\vec{B}$
(c) $\vec{C}$
(d) $\vec{D}$
(e) None of these

Problem 733. Which of the vectors in the graph corresponds to $(3, -3)$?

(a) $\vec{A}$
(b) $\vec{B}$
(c) $\vec{C}$
(d) $\vec{D}$
(e) None of these

6.1.2 Given vector on figure, identify formula

Problem 734. Which of the vectors below corresponds to $\vec{A}$ on the graph at right?

(a) $\hat{i} + \hat{j}$
(b) $-\hat{i} + \hat{j}$
(c) $\hat{i} - \hat{j}$
(d) $-\hat{i} - \hat{j}$
(e) None of these
**Problem 735.** Which of the vectors below corresponds to $\vec{B}$ on the graph at right?

(a) $\hat{i} + \hat{j}$  
(b) $-\hat{i} + \hat{j}$  
(c) $\hat{i} - \hat{j}$  
(d) $-\hat{i} - \hat{j}$  
(e) None of these

**Problem 736.** Which of the vectors below corresponds to $\vec{C}$ on the graph at right?

(a) $\hat{i} + \hat{j}$  
(b) $-\hat{i} + \hat{j}$  
(c) $\hat{i} - \hat{j}$  
(d) $-\hat{i} - \hat{j}$  
(e) None of these

**Problem 737.** Which of the vectors below corresponds to $\vec{D}$ on the graph at right?

(a) $\hat{i} + \hat{j}$  
(b) $-\hat{i} + \hat{j}$  
(c) $\hat{i} - \hat{j}$  
(d) $-\hat{i} - \hat{j}$  
(e) None of these
Problem 738. Which of the vectors below corresponds to $\vec{A}$ on the graph at right?
(a) $3\hat{i} + \hat{j}$  
(b) $-3\hat{i} + \hat{j}$  
(c) $2\hat{i} - \hat{j}$  
(d) $-\hat{i} - 3\hat{j}$  
(e) None of these

Problem 739. Which of the vectors below corresponds to $\vec{A}$ on the graph at right?
(a) $2\hat{i} + 6\hat{j}$  
(b) $3\hat{i} + \hat{j}$  
(c) $2\hat{i} - \hat{j}$  
(d) $-\hat{i} - 3\hat{j}$  
(e) None of these

Problem 740. Which of the vectors below corresponds to $\vec{A}$ on the graph at right?
(a) $2\hat{i} - \hat{j}$  
(b) $\hat{i} - 2\hat{j}$  
(c) $-2\hat{i} + \hat{j}$  
(d) $-\hat{i} + 2\hat{j}$  
(e) None of these
Problem 741. Which of the vectors below corresponds to $\vec{A}$ on the graph at right?

(a) $6\hat{i} - 3\hat{j}$  
(b) $-3\hat{i} + 6\hat{j}$  
(c) $-6\hat{i} + 3\hat{j}$  
(d) $3\hat{i} - 6\hat{j}$  
(e) None of these

Problem 742. Which of the vectors below corresponds to $\vec{A}$ on the graph at right?

(a) $\langle -1, 2 \rangle$  
(b) $\langle -2, 1 \rangle$  
(c) $\langle -1, 2 \rangle$  
(d) $\langle -2, -1 \rangle$  
(e) None of these

Problem 743. Which of the vectors below corresponds to $\vec{A}$ on the graph at right?

(a) $\langle -1, 2 \rangle$  
(b) $\langle 1, -2 \rangle$  
(c) $\langle -2, 1 \rangle$  
(d) $\langle 2, -1 \rangle$  
(e) None of these
Problem 744. Which of the vectors below corresponds to \( \vec{A} \) on the graph at right?
(a) \((-6, -3)\)  
(b) \((-3, -6)\)  
(c) \((6, -3)\)  
(d) \((3, -6)\)  
(e) None of these

Problem 745. Which of the vectors below corresponds to \( \vec{A} \) on the graph at right?
(a) \((-5, -10)\)  
(b) \((5, -10)\)  
(c) \((-10, 5)\)  
(d) \((10, -5)\)  
(e) None of these

Problem 746. Which of the vectors below corresponds to \( \vec{A} \) on the graph at right?
(a) \((-2, 4)\)  
(b) \((2, 4)\)  
(c) \((-4, 2)\)  
(d) \((4, 2)\)  
(e) None of these
Problem 747. Which of the vectors below corresponds to \( \vec{A} \) on the graph at right?

(a) \((-1, 2)\)  
(b) \((-2, 1)\)  
(c) \((1, -2)\)  
(d) \((2, -1)\)  
(e) None of these

Problem 748. Which of the vectors below corresponds to \( \vec{A} \) on the graph at right?

(a) \((2, 1)\)  
(b) \((1, 2)\)  
(c) \((-2, 1)\)  
(d) \((-1, 2)\)  
(e) None of these

Problem 749. Which of the vectors below corresponds to \( \vec{A} \) on the graph at right?

(a) \((-2, -1)\)  
(b) \((-1, -2)\)  
(c) \((2, -1)\)  
(d) \((1, -2)\)  
(e) None of these
6.1.3 Given direction, identify on figure

**Problem 750.** Which of the vectors in the graph corresponds to the direction 20° east of north?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

**Problem 751.** Which of the vectors in the graph corresponds to the direction 20° east of south?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

**Problem 752.** Which of the vectors in the graph corresponds to the direction 20° west of north?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these
Problem 753. Which of the vectors in the graph corresponds to the direction 20° west of south?
(a) $\vec{A}$  (b) $\vec{B}$  (c) $\vec{C}$  (d) $\vec{D}$  (e) None of these

Problem 754. Which of the vectors in the graph corresponds to the direction 30° south of west?
(a) $\vec{A}$  (b) $\vec{B}$  (c) $\vec{C}$  (d) $\vec{D}$  (e) None of these

Problem 755. Which of the vectors in the graph corresponds to the direction 30° north of west?
(a) $\vec{A}$  (b) $\vec{B}$  (c) $\vec{C}$  (d) $\vec{D}$  (e) None of these
Problem 756. Which of the vectors in the graph corresponds to the direction 60° south of west?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 757. Which of the vectors in the graph corresponds to the direction 60° north of west?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these

Problem 758. What is the direction of vector $\vec{A}$ in the graph at right?

(a) 30° north of east  
(b) 30° south of east  
(c) 30° north of west  
(d) 30° south of west  
(e) None of these
Problem 759. What is the direction of vector $\vec{A}$ in the graph at right?

(a) $30^\circ$ north of east
(b) $30^\circ$ south of east
(c) $30^\circ$ north of west
(d) $30^\circ$ south of west
(e) None of these

Problem 760. What is the direction of vector $\vec{A}$ in the graph at right?

(a) $30^\circ$ north of east
(b) $30^\circ$ south of east
(c) $30^\circ$ north of west
(d) $30^\circ$ south of west
(e) None of these

Problem 761. What is the direction of vector $\vec{A}$ in the graph at right?

(a) $30^\circ$ north of east
(b) $30^\circ$ south of east
(c) $30^\circ$ north of west
(d) $30^\circ$ south of west
(e) None of these
Problem 762. What is the direction of vector $\vec{A}$ in the graph at right?
(a) 30° east of north
(b) 60° east of north
(c) 30° west of north
(d) 60° west of north
(e) None of these

Problem 763. What is the direction of vector $\vec{A}$ in the graph at right?
(a) 30° east of north
(b) 60° east of north
(c) 30° west of north
(d) 60° west of north
(e) None of these

Problem 764. What is the direction of vector $\vec{A}$ in the graph at right?
(a) 30° east of north
(b) 60° east of north
(c) 30° west of north
(d) 60° west of north
(e) None of these
Problem 765. What is the direction of vector \( \vec{A} \) in the graph at right?
(a) 30° east of north
(b) 60° east of north
(c) 30° west of north
(d) 60° west of north
(e) None of these

6.1.4 Given vectors on figure, name quadrant of sum/difference

Problem 766. The vectors \( \vec{A} \) and \( \vec{B} \) are labelled on the figure at right. Sketch and label the vector \( \vec{A} - \vec{B} \). What quadrant does \( \vec{A} - \vec{B} \) lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the x- or y-axis

Problem 767. The vectors \( \vec{A} \) and \( \vec{B} \) are labelled on the figure at right. Sketch and label the vector \( \vec{A} + \vec{B} \). What quadrant does \( \vec{A} + \vec{B} \) lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the x- or y-axis
Problem 768. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$. What quadrant does $\vec{A} - \vec{B}$ lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis

Problem 769. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{B} - \vec{A}$. What quadrant does $\vec{B} - \vec{A}$ lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis

Problem 770. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B}$. What quadrant does $\vec{A} + \vec{B}$ lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis
Problem 771. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B}$. What quadrant does $\vec{A} + \vec{B}$ lie in?
   (a) Quadrant I  
   (b) Quadrant II 
   (c) Quadrant III 
   (d) Quadrant IV 
   (e) It lies on the $x$- or $y$-axis

Problem 772. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{B} - \vec{A}$. What quadrant does $\vec{B} - \vec{A}$ lie in?
   (a) Quadrant I  
   (b) Quadrant II 
   (c) Quadrant III 
   (d) Quadrant IV 
   (e) It lies on the $x$- or $y$-axis

Problem 773. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$. What quadrant does $\vec{A} - \vec{B}$ lie in?
   (a) Quadrant I  
   (b) Quadrant II 
   (c) Quadrant III 
   (d) Quadrant IV 
   (e) It lies on the $x$- or $y$-axis
Problem 774. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$. What quadrant does $\vec{A} - \vec{B}$ lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis

Problem 775. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B}$. What quadrant does $\vec{A} + \vec{B}$ lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis

Problem 776. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$. What quadrant does $\vec{A} - \vec{B}$ lie in?
(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis
Problem 777. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B}$. What quadrant does $\vec{A} + \vec{B}$ lie in?

(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis

Problem 778. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{B} - \vec{A}$. What quadrant does $\vec{B} - \vec{A}$ lie in?

(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) It lies on the $x$- or $y$-axis

6.1.5 Mixing it up

Problem 779. Five vectors are graphed on the diagram at right and described below. Write the letter of each vector beside its description.

____ $3\mathbf{i} - \mathbf{j}$
____ $-4\mathbf{i} + 2\mathbf{j}$
____ $2\mathbf{i} - 3\mathbf{j}$
____ $\mathbf{i} + 4\mathbf{j}$
____ $-3\mathbf{i} - 3\mathbf{j}$
Problem 780. Five vectors are graphed on the diagram at right and described below. Write the letter of each vector beside its description.

___ \langle 2, 2 \rangle
___ \langle -3, 1 \rangle
___ \langle 3, -4 \rangle
___ \langle -2, -1 \rangle
___ \langle -2, 4 \rangle

Problem 781. Five velocity vectors are graphed on the diagram at right and described below. Write the letter of each vector beside its magnitude and direction.

___ 14 m/s; 0.79 rad south of east
___ 45 m/s; 1.11 rad south of east
___ 41 m/s; 1.33 rad south of west
___ 45 m/s; 0.46 rad north of west
___ 36 m/s; 0.59 rad south of west

Problem 782. Five velocity vectors are graphed on the diagram at right and described below. Write the letter of each vector beside its magnitude and direction.

___ 22 mph; 64° north of east
___ 50 mph; 37° south of east
___ 36 mph; 34° south of west
___ 32 mph; 18° north of east
___ 28 mph; 45° south of east
Problem 783. The vector $\vec{A}$ is labelled on the figure at right. Sketch and label the vector $-\vec{A}$.

Problem 784. The vector $\vec{A}$ is labelled on the figure at right. Sketch and label the vector $-\vec{A}$.

Problem 785. Which of the vectors in the graph corresponds to $\hat{i} - 3\hat{j}$?

(a) $\vec{A}$  
(b) $\vec{B}$  
(c) $\vec{C}$  
(d) $\vec{D}$  
(e) None of these
**Problem 786.** Five vectors are graphed on the diagram at right and described below. Write the letter of each vector beside its description.

\[ \langle 2, 2 \rangle \]
\[ \langle -3, 1 \rangle \]
\[ \langle 3, -4 \rangle \]
\[ \langle -2, -1 \rangle \]
\[ \langle -2, 4 \rangle \]

**Problem 787.** Which of the vectors in the graph corresponds to \(-6\hat{i} - 3\hat{j}\)?

(a) \(\vec{A}\)  
(b) \(\vec{B}\)  
(c) \(\vec{C}\)  
(d) \(\vec{D}\)  
(e) None of these

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### 6.2 Geometric Vector Addition and Subtraction

**Problem 788.** The vectors \(\vec{A}\) and \(\vec{B}\) are labelled on the figure at right. Sketch and label the vector \(\vec{A} + \vec{B}\).
Problem 789. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$.

Problem 790. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$.

Problem 791. The vectors $\vec{X}$, $\vec{Y}$, and $\vec{Z}$ are labelled on the figure at right. Which of the following equations is true?

(a) $\vec{Z} = \vec{X} - \vec{Y}$
(b) $\vec{Z} = \vec{Y} - \vec{X}$
(c) $\vec{Z} = \vec{X} + \vec{Y}$
(d) $\vec{Z} = -\vec{X} - \vec{Y}$
(e) None of these
Problem 792. The vectors $\vec{X}$, $\vec{Y}$, and $\vec{Z}$ are labelled on the figure at right. Consider the vector $-\vec{Z}$. From the figure it is clear that

$$-\vec{Z} = \pm \vec{X} \pm \vec{Y}.$$ 

Which of the following equations is true?

(a) $-\vec{Z} = -\vec{X} + \vec{Y}$
(b) $-\vec{Z} = \vec{X} + \vec{Y}$
(c) $-\vec{Z} = -\vec{X} - \vec{Y}$
(d) $-\vec{Z} = \vec{X} - \vec{Y}$
(e) None of these

Problem 793. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B}$.

Problem 794. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$. 
**Problem 795.** The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B}$.

**Problem 796.** The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$.

**Problem 797.** The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B}$.
Problem 798. The vectors $\vec{A}$ and $\vec{B}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} - \vec{B}$.

Problem 799. The vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B} + \vec{C}$.

Problem 800. The vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are labelled on the figure at right. Sketch and label the vector $\vec{A} + \vec{B} - \vec{C}$.
6.3 Position vs. Displacement Vector Problems

Problem 801. On the graph below, draw and label the position vectors $\vec{r}_i$ and $\vec{r}_f$ corresponding to the points $P_i$ and $P_f$ on the graph, and the displacement vector $\Delta \vec{r} \equiv \vec{r}_f - \vec{r}_i$.

Problem 802. At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. On the axes at right, sketch and label the spider’s displacement vectors:
- $\vec{A}$ at 12:15
- $\vec{B}$ at 12:30
- $\vec{C}$ at 12:45
- $\vec{D}$ at 1:00

Problem 803. At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. Which of the vectors on the figure at right is the spider’s displacement vector at 12:15?
(a) $\vec{C}$  
(b) $\vec{D}$  
(c) $\vec{G}$  
(d) $\vec{H}$  
(e) None of these
Problem 804. At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. Which of the vectors on the figure at right is the spider’s displacement vector at 12:30?

(a) The zero vector $\vec{0}$  
(b) $\vec{A}$  
(c) $\vec{B}$  
(d) $\vec{E}$  
(e) None of these

Problem 805. At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. Which of the vectors on the figure at right is the spider’s displacement vector at 12:45?

(a) $\vec{B}$  
(b) $\vec{F}$  
(c) $\vec{G}$  
(d) $\vec{H}$  
(e) None of these

Problem 806. At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. Which of the vectors on the figure at right is the spider’s displacement vector at 1:00?

(a) The zero vector $\vec{0}$  
(b) $\vec{A}$  
(c) $\vec{B}$  
(d) $\vec{E}$  
(e) None of these
**Problem 807.** At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. If the origin is at the center of the clock, which of the vectors on the figure at right is the spider’s position vector at 12:15?

(a) \( \vec{B} \)  
(b) \( \vec{C} \)  
(c) \( \vec{G} \)  
(d) \( \vec{H} \)  
(e) None of these

**Problem 808.** At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. If the origin is at the center of the clock, which of the vectors on the figure at right is the spider’s position vector at 12:30?

(a) The zero vector \( \vec{0} \)  
(b) \( \vec{A} \)  
(c) \( \vec{E} \)  
(d) \( \vec{F} \)  
(e) None of these

**Problem 809.** At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. If the origin is at the center of the clock, which of the vectors on the figure at right is the spider’s position vector at 12:45?

(a) \( \vec{B} \)  
(b) \( \vec{F} \)  
(c) \( \vec{G} \)  
(d) \( \vec{H} \)  
(e) None of these
Problem 810. At exactly 12:00, a spider climbs onto the tip of a clock’s minute hand, where it remains for the next hour. If the origin is at the center of the clock, which of the vectors on the figure at right is the spider’s position vector at 1:00?

(a) The zero vector \( \vec{0} \)  
(b) \( \vec{A} \)  
(c) \( \vec{B} \)  
(d) \( \vec{E} \)  
(e) None of these
6.4 Finding Components of Vectors

Problem 811. The vector $\vec{A}$ has magnitude 5.9 and direction $\theta = 0.34$ rad. Find the $x$- and $y$-components of $\vec{A}$.

Problem 812. The vector $\vec{A}$ has magnitude 83 and direction $\theta = 1.98$ rad. Find the $x$- and $y$-components of $\vec{A}$.

Problem 813. The vector $\vec{A}$ has magnitude 41 and direction $\theta = 3.51$ rad. Find the $x$- and $y$-components of $\vec{A}$.

Problem 814. The vector $\vec{A}$ has magnitude 7.7 and direction $\theta = 5.66$ rad. Find the $x$- and $y$-components of $\vec{A}$.

Problem 815. The vector $\vec{A}$ has magnitude 0.88 and direction $37^\circ$. Find the $x$- and $y$-components of $\vec{A}$.

Problem 816. The vector $\vec{A}$ has magnitude 1.28 and direction $98^\circ$. Find the $x$- and $y$-components of $\vec{A}$.

Problem 817. The vector $\vec{A}$ has magnitude 110 and direction $219^\circ$. Find the $x$- and $y$-components of $\vec{A}$.

Problem 818. The vector $\vec{A}$ has magnitude 37 and direction $304^\circ$. Find the $x$- and $y$-components of $\vec{A}$.

Problem 819. $\vec{A} = \langle 5.1, 8.8 \rangle$. Find the magnitude and direction of $\vec{A}$. Give the direction in radians.

Problem 820. $\vec{A} = \langle 16, -41 \rangle$. Find the magnitude and direction of $\vec{A}$. Give the direction in radians.

Problem 821. $\vec{A} = \langle -0.81, -1.13 \rangle$. Find the magnitude and direction of $\vec{A}$. Give the direction in degrees.

Problem 822. $\vec{A} = \langle -216, 82 \rangle$. Find the magnitude and direction of $\vec{A}$. Give the direction in degrees.

Problem 823. $\vec{A} = 4.8i + 7.0j$. Find the magnitude and direction of $\vec{A}$. Give the direction in radians.

Problem 824. $\vec{A} = -120i - 220j$. Find the magnitude and direction of $\vec{A}$. Give the direction in radians.

Problem 825. $\vec{A} = -29i + 12j$. Find the magnitude and direction of $\vec{A}$. Give the direction in degrees.

Problem 826. $\vec{A} = 0.16i - 0.23j$. Find the magnitude and direction of $\vec{A}$. Give the direction in degrees.
6.5 Algebraic Vector Addition and Subtraction

Problem 827.  \( \vec{A} = 3\hat{i} - 2\hat{j} \) and \( \vec{B} = 4\hat{i} + 5\hat{j} \). Find \( \vec{A} + \vec{B} \).

Problem 828.  \( \vec{A} = -\hat{i} + 7\hat{j} \) and \( \vec{B} = 2\hat{i} + 3\hat{j} \). Find \( \vec{A} - \vec{B} \).

Problem 829.  \( \vec{A} = \langle 8, 3 \rangle \) and \( \vec{B} = \langle 1, 2 \rangle \). Find \( \vec{A} + \vec{B} \).

Problem 830.  \( \vec{A} = \langle 2, 6 \rangle \) and \( \vec{B} = \langle -4, 5 \rangle \). Find \( \vec{A} - \vec{B} \).

Problem 831.  \( \vec{A} = \langle 4, -1 \rangle \) and \( \vec{B} = \langle -5, -3 \rangle \). Find \( \vec{A} + \vec{B} \).

(a) \( \langle -9, -4 \rangle \)  (b) \( \langle 1, 2 \rangle \)
(c) \( \langle 3, -8 \rangle \)  (d) \( \langle -1, -4 \rangle \)
(e) None of these

Problem 832.  \( \vec{A} = \langle -2, 5 \rangle \) and \( \vec{B} = \langle 3, -3 \rangle \). Find \( \vec{A} + \vec{B} \).

(a) \( \langle 5, 8 \rangle \)  (b) \( \langle 1, 2 \rangle \)
(c) \( \langle -5, 8 \rangle \)  (d) \( \langle -6, -15 \rangle \)
(e) None of these

Problem 833.  \( \vec{A} = \langle -2, 5 \rangle \) and \( \vec{B} = \langle 7, -3 \rangle \). Find \( \vec{A} - \vec{B} \).

(a) \( \langle 5, 2 \rangle \)  (b) \( \langle -9, 2 \rangle \)
(c) \( \langle -9, 8 \rangle \)  (d) \( \langle 5, -2 \rangle \)
(e) None of these

Problem 834.  \( \vec{A} \) has magnitude 7.2 and direction 1.1 rad.  \( \vec{B} \) has magnitude 5.5 and direction 2.9 rad. Find \( \vec{A} + \vec{B} \). Give your answer in component form: \( \langle C_x, C_y \rangle \).

Problem 835.  \( \vec{A} \) has magnitude 0.83 and direction 22°.  \( \vec{B} \) has magnitude 1.42 and direction 157°. Find \( \vec{A} - \vec{B} \). Give your answer in terms of unit vectors: \( C_x\hat{i} + C_y\hat{j} \).

Problem 836.  \( \vec{A} \) has magnitude 115 and direction 3.8 rad.  \( \vec{B} \) has magnitude 303 and direction 5.1 rad. Find the magnitude and direction of \( \vec{A} + \vec{B} \).

Problem 837.  \( \vec{A} \) has magnitude 57 and direction 43°.  \( \vec{B} \) has magnitude 22 and direction 117°. Find the magnitude and direction of \( \vec{A} - \vec{B} \).

Problem 838.  If \( \vec{A} = \langle 3.3, 5.7 \rangle \), what is the magnitude of \( \vec{A} \)? Round your answer to the nearest 0.1.

(a) 5.3  (b) 5.9
(c) 6.6  (d) 7.2
(e) None of these

Problem 839.  If \( \vec{A} = \langle 3, 4 \rangle \), what is the magnitude of \( \vec{A} \)?

(a) 5  (b) 6
(c) 25  (d) 7
(e) None of these
Problem 840. If $\vec{A} = 3\hat{i} - 4\hat{j}$, what is the magnitude of $\vec{A}$?

(a) -1 
(b) 2 
(c) 4 
(d) 5 
(e) None of these
6.6 Concept questions: vectors

The first three problems deal with trigonometric functions.

**Problem 841.** \( \cos(5 \text{ m/s}) = ? \)

**Problem 842.** Consider the expression: \( \sin(\omega^* t^*) = ? \), where \( t^* \) has units of time. What must the units of \( \omega^* \) be in order for the argument of the sine function to make sense?

**Problem 843.** Why is an angle a dimensionless number when measured in radians?

**Problem 844.** True or false: If we add the scalars 1 m + 1 m, we get 2 m.

**Problem 845.** True or false: If we add two vectors, each with length 1 m, we get a vector with length 2 m.

**Problem 846.** Can you find a vector \( \vec{v} = \langle v_1, v_2 \rangle \) such that \( \vec{v} \neq \vec{0} \), but \( \|\vec{v}\| = 0 \)?

**Problem 847.** Is it possible for the magnitude of a vector to be smaller than any of its components?

**Problem 848.** True or false: Let \( \vec{v} = \langle a, b \rangle \). If \( \|\vec{v}\| > 0 \), then \( a > 0 \) and \( b > 0 \).

**Problem 849.** If \( \vec{A} = A_x \hat{i} + A_y \hat{j} \) and makes an angle \( \theta \) with the positive \( x \)-axis, express \( \sin \theta \) in terms of \( A_x \) and \( A_y \).

**Problem 850.** If \( \vec{A} = A_x \hat{i} + A_y \hat{j} \) and makes an angle \( \theta \) with the positive \( x \)-axis, express \( \cos \theta \) in terms of \( A_x \) and \( A_y \).

**Problem 851.** True or false: If \( \vec{A} = A_x \hat{i} + A_y \hat{j} \), then the equation for the magnitude satisfies \( A = A_x + A_y \)? Justify your answer.

**Problem 852.** If \( \vec{A} = A_x \hat{i} + A_y \hat{j} \) and makes an angle \( \theta \) with the positive \( x \)-axis, express \( \theta \) in terms of \( A_x \) and \( A_y \). Assume that \( A_x > 0 \) and \( A_y > 0 \).

**Problem 853.** True or false: If \( \vec{A} = A_x \hat{i} + A_y \hat{j} \), then the equation for the magnitude satisfies \( \|\vec{A}\| \geq |A_x| \)? Justify your answer.

**Problem 854.** True or false: Let \( \vec{r} = \langle x, y \rangle = x \hat{i} + y \hat{j} \). If \( \|\vec{r}\| > 0 \), then \( x > 0 \) and \( y > 0 \). Justify your answer.

**Problem 855.** Can you find a situation where the average velocity vector is equal to the zero vector, but the average speed is not zero? In symbols:

\[
\text{Can we have } \vec{v}_{\text{ave}} = \frac{\Delta \vec{r}}{\Delta t} = \vec{0} \text{ and speed } = \frac{\text{total distance travelled}}{\Delta t} \neq 0 ?
\]

Give an example, or explain why one doesn’t exist.
Problem 856. Consider the statements (i) and (ii). Are the statements true or false?
(i) If $k$ is a scalar, then $k + k = 2k$
(ii) Let $\vec{v} = \langle a, b \rangle$. If $\|v\| > 0$, then $a > 0$ and $b > 0$.

Choose the correct answer from below.
(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 857. Consider the statements (i) and (ii). Are the statements true or false?
(i) If $\vec{v}$ is a vector, then $\|2\vec{v}\| = 2\|\vec{v}\|$.
(ii) Let $\vec{v} = \langle a, b \rangle$. If $\|v\| = 0$, then $a = 0$ and $b = 0$.

Choose the correct answer from below.
(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 858. Consider the statements (i) and (ii). Are the statements true or false?
(i) If $\vec{v}$ is a vector, then $\|-\vec{v}\| = \|\vec{v}\|$.
(ii) Let $\vec{v} = \langle a, b \rangle$. Then $2\vec{v} = \langle 2a, 2b \rangle$.

Choose the correct answer from below.
(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 859. Consider the statements (i) and (ii). Are the statements true or false?
(i) If $\vec{v}$ is a vector, then $\|-\vec{v}\| = \|\vec{v}\|$.
(ii) Let $\vec{v} = \langle a, b \rangle$. Then $-2\vec{v} = -2a - 2b$.

Choose the correct answer from below.
(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 860. Consider the statements (i) and (ii). Are the statements true or false?
(i) If $\vec{u}$ and $\vec{v}$ are vectors, then $\|\vec{u} + \vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$.
(ii) Let $\vec{v} = \langle a, b \rangle$. Then $\|\vec{v}\| = \sqrt{a^2 + b^2}$.

Choose the correct answer from below.
(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false
Problem 861. Consider the statements (i) and (ii). Are the statements true or false?

(i) If $\vec{u}$ is a vector and $k$ is a scalar, then $\|k\vec{u}\| = k\|\vec{u}\|$.

(ii) Let $\vec{v} = \langle a, b \rangle$. Then $\|\vec{v}\| = \sqrt{a + b}$.

Choose the correct answer from below.

(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 862. Consider the statements (i) and (ii). Are the statements true or false?

(i) If $\vec{u}$ is a vector, $k$ is a scalar, and $k > 0$, then $\|k\vec{u}\| = k\|\vec{u}\|$.

(ii) Let $\vec{v}_1 = \langle a_1, b_1 \rangle$ and $\vec{v}_2 = \langle a_2, b_2 \rangle$. Then $v_1 + v_2 = \langle a_1 + a_2, b_1 + b_2 \rangle$.

Choose the correct answer from below.

(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 863. Consider the statements (i) and (ii). Are the statements true or false?

(i) If $\vec{u}$ is a vector, then $\| - 2\vec{u}\| = 2\|\vec{u}\|$.

(ii) Let $\vec{v}_1 = \langle a_1, b_1 \rangle$ and $\vec{v}_2 = \langle a_2, b_2 \rangle$. Then $v_1 + v_2 = \langle a_1 + b_1, a_2 + b_2 \rangle$.

Choose the correct answer from below.

(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 864. Consider the statements (i) and (ii). Are the statements true or false?

(i) If $\vec{u}$ is a vector, then $\vec{u} + (-\vec{u}) = \vec{0}$.

(ii) Let $\vec{v}_1 = \langle a_1, b_1 \rangle$ and $\vec{v}_2 = \langle a_2, b_2 \rangle$. Then $v_1 - v_2 = \langle a_1 - b_1, a_2 - b_2 \rangle$.

Choose the correct answer from below.

(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false

Problem 865. Consider the statements (i) and (ii). Are the statements true or false?

(i) If $\vec{u}$ is a vector, then $\vec{u} - 2\vec{u} = \vec{u}$.

(ii) Let $\vec{v}_1 = \langle a_1, b_1 \rangle$ and $\vec{v}_2 = \langle a_2, b_2 \rangle$. Then $v_1 - v_2 = \langle a_1 - a_2, b_1 - b_2 \rangle$.

Choose the correct answer from below.

(a) (i) is true; (ii) is false
(b) (i) is false; (ii) is true
(c) Both statements are true
(d) Both statements are false
6.7 Applications of Vectors

6.7.1 Breaking vectors into components

Problem 866. An airplane is flying at a speed of $V$ in a direction $22^\circ$ north of east. Which expression gives the northward component of the plane’s velocity?

(a) $V \cos 22^\circ$  
(b) $V/ \cos 22^\circ$
(c) $V \sin 22^\circ$  
(d) $V/ \sin 22^\circ$
(e) None of these

Problem 867. An airplane is flying at a speed of $V$ in a direction $22^\circ$ north of east. Which expression gives the eastward component of the plane’s velocity?

(a) $V \cos 22^\circ$  
(b) $V/ \cos 22^\circ$
(c) $V \sin 22^\circ$  
(d) $V/ \sin 22^\circ$
(e) None of these

Problem 868. A car is driving at a speed of $V$ up a hill that makes an angle of $22^\circ$ with the horizontal. What is the vertical component of the car’s velocity?

(a) $V \cos 22^\circ$  
(b) $V/ \cos 22^\circ$
(c) $V \sin 22^\circ$  
(d) $V/ \sin 22^\circ$
(e) None of these

Problem 869. An airplane is flying at a speed of 190 mph in a direction $24^\circ$ north of east. What is the northward component of the plane’s velocity? Round your answer to the nearest mph.

(a) 77 mph  
(b) 101 mph
(c) 161 mph  
(d) 175 mph
(e) None of these

Problem 870. An airplane is flying at a speed of 310 mph in a direction $77^\circ$ north of east. What is the eastward component of the plane’s velocity? Round your answer to the nearest mph.

(a) 70 mph  
(b) 155 mph
(c) 180 mph  
(d) 294 mph
(e) None of these

Problem 871. An airplane is flying at 214 m/s in a direction $37^\circ$ north of east. What is the eastward component of its velocity? Round your answer to the nearest m/s.

(a) 129 m/s  
(b) 134 m/s
(c) 161 m/s  
(d) 171 m/s
(e) None of these

Problem 872. An airplane is flying at 220 mph in a direction $30^\circ$ north of east. What is the northward component of its velocity?
Problem 873. An airplane is flying at 220 mph in a direction 30° east of north. What is the northward component of its velocity? Round your answer to the nearest mph.

Problem 874. A hill has a slope of 0.30 radians above the horizontal. If you are driving up that hill at 22 km/hr, how fast are you gaining elevation?

Problem 875. A car is driving at 35 mph up a hill with a slope of 8° above the horizontal. What is the horizontal component of the car’s velocity? Round your answer to the nearest 0.1 mph.

(a) 4.9 mph  
(c) 7.5 mph  
(e) None of these

Problem 876. A gun is fired at an elevation of 39° above the horizontal. The shell emerges from the muzzle at 320 m/s. What is the vertical component of the shell’s velocity? Round your answer to the nearest m/s.

(a) 85 m/s  
(c) 249 m/s  
(e) None of these

(b) 201 m/s  
(d) 308 m/s

Problem 877. (Lab Problem) A spring cannon is fired at an elevation of 34° above the horizontal. The ball emerges from the cannon at 14.4 m/s. What is the horizontal component of the ball’s velocity? Round your answer to the nearest 0.1 m/s.

(a) 8.1 m/s  
(c) 11.9 m/s  
(e) None of these

(b) 9.7 m/s  
(d) 12.2 m/s

Problem 878. You are on an airplane flying upward at an angle of $\theta$ above the horizontal, at 110 m/s. You have a cold, and if you gain altitude faster than 25 m/s, your eardrums will explode. What is the maximum safe value for $\theta$? Round your answer to the nearest degree.

(a) 12°  
(c) 14°  
(e) None of these

(b) 13°  
(d) 16°

Problem 879. (Lab Problem) A spring cannon is fired at an elevation of 34° above the horizontal. The ball emerges from the cannon at 14.4 m/s. What is the horizontal component of the ball’s velocity? Round your answer to the nearest 0.1 m/s.

(a) 8.1 m/s  
(c) 11.9 m/s  
(e) None of these

(b) 9.7 m/s  
(d) 12.2 m/s
**Problem 880.** A bird is flying at 20 m/s in a direction 30° south of west. If we take the $x$-axis pointing east and the $y$-axis pointing north, then what is the $y$-component of its velocity $\vec{v} = (v_x, v_y)$? Round your answer to the nearest m/s.

(a) $10\sqrt{3}$ m/s  
(b) 10 m/s  
(c) $-10\sqrt{3}$ m/s  
(d) -10 m/s  
(e) None of these

**Problem 881.** Your Civil War cannon has a muzzle velocity of $v_0 = 1220$ ft/s. If it is fired at an elevation of 6° above the horizontal, what is the vertical component $v_{0y}$ of the velocity? Round your answer to the nearest ft/s.

**Problem 882.** An airplane is flying at 330 km/hr, in a direction 0.19 rad above the horizontal. At what speed is the airplane gaining altitude?

**Problem 883.** A train is moving at 68 mph in a direction 22° north of west. What is the train’s speed westward?

**Problem 884.** (Position) Pirates have buried their treasure on campus, and you have acquired a copy of their map. (Like everyone else, pirates need to be careful about that “Reply All” button.) It tells you to start at the main entrance to the physics building and take 120 steps in a direction 0.38 rad north of east; from there, you should go another 190 steps in a direction 0.33 rad south of east. How far from the entrance is the treasure, and in what direction?

**Problem 885.** (Position) You have discovered the Lost Belgian Mine, which runs directly northward into a mountainside. From the opening, you go 47 m in a direction 0.28 rad below the horizontal. From there, you go another 66 m in a direction 0.19 rad below the horizontal. At this point, what is your horizontal distance from the opening, and how far are you above or below it?

**Problem 886.** A goose is directly east of an airplane. The airplane is flying at a speed of 160 mph, in a direction 15° north of east. The goose is flying at a speed of 45 mph, in a direction $\theta$ north of west. What must $\theta$ be in order for the goose to collide with the airplane? Give your answer in degrees.

**Problem 887.** You are tracking a submarine by determining its position when it sends radio messages. When you first detect the sub, it is 57 km west and 24 km north of you. When it sends its next message 37 min later, it is 51 km west and 13 km north of you. What are the magnitude and direction of the sub’s average velocity during that time? (Give the magnitude in km/hr, and the direction in radians, relative to east or west: e.g. $\theta$ rad north of east, with $0 < \theta < \pi/2$.)

**Problem 888.** A cockroach has a tiny embedded radio transponder, allowing you to track its position and velocity. Just before your lab partner attempts to stomp on it, it is moving at 8 cm/s to the east and 5 cm/s to the south. 1.3 seconds later, it is scurrying away at 31 cm/s to the west and 19 cm/s to the south. What are the components of the roach’s average acceleration during that time?
Problem 889. A car is travelling eastward along a highway at 60 mph. It passes under a hawk that is flying northward at 42 mph. From the point of view of an observer in the car, what are the hawk’s speed and direction? Give the direction in degrees relative to north, e.g. $\theta^\circ$ east of north.

Problem 890. You can paddle a kayak in still water at 2.6 m/s. The Santa Cruz River is 55 m wide, and flows northward at 1.9 m/s. If you launch your kayak on the east bank, and point it directly westward, what will your speed and direction be relative to an observer standing on the bank? Give your direction in radians relative to west, e.g. $\theta$ rad south of west.

Problem 891. An airplane can fly at 410 km/hr in still air. The pilot wishes to fly it from Smithpur to Jonesgorod, 530 km to the north. The wind is blowing at 55 km/hr from west to east. In what direction should the pilot point the plane to fly directly to Jonesgorod? How long will the journey take? Give your direction in radians relative to north, e.g. $\theta$ rad east of north.

Problem 892. A river has a current with a speed of 5 ft/s. You can swim at 4 ft/s in still water. At what angle to the cross-river direction do you need to swim to reach a point directly across the river from you?

Problem 893. A train is going directly north at 70 mph. An action-movie star is atop the train, running southward at 12 mph. Relative to someone standing on the ground, how fast is the action-movie star moving, and in what direction?
### 6.7.2 Introduction to force vectors

**Working definition:** You can think of a *force* as a push or a pull. Since a push or pull has both magnitude and direction, it follows that a force is a vector.  \(^1\)

We’ll learn more about forces later on in the course when you learn about Newton’s Laws. In particular, Newton’s 2\(^{nd}\) Law \(F_{\text{net}} = ma\) requires computing the net force (sometimes called the resultant force) acting on an object of mass \(m\). Since a force is a vector, the net force is just the resultant vector. In more than one spatial dimension we find the resultant vector by breaking the individual force vectors acting on the object into component vectors and then finding the net force in each of the \(x\) and \(y\) directions.

The purposes of these exercises is to give you an early exposure to the concept of breaking forces into their respective components and summing them (keeping proper \(\pm\) sign conventions) to find the net force in the \(x\) and \(y\) directions, denoted \(F_{\text{net},x}\) and \(F_{\text{net},y}\).

**Problem 894.** A rope is used to pull a block up a ramp. The ramp makes an angle of 25\(^{\circ}\) to the horizontal; the rope makes an angle of 33\(^{\circ}\) with the ramp. The rope is pulled with a force \(\vec{F}\) whose magnitude is 317 N. What is the horizontal component of \(\vec{F}\)? Round your answer to the nearest newton.

- (a) 168 N
- (b) 185 N
- (c) 203 N
- (d) 224 N
- (e) None of these

**Problem 895.** A rope is used to pull a block up a ramp. The ramp makes an angle of 25\(^{\circ}\) to the horizontal; the rope makes an angle of 33\(^{\circ}\) with the ramp. The rope is pulled with a force \(\vec{F}\) whose magnitude is 317 N. What is the vertical component of \(\vec{F}\)? Round your answer to the nearest newton.

- (a) 218 N
- (b) 242 N
- (c) 269 N
- (d) 296 N
- (e) None of these

\(^1\)Actually, this is not obvious and must be proven mathematically.
Problem 896. A rope is used to pull a block up a ramp. The ramp makes an angle of 22° to the horizontal; the rope makes an angle of 31° with the ramp. The rope is pulled with a force \( \vec{F} \) whose magnitude is 428 N. What is the horizontal component of \( \vec{F} \)? Round your answer to the nearest newton.
(a) 209 N  (b) 232 N  
(c) 258 N  (d) 283 N 
(e) None of these

Problem 897. Two forces \( \vec{F}_1 \) and \( \vec{F}_2 \) are acting on a particle, as shown at right. \( \vec{F}_1 \) makes an angle of 35° with the x-axis and has magnitude 87 N. \( \vec{F}_2 \) makes an angle of 18° with the y-axis and has magnitude 181 N. Use the component method to find the net force in the x-direction. Round your answer to the nearest newton. (The picture is not drawn to scale.)
(a) 12 N  (b) 13 N  
(c) 15 N  (d) 16 N 
(e) None of these

Problem 898. Two forces \( \vec{F}_1 \) and \( \vec{F}_2 \) are acting on a particle, as shown at right. \( \vec{F}_1 \) makes an angle of 35° with the x-axis and has magnitude 87 N. \( \vec{F}_2 \) makes an angle of 18° with the y-axis and has magnitude 181 N. Use the component method to find the net force in the y-direction. Round your answer to the nearest newton. (The picture is not drawn to scale.)
(a) 200 N  (b) 222 N  
(c) 244 N  (d) 269 N 
(e) None of these

Problem 899. Two forces \( \vec{F}_1 \) and \( \vec{F}_2 \) are acting on a particle, as shown at right. \( \vec{F}_1 \) makes an angle of 35° with the x-axis and has magnitude 87 N. \( \vec{F}_2 \) makes an angle of 18° with the y-axis and has magnitude 181 N. Find the magnitude of the net force on the particle. Round your answer to the nearest newton. (The picture is not drawn to scale.)
(a) 200 N  (b) 223 N  
(c) 245 N  (d) 269 N 
(e) None of these
Problem 900. Two forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a particle, as shown at right. $\vec{F}_1$ makes an angle of $35^\circ$ with the $x$-axis and has magnitude 87 N. $\vec{F}_2$ makes an angle of $18^\circ$ with the $y$-axis and has magnitude 181 N. If $\vec{F}_{\text{net}}$ is the net force on the particle, find the angle that $\vec{F}_{\text{net}}$ makes with the $x$-axis. Round your answer to the nearest degree. (The picture is not drawn to scale.)

(a) $83^\circ$  
(b) $86^\circ$  
(c) $89^\circ$  
(d) $92^\circ$  
(e) None of these

Problem 901. You are pulling a crate of physics books down the hallway, using a rope that makes an angle of $34^\circ$ to the horizontal. If you are pulling on the rope with a force of 440 N, what is the horizontal component of the force? Round your answer to the nearest newton.

(a) 373 N  
(b) 233 N  
(c) 365 N  
(d) 246 N  
(e) None of these

Problem 902. You are pulling a crate of physics books down the hallway, using a rope that makes an angle of $26^\circ$ to the horizontal. If you are pulling on the rope with a force of 290 N, what is the horizontal component of the force? Round your answer to the nearest newton.

(a) 127 N  
(b) 188 N  
(c) 221 N  
(d) 261 N  
(e) None of these

Problem 903. Your car is stuck in a muddy road, and you and a friend are trying to pull it out with ropes. You are pulling with a force of 820 N in a direction $0.45$ rad east of north; your friend is pulling with a force of 1040 N in a direction $0.32$ rad west of north. What is the northward force on the car?

Problem 904. A boat is held in the middle of a river by two ropes, one going to either bank. Each rope makes an angle of $79^\circ$ to the upstream direction of the river. The current exerts a downstream force of 550 lbs on the boat. How much force does each rope have to exert to hold the boat in place?

Problem 905. You are riding your bicycle straight north. The wind is exerting a force of 48 lbs on you, in a direction $39^\circ$ east of south. What is the southward component of the wind’s force?

Problem 906. A dietitian is pulling a crate of romaine lettuce, using a rope that makes an angle of $0.41$ rad to the horizontal. To move the crate requires a horizontal force of 240 N. What is the force with which the dietitian needs to pull on the rope?
Part III

Kinematics
7 One-dimensional linear kinematics

7.1 Concept questions: kinematics

Problem 907. Is it possible for an object to have nonzero velocity and zero speed? Give an example, or explain why it’s not possible.

Problem 908. Is it possible for an object to have nonzero speed and zero velocity? Give an example, or explain why it’s not possible.

Problem 909. Is it possible for an object to have zero average velocity and nonzero average speed? Give an example, or explain why it’s not possible.

Problem 910. Is it possible for an object to have zero average speed and nonzero average velocity? Give an example, or explain why it’s not possible.

Problem 911. You are given a ticket for driving at 37 mph in a school zone with a speed limit of 15 mph. At your hearing, you inform the judge that since you were going in reverse, your speed was $-37$ mph; and since $-37 < 15$, you were going below the speed limit and should not be penalized. Unfortunately for you, the policeman has taken physics. What does he tell the judge?

Problem 912. You drive in a straight line from your home to your physics classroom. Your driveway is 30 m long; you drive down it at 5 m/s. You then drive 300 m down Smith Street at 15 m/s. After that, you drive 4.8 km down Jones Road to your classroom at 30 m/s. Once you get there, you realize that it’s Saturday, and that you don’t have to come to class. You turn around and return home along the same route: at 40 m/s along Jones Road; at 20 m/s along Smith Street; and at 10 m/s up your driveway, back to where you started. What is your average velocity for the round trip?

Problem 913. Assume that the direction in which a car faces is positive. Give an example of a situation in which the car has:

(a) positive velocity and positive acceleration
(b) positive velocity and negative acceleration
(c) negative velocity and positive acceleration
(d) negative velocity and negative acceleration
(e) positive velocity and zero acceleration
(f) zero velocity and negative acceleration

Problem 914. You and your lab partner are designing a fountain for the front of the physics building. You have a pressurized tank with a nozzle that squirts water out at 5.2 m/s. You claim that if that nozzle is pointed upward, it will produce a beautiful spray of water. Your lab partner disagrees: he says that since the acceleration due to gravity is 9.8, and 5.2 < 9.8, the water will barely dribble out of the nozzle and run back down the sides of the pipe. Is he right? Why or why not?
Problem 915. A car is being driven erratically along a straight stretch of highway. The graph below shows its position $x$ as a function of time $t$. For each interval between consecutive points (A-B, B-C, etc.), describe whether the velocity of the car is positive, negative, or zero; and whether the acceleration is positive, negative, or zero.

![Graph showing position $x$ as a function of time $t$.](image)

7.2 Qualitative kinematics: descriptions

Problem 916. A car is facing in the positive direction. It is moving forward at 65 mph; the driver has just seen a deer on the road and applied the brakes. Which of the following describes the car’s situation?

(a) positive velocity; positive acceleration
(b) positive velocity; zero acceleration
(c) positive velocity; negative acceleration
(d) negative velocity; positive acceleration
(e) none of these

Problem 917. A car is facing in the positive direction. It is rolling backward down a hill, and its speed is increasing. Which of the following describes the car’s situation?

(a) positive velocity; positive acceleration
(b) positive velocity; negative acceleration
(c) negative velocity; positive acceleration
(d) negative velocity; negative acceleration
(e) none of these
Problem 918. A car is facing in the positive direction. It is rolling backward down a hill; the driver has just applied the brakes to slow it down. Which of the following describes the car’s situation?

(a) positive velocity; zero acceleration  
(b) positive velocity; negative acceleration  
(c) negative velocity; positive acceleration  
(d) negative velocity; negative acceleration  
(e) none of these

Problem 919. A car is facing in the positive direction. It is moving forward, and the driver is speeding up to merge onto the freeway. Which of the following describes the car’s situation?

(a) positive velocity; positive acceleration  
(b) positive velocity; zero acceleration  
(c) zero velocity; positive acceleration  
(d) negative velocity; zero acceleration  
(e) none of these

Problem 920. A car is facing in the positive direction. The driver is going at a steady speed of 55 mph on a highway. Which of the following describes the car’s situation?

(a) positive velocity; zero acceleration  
(b) positive velocity; negative acceleration  
(c) positive velocity; positive acceleration  
(e) none of these

Problem 921. A car is facing in the positive direction. It was going up a hill when the engine failed; the car slowed down as it rolled up the hill, then started rolling backward. Which of the following describes the car’s situation at the point where it started rolling backward?

(a) positive velocity; positive acceleration  
(b) negative velocity; negative acceleration  
(c) zero velocity; positive acceleration  
(d) zero velocity; negative acceleration  
(e) none of these

Problem 922. In this problem, use upward as the positive direction. A rocket is flying straight upward; its speed increases as it rises. Which of the following describes the rocket’s situation?

(a) positive velocity; positive acceleration  
(b) positive velocity; negative acceleration  
(c) zero velocity; positive acceleration  
(d) zero velocity; negative acceleration  
(e) none of these
Problem 923. In this problem, use downward as the positive direction. A ball has been dropped from the top of a building; it has not yet reached the ground. Which of the following describes the ball’s situation?

(a) positive velocity; positive acceleration
(b) positive velocity; negative acceleration
(c) zero velocity; negative acceleration
(d) negative velocity; positive acceleration
(e) none of these

Problem 924. In this problem, use downward as the positive direction. A ball has been dropped from the top of a building, has hit the ground and bounced, and is now moving upward. Which of the following describes the ball’s situation?

(a) positive velocity; positive acceleration
(b) positive velocity; negative acceleration
(c) positive velocity; zero acceleration
(d) negative velocity; positive acceleration
(e) none of these

Problem 925. In this problem, use downward as the positive direction. An elevator has just left the 39th floor, heading downward and speeding up as it goes. Which of the following describes the elevator’s situation?

(a) positive velocity; positive acceleration
(b) positive velocity; negative acceleration
(c) negative velocity; negative acceleration
(d) negative velocity; positive acceleration
(e) none of these

Problem 926. In this problem, use downward as the positive direction. An elevator is going down from the 39th floor to the 3rd. It is now at the level of the fourth floor, and is slowing down as it approaches the third. Which of the following describes the elevator’s situation?

(a) positive velocity; positive acceleration
(b) positive velocity; negative acceleration
(c) negative velocity; negative acceleration
(d) negative velocity; positive acceleration
(e) none of these

Problem 927. In this problem, use downward as the positive direction. An elevator is on its way from the 89th floor to the 4th. It has reached its maximum downward speed, and has not yet started slowing down. Which of the following describes the elevator’s situation?

(a) positive velocity; negative acceleration
(b) positive velocity; zero acceleration
(c) negative velocity; negative acceleration
(d) negative velocity; positive acceleration
(e) none of these
Problem 928. In this problem, use downward as the positive direction. An elevator has just left the 3rd floor, heading upward and speeding up as it goes. Which of the following describes the elevator’s situation?

(a) positive velocity; positive acceleration  
(b) positive velocity; negative acceleration  
(c) negative velocity; negative acceleration  
(d) negative velocity; zero acceleration  
(e) none of these

Problem 929. If the upward direction is positive, which situation involves positive velocity and positive acceleration?

(a) An elevator moving upward at a constant speed  
(b) An elevator speeding up as it moves downward  
(c) An elevator speeding up as it moves upward  
(d) An elevator slowing down as it moves downward  
(e) none of these

Problem 930. If the upward direction is positive, which situation involves negative velocity and negative acceleration?

(a) An elevator moving upward at a constant speed  
(b) An elevator speeding up as it moves downward  
(c) An elevator speeding up as it moves upward  
(d) An elevator slowing down as it moves downward  
(e) none of these

Problem 931. If the upward direction is positive, which situation involves positive velocity and zero acceleration?

(a) An elevator moving upward at a constant speed  
(b) An elevator speeding up as it moves downward  
(c) An elevator speeding up as it moves upward  
(d) An elevator slowing down as it moves downward  
(e) none of these

Problem 932. If the upward direction is positive, which situation involves negative velocity and positive acceleration?

(a) An elevator moving upward at a constant speed  
(b) An elevator speeding up as it moves downward  
(c) An elevator speeding up as it moves upward  
(d) An elevator slowing down as it moves downward  
(e) none of these

Problem 933. If north is the positive direction, which situation involves positive velocity and negative acceleration?

(a) A car speeding up as it moves southward  
(b) A car moving southward at a constant speed  
(c) A car slowing down as it moves northward  
(d) A car slowing down as it moves southward  
(e) none of these
**Problem 934.** If north is the positive direction, which situation involves negative velocity and negative acceleration?

(a) A car speeding up as it moves southward  
(b) A car moving southward at a constant speed  
(c) A car slowing down as it moves northward  
(d) A car slowing down as it moves southward  
(e) none of these

**Problem 935.** If north is the positive direction, which situation involves negative velocity and positive acceleration?

(a) A car speeding up as it moves southward  
(b) A car moving southward at a constant speed  
(c) A car slowing down as it moves northward  
(d) A car slowing down as it moves southward  
(e) none of these

**Problem 936.** If north is the positive direction, which situation involves negative velocity and zero acceleration?

(a) A car speeding up as it moves southward  
(b) A car moving southward at a constant speed  
(c) A car slowing down as it moves northward  
(d) A car slowing down as it moves southward  
(e) none of these

**Problem 937.** If the positive direction is downward, which situation involves negative velocity and negative acceleration?

(a) A rocket slowing down as it moves upward  
(b) A rocket moving downward at a constant speed  
(c) A rocket moving upward at a constant speed  
(d) A rocket speeding up as it moves upward  
(e) none of these

**Problem 938.** If the positive direction is downward, which situation involves positive velocity and zero acceleration?

(a) A rocket slowing down as it moves upward  
(b) A rocket moving downward at a constant speed  
(c) A rocket moving upward at a constant speed  
(d) A rocket speeding up as it moves upward  
(e) none of these

**Problem 939.** If the positive direction is downward, which situation involves negative velocity and zero acceleration?

(a) A rocket slowing down as it moves upward  
(b) A rocket moving downward at a constant speed  
(c) A rocket moving upward at a constant speed  
(d) A rocket speeding up as it moves upward  
(e) none of these
Problem 940. If the positive direction is downward, which situation involves negative velocity and positive acceleration?

(a) A rocket slowing down as it moves upward
(b) A rocket moving downward at a constant speed
(c) A rocket moving upward at a constant speed
(d) A rocket speeding up as it moves upward
(e) none of these
7.3 Qualitative kinematics: from graph

Figure 1: A car is being driven erratically along a straight stretch of highway. The graph shows its position $x$ as a function of the time $t$.

Problem 941. For the interval between points A and B in figure 1, the car’s velocity is:
(a) positive   (b) negative   (c) zero

Problem 942. For the interval between points B and C in figure 1, the car’s velocity is:
(a) positive   (b) negative   (c) zero

Problem 943. For the interval between points C and D in figure 1, the car’s velocity is:
(a) positive   (b) negative   (c) zero

Problem 944. For the interval between points D and E in figure 1, the car’s velocity is:
(a) positive   (b) negative   (c) zero

Problem 945. For the interval between points E and F in figure 1, the car’s velocity is:
(a) positive   (b) negative   (c) zero

Problem 946. For the interval between points F and G in figure 1, the car’s velocity is:
(a) positive   (b) negative   (c) zero

Problem 947. For the interval between points G and H in figure 1, the car’s velocity is:
(a) positive   (b) negative   (c) zero
**Problem 948.** For the interval between points A and B in figure 1, the car’s acceleration is:

(a) positive  (b) negative  (c) zero

**Problem 949.** For the interval between points B and C in figure 1, the car’s acceleration is:

(a) positive  (b) negative  (c) zero

**Problem 950.** For the interval between points C and D in figure 1, the car’s acceleration is:

(a) positive  (b) negative  (c) zero

**Problem 951.** For the interval between points D and E in figure 1, the car’s acceleration is:

(a) positive  (b) negative  (c) zero

**Problem 952.** For the interval between points E and F in figure 1, the car’s acceleration is:

(a) positive  (b) negative  (c) zero

**Problem 953.** For the interval between points F and G in figure 1, the car’s acceleration is:

(a) positive  (b) negative  (c) zero

**Problem 954.** For the interval between points G and H in figure 1, the car’s acceleration is:

(a) positive  (b) negative  (c) zero
Figure 2: A car is being driven erratically along a straight stretch of highway. The graph shows its position $x$ as a function of the time $t$.

**Problem 955.** For the interval between points A and B in figure 2, the car’s velocity is:
(a) positive  (b) negative  (c) zero

**Problem 956.** For the interval between points B and C in figure 2, the car’s velocity is:
(a) positive  (b) negative  (c) zero

**Problem 957.** For the interval between points C and D in figure 2, the car’s velocity is:
(a) positive  (b) negative  (c) zero

**Problem 958.** For the interval between points D and E in figure 2, the car’s velocity is:
(a) positive  (b) negative  (c) zero

**Problem 959.** For the interval between points E and F in figure 2, the car’s velocity is:
(a) positive  (b) negative  (c) zero

**Problem 960.** For the interval between points F and G in figure 2, the car’s velocity is:
(a) positive  (b) negative  (c) zero

**Problem 961.** For the interval between points G and H in figure 2, the car’s velocity is:
(a) positive  (b) negative  (c) zero

**Problem 962.** For the interval between points A and B in figure 2, the car’s acceleration is:
(a) positive  (b) negative  (c) zero
Problem 963. For the interval between points B and C in figure 2, the car’s acceleration is:
(a) positive  (b) negative  (c) zero

Problem 964. For the interval between points C and D in figure 2, the car’s acceleration is:
(a) positive  (b) negative  (c) zero

Problem 965. For the interval between points D and E in figure 2, the car’s acceleration is:
(a) positive  (b) negative  (c) zero

Problem 966. For the interval between points E and F in figure 2, the car’s acceleration is:
(a) positive  (b) negative  (c) zero

Problem 967. For the interval between points F and G in figure 2, the car’s acceleration is:
(a) positive  (b) negative  (c) zero

Problem 968. For the interval between points G and H in figure 2, the car’s acceleration is:
(a) positive  (b) negative  (c) zero
7.4 Introducing the fundamental one-dimensional kinematic equations

Assumptions: Unless instructed otherwise, assume that air resistance does not occur; and that projectiles (cannonballs, golf balls, arrows, etc.) are launched from ground level.

All of the following problems in this problem set can be solved using one of the following three fundamental equations:

In practice there are three different sets of subscripts used on physical variables:

1. (subscripts: \( i \) for initial values, \( f \) for final values) Example: initial position \( x_i \); final position \( x_f \)

2. (subscripts: zero (pronounced \textit{nought}) for initial values, no subscript for final values) Example: initial position \( x_0 \); final position \( x \)

3. (subscripts: 1 for initial values, 2 for final values) Example: initial position \( x_1 \); final position \( x_2 \)

The general fundamental kinematic equations in the first type of subscript notation are:

**Fundamental Equation 1:**
\[
v_f = v_i + a(t_f - t_i)
\]

**Fundamental Equation 2:**
\[
v_f^2 = v_i^2 + 2a(x_f - x_i)
\]

**Fundamental Equation 3:**
\[
x_f = x_i + v_i(t_f - t_i) + \frac{1}{2}a(t_f - t_i)^2
\]

where \( a \) is acceleration, \( v \) is velocity, \( x \) is position, \( t \) is time, and the subscript \( i \) and \( f \) denotes initial and final values, respectively. Typically, we take the initial time \( t_i = 0 \) since, in practice, when you time an experiment you always start with the stopwatch set to zero.

**Warning:** These equations are only valid if the acceleration is constant. Assume acceleration is constant unless stated otherwise.

The general fundamental kinematic equations in the second type of subscript notation are:

**Fundamental formulas for motion in 1-D**

**Equation 1:**
\[
v = v_0 + at \quad \text{(use when: position not in problem statement)}
\]

**Equation 2:**
\[
v^2 = v_0^2 + 2a(x - x_0) \quad \text{(use when: time not in problem statement)}
\]

**Equation 3:**
\[
x = x_0 + v_0t + \frac{1}{2}at^2 \quad \text{(use when: want position as a fcn of time)}
\]
The general fundamental kinematic equations in the third type of subscript notation are:

**Fundamental formulas for motion in 1-D**

**Equation 1:** \( v_2 = v_1 + at_2 \)

**Equation 2:** \( v_2^2 = v_1^2 + 2a(x_2 - x_1) \)

**Equation 3:** \( x_2 = x_1 + v_1 t_2 + \frac{1}{2} at_2^2 \)

This final form has the potential of leading to much confusion. For this reason we will not use this form. But you should be aware that it can be found in textbooks everywhere.

### 7.4.1 Dimensional consistence of the fundamental equations

**Problem 969.** Verify that the 1\(^{st}\) fundamental equation is dimensionally consistent.

**Problem 970.** Verify that the 2\(^{nd}\) fundamental equation is dimensionally consistent.

**Problem 971.** Verify that the 3\(^{rd}\) fundamental equation is dimensionally consistent.

### 7.5 Quantitative kinematics: horizontal

**Problem 972.** Beginning at rest, a car accelerates down a straight road at 1.03 m/s\(^2\) for 6.7 s. What is the car’s final speed? Round your answer to the nearest 0.1 m/s.

(a) 6.9 m/s  
(b) 7.6 m/s  
(c) 8.4 m/s  
(d) 9.2 m/s  
(e) None of these

**Problem 973.** Beginning at rest, a car accelerates down a straight road at 2.18 m/s\(^2\) for 14.8 s. What is the car’s final speed? Round your answer to the nearest 0.1 m/s.

(a) 6.8 m/s  
(b) 32.3 m/s  
(c) 35.2 m/s  
(d) 70.3 m/s  
(e) None of these

**Problem 974.** A boat can accelerate at 2.9 m/s\(^2\). If it begins at rest, how long must it accelerate before it reaches a speed of 14.4 m/s? Round your answer to the nearest 0.1 s.

(a) 5.0 s  
(b) 5.5 s  
(c) 6.0 s  
(d) 6.6 s  
(e) None of these
Problem 975. A boat is initially going northward at 7.4 m/s. It accelerates at 0.36 m/s² northward for 22 s. At the end of this time, how far north has it travelled? Round your answer to the nearest 10 m.

(a) 200 m  (b) 220 m  (c) 250 m  (d) 270 m  (e) None of these

Problem 976. A car can accelerate at 1.31 m/s². If it begins at rest, how long must it accelerate before it reaches a speed of 20.4 m/s? Round your answer to the nearest 0.1 s.

(a) 15.6 s  (b) 17.1 s  (c) 18.8 s  (d) 20.7 s  (e) None of these

Problem 977. Beginning at rest, a car accelerates forward for 5.2 s, reaching a speed of 15.0 m/s. What is the car’s acceleration? Round your answer to the nearest 0.1 m/s².

(a) 2.6 m/s²  (b) 2.9 m/s²  (c) 3.2 m/s²  (d) 3.5 m/s²  (e) None of these

Problem 978. A ship is initially moving northward at 12.1 m/s. It accelerates northward at 1.91 m/s² for 4.9 s. What is its final speed? Round your answer to the nearest 0.1 m/s.

(a) 15.7 m/s  (b) 21.5 m/s  (c) 45.9 m/s  (d) 56.6 m/s  (e) None of these

Problem 979. A car is initially moving northward at 5.9 m/s. It accelerates northward at 0.88 m/s² for 12.1 s. What is its final speed? Round your answer to the nearest 0.1 m/s.

(a) 13.4 m/s  (b) 14.9 m/s  (c) 16.5 m/s  (d) 18.2 m/s  (e) None of these

Problem 980. A ship is initially moving eastward at 4.2 m/s. It accelerates over the course of 27 s, at the end of which it is moving eastward at 10.3 m/s. What is the ship’s acceleration? Round your answer to the nearest 0.01 m/s².

(a) 0.20 m/s²  (b) 0.23 m/s²  (c) 0.25 m/s²  (d) 0.27 m/s²  (e) None of these
Problem 981. A train can accelerate at $1.04\,\text{m/s}^2$. It is initially travelling eastward at $10.0\,\text{m/s}$. How long must it accelerate before it reaches an eastward speed of $30.8\,\text{m/s}$? Round your answer to the nearest 0.1 s.

(a) 18.0 s  (b) 19.1 s  
(c) 20.0 s  (d) 24.2 s  
(e) None of these

Problem 982. A train can accelerate from a complete stop to a speed of $31\,\text{m/s}$ in 145 s. What is the train’s acceleration? Assume constant acceleration. Round your answer to two significant figures.

(a) $0.15\,\text{m/s}^2$  (b) $0.21\,\text{m/s}^2$  
(c) $0.48\,\text{m/s}^2$  (d) $0.68\,\text{m/s}^2$  
(e) None of these

Problem 983. A ship is initially moving northward at $16.7\,\text{m/s}$. It accelerates southward at $2.24\,\text{m/s}^2$ for 3.8 s. What is its final speed? Round your answer to the nearest 0.1 m/s.

(a) 8.2 m/s  (b) 21.7 m/s  
(c) 25.2 m/s  (d) 32.9 m/s  
(e) None of these

Problem 984. A car is initially moving northward at $9.5\,\text{m/s}$. It accelerates southward at $3.34\,\text{m/s}^2$ for 5.9 s. What is its final speed? Round your answer to the nearest 0.1 m/s.

(a) 4.3 m/s  (b) 7.6 m/s  
(c) 10.2 m/s  (d) 29.2 m/s  
(e) None of these

Problem 985. A train is initially rolling backward at $6.9\,\text{m/s}$. It accelerates forward at $0.58\,\text{m/s}^2$. How long must it accelerate before it is moving forward at $9.2\,\text{m/s}$? Round your answer to the nearest second.

(a) 22 s  (b) 25 s  
(c) 28 s  (d) 31 s  
(e) None of these

Problem 986. A ship is initially moving backward at $5.6\,\text{m/s}$. It accelerates over the course of 63 s, at the end of which it is moving forward at $11.0\,\text{m/s}$. What is the ship’s acceleration? Round your answer to the nearest 0.01 m/s$^2$.

(a) $0.09\,\text{m/s}^2$  (b) $0.11\,\text{m/s}^2$  
(c) $0.24\,\text{m/s}^2$  (d) $0.26\,\text{m/s}^2$  
(e) None of these
Problem 987. Beginning at rest, a car accelerates down a straight road at 3.3 m/s\(^2\) for 9.8 s. At the end of this time, how far is the car from its starting point? Round your answer to the nearest 10 m.

(a) 130 m  (b) 140 m  
(c) 160 m  (d) 170 m  
(e) None of these

Problem 988. Beginning at rest, a car accelerates down a straight road at 5.6 m/s\(^2\) for 2.9 s. At the end of this time, how far is the car from its starting point? Round your answer to the nearest meter.

(a) 24 m  (b) 26 m  
(c) 28 m  (d) 31 m  
(e) None of these

Problem 989. A boat is initially going northward at 15.3 m/s. It accelerates at 0.93 m/s\(^2\) northward for 8.3 s. At the end of this time, how far north has it travelled? Round your answer to the nearest 10 m.

(a) 150 m  (b) 160 m  
(c) 170 m  (d) 180 m  
(e) None of these

Problem 990. A boat is initially going northward at 8.8 m/s. It accelerates at 3.1 m/s\(^2\) northward for 9.9 s. At the end of this time, how far north has it travelled? Round your answer to the nearest 10 m.

(a) 240 m  (b) 260 m  
(c) 280 m  (d) 300 m  
(e) None of these

Problem 991. A train is initially rolling southward at 7.9 m/s. It accelerates northward at 0.41 m/s\(^2\) for 15 s. At the end of this time, where is it relative to its starting point? Round your answer to the nearest meter.

(a) 65 m south  (b) 72 m south  
(c) 65 m north  (d) 72 m north  
(e) None of these

Problem 992. A boat is initially drifting southward at 4.2 m/s. It accelerates northward at 0.84 m/s\(^2\) for 12 s. At the end of this time, where is it relative to its starting point? Round your answer to the nearest meter.

(a) 10 m south  (b) 12 m south  
(c) 10 m north  (d) 12 m north  
(e) None of these
Problem 993. A car is initially 30 m south of an intersection and moving northward at 6.0 m/s. It accelerates northward at 2.1 m/s$^2$ for 8.3 s. At the end of this time, where is it with respect to the intersection? Round your answer to the nearest meter.

(a) 92 m south  
(b) 104 m south  
(c) 92 m north  
(d) 104 m north  
(e) None of these

Problem 994. Beginning at rest, a ship accelerates at 0.45 m/s$^2$ until it has gone 150 m. At the end of this time, how fast is the ship moving? Round your answer to the nearest m/s.

(a) 9 m/s  
(b) 10 m/s  
(c) 12 m/s  
(d) 13 m/s  
(e) None of these

Problem 995. Beginning at rest, a train accelerates at 0.13 m/s$^2$ until it has gone 220 m. At the end of this time, how fast is the train moving? Round your answer to the nearest 0.1 m/s.

(a) 6.1 m/s  
(b) 6.8 m/s  
(c) 7.6 m/s  
(d) 8.3 m/s  
(e) None of these

Problem 996. Beginning at rest, a boat accelerates at 0.13 m/s$^2$ until it has reached a speed of 5.6 m/s. How far has the boat travelled in this time? Round your answer to the nearest meter.

(a) 88 m  
(b) 98 m  
(c) 109 m  
(d) 121 m  
(e) None of these

Problem 997. A train is moving at 31 m/s when the engineer is informed that there is a puppy on the track in front of him. If the train is capable of decelerating at 0.80 m/s$^2$, how far does it go before it comes to a stop? Round your answer to the nearest 10 m.

(a) 490 m  
(b) 540 m  
(c) 600 m  
(d) 660 m  
(e) None of these

Problem 998. A train can decelerate from an initial speed of 30 m/s to a complete stop in 120 s. What is the train’s acceleration? Assume constant acceleration and take the positive x-axis to be in the direction of motion. Round your answer to two significant figures.

(a) -0.15 m/s$^2$  
(b) 0.15 m/s$^2$  
(c) -0.25 m/s$^2$  
(d) 0.25 m/s$^2$  
(e) None of these
Problem 999. A car is initially moving at 18.9 m/s. It accelerates forward at 2.09 m/s\(^2\) until it has reached a speed of 29.3 m/s. How far has it gone in this time? Round your answer to the nearest meter.

(a) 108 m  (b) 120 m  
(c) 132 m  (d) 145 m  
(e) None of these

Problem 1000. During World War I, the Germans used the Paris Gun to shell that city from 75 miles away. The barrel of the gun was 28 m long, and the shells had a muzzle velocity of 1600 m/s. Assuming that the acceleration of the shell was constant over the whole length of the barrel, what was that acceleration? Round your answer to the nearest 1000 m/s\(^2\).

(a) 46,000 m/s\(^2\)  (b) 50,000 m/s\(^2\)  
(c) 55,000 m/s\(^2\)  (d) 61,000 m/s\(^2\)  
(e) None of these

Problem 1001. A ship has a maximum speed in still water of 13.0 m/s. To be operated legally on the Rhine River, it must be able to come to a complete stop in 350 m. What acceleration would the ship need to do this? Round your answer to the nearest 0.01 m/s\(^2\).

(a) 0.20 m/s\(^2\)  (b) 0.22 m/s\(^2\)  
(c) 0.24 m/s\(^2\)  (d) 0.27 m/s\(^2\)  
(e) None of these

Problem 1002. A radio-controlled airplane needs to reach a speed of 20 m/s to take off. Assuming a constant acceleration and a runway length of 20 m, what is the minimum acceleration that the plane needs if it starts from rest?

(a) 1 m/s\(^2\)  (b) 5 m/s\(^2\)  
(c) 10 m/s\(^2\)  (d) 20 m/s\(^2\)  
(e) None of these

Problem 1003. A certain bi-plane needs to reach a speed of 50 m/s to take off. Assuming the plane starts from rest with a constant acceleration of 2.5 m/s\(^2\), what is the minimum runway length the jet needs to takeoff?

(a) 250 m  (b) 500 m  
(c) 750 m  (d) 1000 m  
(e) None of these
Problem 1004. A young physics student buys a pre-made remote-controlled air plane. The instructions say that the plane needs a minimum speed of 10 m/s to takeoff. The instructions also give the thrust-to-weight ratio of the plane when it’s at full throttle. From this information he computes the maximum acceleration to be 3 m/s\(^2\). Allowing for friction between the wheels and the runway the student estimates that the maximum acceleration on the ground is 2.5 m/s\(^2\). Assuming the plane starts from rest and has a constant acceleration 2.5 m/s\(^2\) (he keeps the throttle wide open at takeoff), what is the minimum runway length the jet needs to takeoff?

(a) 10 m  
(b) 20 m  
(c) 50 m  
(d) 100 m  
(e) None of these

Problem 1005. A typical airline jet needs to reach a speed of 360 km/h to take off. Assuming a constant acceleration and a short runway length of 1.0 km, what is the minimum acceleration that the jet needs if it starts from rest? Write your answer in terms of \(g\), where \(g \approx 10 \text{ m/s}^2\).

Problem 1006. An overloaded transport jet needs to reach a speed of 500 km/h to take off. Assuming a constant acceleration and a runway length of 2.0 km, what is the minimum acceleration that the jet needs if it starts from rest? Write your answer in terms of \(g\), where \(1g = 9.8 \text{ m/s}^2\). Round your answer to the nearest 0.01\(g\).

Problem 1007. (This problem requires solving a system of equations) A policeman is lurking behind a billboard beside the highway when someone speeds by him at 38 m/s. The policeman immediately accelerates at a constant rate of 6.7 m/s\(^2\) until he has caught up with the speeder. How long does it take for the policeman to catch up? Round your answer to the nearest second.

(a) 11 s  
(b) 12 s  
(c) 13 s  
(d) 15 s  
(e) None of these

7.6 Quantitative kinematics: vertical

Problem 1008. An astronaut drops a watermelon from the top of a high cliff on the Moon, where the surface gravity is 1.63 m/s\(^2\). After 3.3 s, how fast is the watermelon moving? Round your answer to the nearest 0.1 m/s.

(a) 3.9 m/s  
(b) 4.4 m/s  
(c) 4.8 m/s  
(d) 5.4 m/s  
(e) None of these
Problem 1009. Using a stopwatch, you determine that a television dropped from a rooftop takes 2.7 s to reach the ground. How fast is it going when it hits the ground? Round your answer to the nearest m/s.

(a) 21 m/s  
(b) 24 m/s  
(c) 26 m/s  
(d) 29 m/s  
(e) None of these

Problem 1010. An astronaut hurls a watermelon straight downward from the top of a high cliff on the Moon, where the surface gravity is 1.63 m/s². The initial speed of the watermelon is 11.8 m/s downward. After 2.4 s, how fast is the watermelon moving? Round your answer to the nearest 0.1 m/s.

(a) 11.5 m/s  
(b) 12.7 m/s  
(c) 14.1 m/s  
(d) 15.7 m/s  
(e) None of these

Problem 1011. While walking in the desert, you discover an old mine. To find out how deep it is, you drop a rock down the shaft. Using the stopwatch on your cell phone, you time how long it takes before you hear the rock hit the bottom. If it takes 3.2 s for the rock to reach the bottom, how deep is the shaft? Round your answer to the nearest meter.

(a) 50 m  
(b) 55 m  
(c) 61 m  
(d) 67 m  
(e) None of these

Problem 1012. A banker drops a sack of pennies from the top of a tall building. After 1.5 s, how far has the sack of pennies fallen? Round your answer to the nearest 0.1 m.

(a) 9.9 m  
(b) 11.0 m  
(c) 12.1 m  
(d) 13.3 m  
(e) None of these

Problem 1013. An astronaut drops a watermelon from the top of a tall cliff on Mercury, where the surface gravity is 3.7 m/s². After 2.4 s, how far has the watermelon fallen? Round your answer to the nearest 0.1 m.

(a) 10.7 m  
(b) 11.7 m  
(c) 12.9 m  
(d) 14.2 m  
(e) None of these

Problem 1014. A dietitian drops a head of iceberg lettuce from a bridge. How long does it take for the lettuce to reach the river 31.2 m below? Round your answer to the nearest 0.1 s.

(a) 2.0 s  
(b) 2.3 s  
(c) 2.5 s  
(d) 2.8 s  
(e) None of these
Problem 1015. You are on the surface of Ganymede, where the acceleration due to gravity is 1.35 m/s². You throw an egg straight upward at 25 m/s. How long does it take for the egg to reach the ground? Round your answer to the nearest second.

(a) 30 s  (b) 33 s  
(c) 37 s  (d) 41 s  
(e) None of these

Problem 1016. An astronaut on Planet X drops a hammer from a cliff 6.8 m high. The hammer hits the ground after 1.3 s. What is the surface gravity of Planet X? Round your answer to the nearest 0.1 m/s².

(a) 8.0 m/s²  (b) 8.9 m/s²  
(c) 9.7 m/s²  (d) 10.7 m/s²  
(e) None of these

Problem 1017. A physics student angrily hurls his cell phone downward from a bridge at 19.9 m/s. After 2.2 s, how far below the bridge will the phone be? Round your answer to the nearest meter.

(a) 67 m  (b) 74 m  
(c) 82 m  (d) 90 m  
(e) None of these

Problem 1018. A banker drops a sack of pennies from the top of a building 18.8 m tall. How fast is the sack moving when it strikes the ground? Round your answer to the nearest 0.1 m/s.

(a) 15.5 m/s  (b) 17.3 m/s  
(c) 19.2 m/s  (d) 21.1 m/s  
(e) None of these

Problem 1019. If we ignore air drag, what is the speed of a hail stone that falls from rest at the top of a cumulus cloud that is 10,000 m high?

(a) 126 m/s  (b) 13.2 m/s  
(c) 67.6 m/s  (d) 443 m/s  
(e) None of these

Problem 1020. A ball is thrown straight upward from ground level, y = 0. At what speed must the ball be thrown if it is to reach a height of \( y_{\text{max}} = 10 \) meters above the ground? Round your answer to the nearest m/s.

Problem 1021. You are standing on a bridge over a canyon. To find out how deep the canyon is, you drop a rock off the bridge and time its fall. The rock hits the river at the canyon bottom after 7.2 s. How deep is the canyon? Round your answer to the nearest meter.

(a) 254 m  (b) 359 m  
(c) 508 m  (d) 718 m  
(e) None of these
7.6.1 The difference between average speed and average velocity in one-dimension

Problem 1022. (Speed vs. Velocity) An object is launched straight up into the air from the ground with an initial vertical velocity of 30 m/s. The object rises to a height of approximately 45 m above the ground in 3 seconds; it then falls back to the ground in 3 more seconds, impacting with a speed of 30 m/s. Approximately what is the average speed of the object during its time in the air?

(a) 0 m/s  (b) 5 m/s
(c) 15 m/s  (d) 30 m/s
(e) None of these

Problem 1023. (Speed vs. Velocity) An object is launched straight upward from ground level with an initial vertical velocity of about 30 m/s. In roughly 3 seconds, the object reaches a maximum height of approximately 45 m above the ground; it takes another 3 seconds to fall back down, striking the ground at 30 m/s. If $v_{ave}$ is the average velocity of the object during its 6-second flight, then the magnitude of the average velocity $v_{ave}$ is:

(a) 0 m/s  (b) 5 m/s
(c) 15 m/s  (d) 30 m/s
(e) None of these
7.7 Similarity Problems

The Idea: With all similarity problems there is always two similar problems (hence the name!) that are modeled by the same equation/equations. The two similar situations are sometimes called experiments, and the equation/equations that describes the situation (the model that governs the behavior of the particle) is/are referred to as the governing equation/equations. Some of the variables/parameters have the same value in both experiments. These parameters are known as common parameters. The variables/parameters that are situation dependent (i.e., different between the two experiments) are labelled by subscripts: a subscript 1 is given to the parameters from the first experiment and a subscript 2 is given to the parameters from the second experiment. The parameters that are in common to both experiments are the link between the two experiments. We typically don’t give these common parameters subscripts.

Problem-Solving Algorithm: Write down what your given, what you want, and determine the governing equation that describes both situations. Identify all of the variables/parameters that are in common to both situations and put them on the right-hand side (RHS) of the equation, and put all of the variables/parameters that are situation-dependent on the LHS of the equation. Then equate the two situations through the common RHS of the equation of the re-written governing equation. This is your common link between the two situations!

Problem 1024. (Similarity Problem) A train has a constant forward acceleration of \( a \). Starting at rest, it reaches a speed of \( v_1 \) at time \( t_1 \). At time \( t_2 = 2t_1 \), how fast is the train moving?

(a) \( \sqrt{2} v_1 \)  
(b) \( 2v_1 \)  
(c) \( 2\sqrt{2} v_1 \)  
(d) \( 4v_1 \)  
(e) None of these

Problem 1025. (Similarity Problem) A physics student drops a cell phone off a very high bridge. At time \( t_1 \) after being released, the phone has fallen a distance \( y_1 \). How far has the phone fallen at time \( t_2 = 3t_1 \)?

(a) \( \sqrt{3} y_1 \)  
(b) \( 3y_1 \)  
(c) \( 3\sqrt{3} y_1 \)  
(d) \( 9y_1 \)  
(e) None of these

Problem 1026. (Similarity Problem) An object dropped from a building with height \( y_1 \) takes time \( t_1 \) to reach the ground. How long would it take for an object dropped from a building with height \( y_2 = 2y_1 \)?

(a) \( \sqrt{2} t_1 \)  
(b) \( 2t_1 \)  
(c) \( 2\sqrt{2} t_1 \)  
(d) \( 4t_1 \)  
(e) None of these
Problem 1027. (Similarity Problem) An object dropped from a building with height \( y_1 \) strikes the ground with speed \( v_1 \). If an object is dropped from a building with height \( y_2 = 4y_1 \), how fast is it moving when it hits the ground?

(a) \( 2v_1 \)  
(b) \( 4v_1 \)  
(c) \( 8v_1 \)  
(d) \( 16v_1 \)  
(e) None of these

Problem 1028. (Similarity Problem) An object dropped from a certain height on Planet X takes time \( t_X \) to reach the ground. An object dropped from the same height on Planet Y takes time \( t_Y \) to fall. Which equation describes the relation between the surface gravities \( g_X \) and \( g_Y \) of the two planets?

(a) \( \frac{g_Y}{g_X} = \frac{t_Y}{t_X} \)  
(b) \( \frac{g_Y}{g_X} = \frac{t_Y^2}{t_X^2} \)  
(c) \( \frac{g_Y}{g_X} = \frac{t_X}{t_Y} \)  
(d) \( \frac{g_Y}{g_X} = \frac{t_X^2}{t_Y^2} \)  
(e) None of these

Problem 1029. (Similarity Problem) Your friend’s hot rod can go from 0 to 60 mph in 6 seconds. Your father’s minivan can only do 0 to 60 mph in 12 seconds. Assuming constant acceleration for each vehicle, what is the ratio of the hot rod’s acceleration \( a_{hr} \) to the minivan’s acceleration \( a_{mv} \)?

(a) \( \frac{a_{hr}}{a_{mv}} = \frac{1}{2} \)  
(b) \( \frac{a_{hr}}{a_{mv}} = \frac{1}{\sqrt{2}} \)  
(c) \( \frac{a_{hr}}{a_{mv}} = \sqrt{2} \)  
(d) \( \frac{a_{hr}}{a_{mv}} = 2 \)  
(e) None of these

Problem 1030. (Similarity Problem) A spherical balloon has a radius of \( r \) when filled with 0.50 gallons of water. If you fill the balloon with 1.0 gallons of water, what will its radius be? Your answer should be exact.

Problem 1031. (Similarity Problem) An unloaded train has an acceleration of \( a_u \). Starting from a standstill, it can accelerate to a speed of \( v \) in time \( t_u \). A fully loaded train has an acceleration of \( a_l = 0.40a_u \). How long does it take the loaded train to accelerate from zero to \( v \)?

Problem 1032. (Similarity Problem) The Smith Building is twice as tall as the Jones Building. A sack of pennies dropped from the top of the Jones Building takes time \( t_J \) to reach the ground. How long does it take for a similar sack dropped from the top of the Smith Building? Your answer should be exact. (Hint: Use the top of the building as the origin, and let downward be the positive \( y \)-direction.)
Problem 1033. (Similarity Problem) The Smith Building is three times as high as the Jones Building. A watermelon dropped from the top of the Smith Building takes $t_s$ seconds to hit the ground. How long does it take for a watermelon dropped from the top of the Jones Building? Your answer should be exact.

Problem 1034. (Similarity Problem) Two identical balls are launched vertically into the air with the same initial velocity $v_0$. One of them is launched from the earth; the other from the surface of Planet X, which has 1/10 of the earth’s gravitational attraction ($g_{\text{earth}} = 10g_X$). The ball thrown on the earth reaches a maximum height of $H_E$; the ball launched on Planet X reaches a maximum height of $H_X$. What is the relationship between $H_E$ and $H_X$? (Assume that air drag has no effect.)

Problem 1035. (Similarity Problem) Two identical balls are launched vertically into the air with the same initial velocity $v_0$. One of them is launched from the earth; the other from the surface of Planet X, which has 1/10 of the earth’s gravitational attraction ($g_E = 10g_X$). The ball thrown on the earth reaches its maximum height after time $t_E$; the ball launched on Planet X reaches its maximum height after time $t_X$. What is the relationship between $t_E$ and $t_X$? (Assume that air drag has no effect.)
7.8 Lab-application problems: One-Dimensional Kinematics

This is a mixture of problem types based on actual equipment found in instructional physic labs everywhere.

7.8.1 Ball-drop apparatus

Problem 1036. (Lab Problem) In a ball-drop apparatus, a ball is held in place above a drop pad. When a knob is turned, the ball is released to fall onto the pad; an electrical circuit through the drop apparatus and the pad is used to measure the time it takes for the ball to fall. If the ball is initially 0.85 m above the pad, how long should it take for the ball to fall to the pad? Round your answer to the nearest 0.01 s.

(a) 0.37 s
(b) 0.42 s
(c) 0.46 s
(d) 0.50 s
(e) None of these

7.8.2 Spring-cannon apparatus

Problem 1037. (Lab Problem) In this problem, use upward as the positive direction. A spring cannon has been pointed straight up, and a ball has been fired from it. The ball has not yet reached the highest point in its flight. Which of the following describes the ball’s situation?

(a) positive velocity; positive acceleration
(b) positive velocity; negative acceleration
(c) positive velocity; zero acceleration
(d) zero velocity; negative acceleration
(e) none of these

Problem 1038. (Lab Problem) In this problem, use upward as the positive direction. A spring cannon has been pointed straight up, and a ball has been fired from it. The ball has reached the highest point in its flight, and its speed is increasing as it falls back toward the ground. Which of the following describes the ball’s situation?

(a) negative velocity; positive acceleration
(b) negative velocity; negative acceleration
(c) negative velocity; zero acceleration
(d) zero velocity; positive acceleration
(e) none of these

Problem 1039. (Lab Problem) A physics student shoots a ball straight upward from a spring cannon, with an initial speed of 15.1 m/s. How long does it take for the ball to reach the highest point in its flight? Round your answer to the nearest 0.1 s.

(a) 1.5 s
(b) 1.7 s
(c) 1.9 s
(d) 2.1 s
(e) None of these
Problem 1040. (Lab Problem) A physics student on Pluto shoots a ball straight upward from a spring cannon, with an initial speed of 9.1 m/s. The ball reaches the highest point in its flight after 11.2 s. What is Pluto’s surface gravity? Round your answer to the nearest 0.01 m/s\(^2\).

(a) 0.59 m/s\(^2\)  (b) 0.66 m/s\(^2\)
(c) 0.73 m/s\(^2\)  (d) 0.81 m/s\(^2\)
(e) None of these

Problem 1041. (Lab Problem) A student shoots a ball straight upward from a spring cannon. The ball reaches the highest point in its flight after 1.2 s. What was the ball’s initial speed? Round your answer to the nearest 0.1 m/s.

(a) 10.6 m/s  (b) 11.8 m/s
(c) 12.9 m/s  (d) 14.2 m/s
(e) None of these

Problem 1042. (Lab Problem) A physics student shoots a ball from a spring cannon straight upward at 15.0 m/s. After 1.1 s, how high will the ball be? Round your answer to the nearest 0.1 m.

(a) 9.5 m  (b) 10.6 m
(c) 11.6 m  (d) 12.8 m
(e) None of these

Problem 1043. (Lab Problem) A physics student shoots a ball from a spring cannon straight upward at 15.0 m/s. After 2.3 s, how high will the ball be? Round your answer to the nearest 0.1 m.

(a) 8.6 m  (b) 9.4 m
(c) 10.4 m  (d) 11.4 m
(e) None of these

Problem 1044. (Lab Problem) A physics student shoots a ball from a spring cannon straight upward. After 1.2 s, the ball is 15.0 m high. What was the initial velocity of the ball?

(a) 18.4 m/s  (b) 20.2 m/s
(c) 22.2 m/s  (d) 24.5 m/s
(e) None of these

Problem 1045. (Lab Problem\(^a\)) A physics student shoots a ball straight upward from a spring cannon. Using a meter stick, the student measures the ball’s maximum height. If the height was found to be 53 cm, what was the ball’s initial velocity as it left the muzzle of the cannon? Round your answer to the nearest 0.1 m/s.

(a) 2.6 m/s  (b) 3.2 m/s
(c) 2.9 m/s  (d) 3.7 m/s
(e) None of these

\(^a\)This is a very accurate method for determining the initial velocity of the ball, especially when you shoot the spring cannon at a high angle of inclination (\(\theta_0 \geq 60^\circ\)).
Problem 1046. (Lab Problem) A physics student is designing a spring cannon. He wants to fire a ball straight upward from ground level; and he wants the ball to have an upward speed of 10.2 m/s when it reaches a height of 6.8 m above the ground. What must the ball’s initial speed be? Round your answer to the nearest 0.1 m/s.

(a) 12.5 m/s  
(b) 13.9 m/s  
(c) 15.4 m/s  
(d) 16.9 m/s  
(e) None of these

Problem 1047. (Lab Problem) A physics student shoots a ball straight upward from a spring cannon, with an initial velocity of 18.2 m/s. At what time will the ball reach a height of 10.4 m on its way up? Round your answer to the nearest 0.01 s.

(a) 0.51 s  
(b) 0.57 s  
(c) 0.63 s  
(d) 0.71 s  
(e) None of these

Problem 1048. (Lab Problem) A physics student shoots a ball straight upward from a spring cannon, with an initial velocity of 14.3 m/s. At what time will the ball reach a height of 8.8 m on its way back down, after it has reached its maximum height? Round your answer to the nearest 0.01 s.

(a) 1.83 s  
(b) 2.04 s  
(c) 2.24 s  
(d) 2.46 s  
(e) None of these

Problem 1049. (Lab Problem) A student shoots a ball straight upward from a spring cannon. The ball reaches the highest point in its flight after 1.5 s. What was the ball’s initial speed? Round your answer to the nearest meter per second.

(a) 11 m/s  
(b) 13 m/s  
(c) 15 m/s  
(d) 17 m/s  
(e) None of these


7.9 Mixing It Up

**Problem 1050.** Harry Potter’s magic flying broomstick is described as having an acceleration of 150 miles per hour in 10 seconds. What is this acceleration in ft/s$^2$? What is it in terms of $g$?

**Problem 1051.** Using a stopwatch, you determine that a television dropped from a rooftop takes 2.7 s to reach the ground. How fast is it going when it hits the ground? Give your answer in m/s.

**Problem 1052.** An experimental aircraft is capable of accelerating horizontally at 4.2 $g$. How long does it take for the aircraft to reach a horizontal speed of 5600 ft/s?

**Problem 1053.** You are on the surface of the Moon, where the acceleration due to gravity is 1.62 m/s$^2$. You throw an egg straight upward at 25 m/s. How long does it take for the egg to reach the ground?

**Problem 1054.** A train is moving at 32 m/s when the engineer is informed that a school bus full of orphans and puppies is on the tracks ahead of him. If the train is capable of decelerating at 0.80 m/s$^2$, how long does it take for the train to come to a halt? How far does it go after the brakes are applied?

**Problem 1055.** Your lab partner accidentally rides his canoe over Horseshoe Falls, which is 53 m high. How long does it take for him to reach the bottom? What is his downward speed when he gets there?

**Problem 1056.** A ship has a maximum speed in still water of 8.8 m/s. To be operated legally on the Rhine River, it must be able to come to a complete stop in 350 m. What acceleration would the ship need to do this?

**Problem 1057.** A ball is thrown straight up from ground level, which we take to be $y = 0$.

(a) At what speed must it be thrown to reach a height $y_{\text{max}}$ meters above the ground?

(b) How long does it take the ball to reach the maximum height $t_{\text{up}}$?
8 One-dimensional rotational kinematics

8.1 Angular speed, period, and frequency

Problem 1058. A Ferris wheel has a radius of 12 m. If a point on the outer rim is moving at 4 m/s, what is its angular speed?

(a) $1/3 \text{ rad/s}$ (b) $3 \text{ rad/s}$
(c) $2\pi/3 \text{ rad/s}$ (d) $6\pi \text{ rad/s}$
(e) None of these

Problem 1059. A reflector is attached to the spoke of a bicycle wheel, 20 cm from the center. If the wheel is turning at 18 rad/s, how fast is the reflector moving? Round your answer to two significant figures.

(a) 110 cm/s (b) 360 cm/s
(c) 640 cm/s (d) 1100 cm/s
(e) None of these

Problem 1060. A bug is clinging to the tip of a wind turbine blade, 45 meters from the center. The blade turns through a complete circle every 7.5 s. What is the bug's angular speed? Round your answer to two significant figures.

(a) 0.84 rad/s (b) 6.0 rad/s
(c) 19 rad/s (d) 38 rad/s
(e) None of these

Problem 1061. The orbit of Jupiter's moon Callisto has a radius of $1.9 \times 10^9$ m. The moon makes a complete circle around Jupiter in $1.4 \times 10^6$ s. What is Callisto's orbital speed? Round your answer to two significant figures.

(a) 6700 m/s (b) 7400 m/s
(c) 8500 m/s (d) 9100 m/s
(e) None of these

Problem 1062. Planet X orbits its star at a distance of $1.5 \times 10^{11}$ m. Its orbital speed is $3.0 \times 10^4$ m/s. How long does it take for the planet to make a complete circle around the star? Round your answer to two significant figures.

(a) $2.3 \times 10^7$ s (b) $2.5 \times 10^7$ s
(c) $2.8 \times 10^7$ s (d) $3.1 \times 10^7$ s
(e) None of these

Problem 1063. An airplane propeller is turning at a rate of 40 revolutions per second. What is the propeller’s angular speed? Round your answer to two significant figures.

(a) 6.4 rad/s (b) 13 rad/s
(c) 130 rad/s (d) 250 rad/s
(e) None of these
Problem 1064. A Ferris wheel takes 40 s to make a complete circle. What is its angular speed? Round your answer to two significant figures.
(a) 0.025 rad/s  
(b) 0.16 rad/s  
(c) 6.4 rad/s  
(d) 13 rad/s  
(e) None of these

Problem 1065. An electric fan turns at a rate of 24 revolutions per second. How long does it take for the fan to turn through 60 radians? Round your answer to two significant figures.
(a) 0.40 s  
(b) 0.73 s  
(c) 1.4 s  
(d) 2.5 s  
(e) None of these

Problem 1066. A wind turbine turns with an angular speed of 1.1 rad/s. How long does it take for the turbine to make 10 complete revolutions? Round your answer to two significant figures.
(a) 9.1 s  
(b) 17 s  
(c) 31 s  
(d) 57 s  
(e) None of these

Problem 1067. (Similarity Problem) Two bugs are clinging to a spoke of a turning bicycle wheel. The first bug is 10 cm from the center of the wheel; the second bug is 20 cm from the center. If the first bug is experiencing an angular speed of $\omega_1$, what is the second bug’s angular speed?
(a) $\omega_1/2$  
(b) $\omega_1$  
(c) $\sqrt{2}\omega_1$  
(d) $2\omega_1$  
(e) None of these

Problem 1068. (Similarity Problem) Two bugs are clinging to a spoke of a turning bicycle wheel. The first bug is 10 cm from the center of the wheel; the second bug is 20 cm from the center. If the first bug is moving at a speed of $v_1$, what is the speed of the second bug?
(a) $v_1/2$  
(b) $v_1$  
(c) $\sqrt{2}v_1$  
(d) $2v_1$  
(e) None of these

Problem 1069. (Similarity problem) A physics professor is riding on a moving merry-go-round. At a distance of $r_1 > 0$ from the center, his angular speed is $\omega_1$. At what distance from the center will his angular speed be $2\omega_1$?
(a) $\sqrt{2}r_1$  
(b) $2r_1$  
(c) $4r_1$  
(d) No solution  
(e) None of these
Problem 1070. *(Similarity problem)* A physics professor is riding on a moving merry-go-round. At a distance of \( r_1 > 0 \) from the center, his speed is \( v_1 \). At what distance from the center will his speed be \( 2v_1 \)?

(a) \( \sqrt{2}r_1 \)  
(b) \( 2r_1 \)  
(c) \( 4r_1 \)  
(d) No solution  
(e) None of these

8.2 Fundamental equation 1

Problem 1071. A ceiling fan is initially at rest. When it is switched on, it experiences an angular acceleration of \( 2.2 \text{ rad/s}^2 \). How long does it take before the fan is turning at \( 7.5 \text{ rad/s} \)? Round your answer to two significant figures.

(a) 1.6 s  
(b) 3.4 s  
(c) 7.5 s  
(d) 17 s  
(e) None of these

Problem 1072. An airplane propeller is initially turning at 220 rad/s. When the engine is switched off, it takes the propeller 18 seconds to come to a complete stop. What is the propeller’s angular acceleration? Round your answer to two significant figures.

(a) \(-8.9 \text{ rad/s}^2\)  
(b) \(-9.9 \text{ rad/s}^2\)  
(c) \(-11 \text{ rad/s}^2\)  
(d) \(-12 \text{ rad/s}^2\)  
(e) None of these

Problem 1073. A wheel is initially at rest. It experiences an angular acceleration of \( 13 \text{ rad/s}^2 \) for 22 s. At the end of this time, what is its angular speed? Round your answer to two significant figures.

(a) 290 rad/s  
(b) 950 rad/s  
(c) 3100 rad/s  
(d) 6300 rad/s  
(e) None of these

Problem 1074. A wind turbine is turning clockwise at 0.85 rad/s. The wind strengthens, causing the turbine to experience a clockwise angular acceleration of 0.044 rad/s\(^2\). After 15 seconds, how fast is the turbine turning? Round your answer to two significant figures.

(a) 0.56 rad/s  
(b) 1.5 rad/s  
(c) 4.1 rad/s  
(d) 6.1 rad/s  
(e) None of these
Problem 1075. A disc is spinning at an initial angular speed of 10 rad/s. It is subjected to an angular acceleration of 2 rad/s$^2$ for 5 seconds. At the end of this time, how fast is the disc spinning?

- (a) 10 rad/s
- (b) 20 rad/s
- (c) 35 rad/s
- (d) 60 rad/s
- (e) None of these

Problem 1076. A propeller is initially turning at 20 rad/s. When the power to it is increased, it speeds up, reaching a speed of 48 rad/s after 4 seconds. What was its angular acceleration?

- (a) 7 rad/s$^2$
- (b) 12 rad/s$^2$
- (c) 80 rad/s$^2$
- (d) 192 rad/s$^2$
- (e) None of these

Problem 1077. A fan is initially turning at 120 rad/s. When it is switched off, it experiences an angular acceleration of $-5$ rad/s$^2$. After 10 seconds, how fast is it turning?

- (a) 24 rad/s
- (b) 50 rad/s
- (c) 70 rad/s
- (d) 96 rad/s
- (e) None of these

Problem 1078. A circular saw is initially turning at 450 rad/s. It is subjected to an angular acceleration of $-30$ rad/s$^2$ until it is turning at 300 rad/s. How long does it take before it reaches this speed?

- (a) $5/3$ s
- (b) 5 s
- (c) $10/3$ s
- (d) 10 s
- (e) None of these

Problem 1079. A windmill is turning at 10 rad/s in a clockwise direction. The wind suddenly shifts, producing an angular acceleration of 0.5 rad/s$^2$ in a counterclockwise direction. After 12 sec, what is the windmill’s angular velocity?

- (a) 6 rad/s counterclockwise
- (b) 16 rad/s counterclockwise
- (c) 6 rad/s clockwise
- (d) 4 rad/s clockwise
- (e) None of these

Problem 1080. A flywheel is turning at 60 rad/s in a counterclockwise direction. It is then subjected to an angular acceleration of 5 rad/s$^2$ in a clockwise direction. After 20 seconds, what is the flywheel’s angular velocity?

- (a) 30 rad/s counterclockwise
- (b) 40 rad/s clockwise
- (c) 100 rad/s clockwise
- (d) 40 rad/s counterclockwise
- (e) None of these
Problem 1081. Starting from rest, a wheel is subjected to an angular acceleration of 3.5 rad/s² for 5 seconds; after that, it turns at a constant angular speed. At that constant speed, how long does it take for the wheel to make one revolution? Round your answer to two significant figures.

(a) 0.32 s  (b) 0.36 s  
(c) 0.39 s  (d) 0.43 s  
(e) None of these

Problem 1082. A shaft is initially turning at 50 revolutions per second. When a brake is applied to it, it slows down and comes to a halt after 5.0 seconds. What angular acceleration does the shaft experience while it is slowing down? Round your answer to two significant figures.

(a) −3.2 rad/s²  (b) −6.4 rad/s²  
(c) −31 rad/s²  (d) −63 rad/s²  
(e) None of these

Problem 1083. A merry-go-round is turning at a constant rate, with a period (the time that it takes for it to make one complete revolution) of 8.0 seconds. The operator increases the power to it, applying an angular acceleration of 0.25 rad/s for 5.0 seconds. At the end of this time, what is the merry-go-round’s period? Round your answer to two significant figures.

(a) 3.1 s  (b) 4.6 s  
(c) 6.8 s  (d) 9.3 s  
(e) None of these

Problem 1084. A Ferris wheel is turning at a rate of one revolution every 10 seconds. The operator applies a brake to it for 8 seconds, after which it is turning at a rate of one revolution every 15 seconds. What angular acceleration does the Ferris wheel experience while it is slowing down? Round your answer to two significant figures.

(a) −0.011 rad/s²  (b) −0.026 rad/s²  
(c) −0.064 rad/s²  (d) −0.16 rad/s²  
(e) None of these

8.3 Fundamental equation 2

Problem 1085. Starting from rest, a flywheel is subjected to an angular acceleration of 2.5 rad/s². After it has turned through 11 radians, how fast is it turning? Round your answer to two significant figures.

(a) 7.4 rad/s  (b) 8.2 rad/s  
(c) 9.0 rad/s  (d) 9.9 rad/s  
(e) None of these
Problem 1086. A windmill is initially held at rest by a brake. When the brake is released, it is subjected to an angular acceleration of 3 rad/s². What angle will it turn through before it reaches an angular speed of 12 rad/s?

(a) 4 rad  
(b) 12 rad  
(c) 16 rad  
(d) 24 rad  
(e) None of these

Problem 1087. A physics instructor is riding on a merry-go-round, which is initially turning at 3 rad/s. He ignites a rocket that he has attached to the merry-go-round, producing an angular acceleration of 8 rad/s². How fast will the merry-go-round be turning after it has turned through 15 rad? Round your answer to the nearest rad/s.

(a) 12 rad/s  
(b) 13 rad/s  
(c) 15 rad/s  
(d) 16 rad/s  
(e) None of these

Problem 1088. A wheel is turning at a rate of 12 rad/s. When a brake is applied to it, it experiences an angular acceleration of \( \alpha \). If the wheel turns through 9 rad before coming to a halt, what is \( \alpha \)?

(a) \(-1/3 \) rad/s²  
(b) \(-4/3 \) rad/s²  
(c) \(-2 \) rad/s²  
(d) \(-8 \) rad/s²  
(e) None of these

Problem 1089. A propeller is turning at 30 rad/s. The power to it is increased, so that after it has turned through 20 radians, it is turning at 45 rad/s. What angular acceleration did the propeller experience? Round your answer to 2 significant figures.

(a) 21 rad/s²  
(b) 23 rad/s²  
(c) 25 rad/s²  
(d) 28 rad/s²  
(e) None of these

Problem 1090. A shaft is initially turning at 10 rad/s. A brake is applied, producing an angular acceleration of \(-0.5\) rad/s². What angle does the shaft turn through before it has slowed down to 6 rad/s?

(a) 36 rad  
(b) 45 rad  
(c) 56 rad  
(d) 64 rad  
(e) None of these

Problem 1091. A windmill is initially spinning at a constant rate. The wind abruptly dies, slowing the windmill down at a rate of 0.25 rad/s². After the wind dies, the windmill turns through 30 rad before stopping. What was the windmill’s initial angular speed? Round your answer to two significant figures.

(a) 3.9 rad/s  
(b) 4.8 rad/s  
(c) 6.0 rad/s  
(d) 7.5 rad/s  
(e) None of these
Problem 1092. A steam turbine is initially turning at a constant rate. The steam pressure to it is increased, producing a positive angular acceleration of 20 rad/s$^2$. After it has turned through 30 rad, it has an angular speed of 50 rad/s. What was its initial angular speed? Round your answer to two significant figures.

(a) 18 rad/s  
(b) 28 rad/s  
(c) 36 rad/s  
(d) 42 rad/s  
(e) None of these

Problem 1093. A wind turbine is initially spinning at a constant rate, making one revolution every 4.5 seconds. The wind increases, subjecting the turbine to an angular acceleration of 0.2 rad/s$^2$. After the turbine has completed 3 revolutions, it is once again turning at a constant rate. What is its new period of revolution? Round your answer to the nearest 0.1 s.

(a) 2.0 s  
(b) 2.7 s  
(c) 3.5 s  
(d) 4.6 s  
(e) None of these

Problem 1094. A Ferris wheel is turning at a rate of one revolution every 8.0 seconds. The operator applies a brake while the wheel makes two revolutions; at the end of this time, it is turning at a rate of one revolution every 15.0 sec. What is the magnitude of the angular acceleration produced by the brake? Round your answer to two significant figures. $\alpha$?

(a) 0.018 rad/s$^2$  
(b) 0.11 rad/s$^2$  
(c) 6.4 rad/s$^2$  
(d) 40 rad/s$^2$  
(e) None of these

8.4 Fundamental equation 3

Problem 1095. A fan is initially turning at 12 rad/s. The power to the fan is increased, applying a rotational acceleration of 3 rad/s$^2$. What angle does the fan turn through over the next 2 sec?

(a) 30 rad  
(b) 36 rad  
(c) 42 rad  
(d) 45 rad  
(e) None of these

Problem 1096. Starting at rest, a wheel experiences an angular acceleration of 2 rad/s$^2$. How much time elapses before the wheel has turned through 16 rad?

(a) 2 s  
(b) $\sqrt{8}$ s  
(c) 4 s  
(d) 8 s  
(e) None of these
Problem 1097. A shaft is turning at 15 rad/s. A brake is applied, producing an angular acceleration of $-0.25 \text{ rad/s}^2$. What angle does the shaft turn through over the next 4 seconds?
(a) 49 rad  (b) 52 rad  
(c) 58 rad  (d) 62 rad  
(e) None of these

Problem 1098. A propeller is spinning at 40 rad/s. When the power is switched off, it experiences a constant angular acceleration $\alpha$. In the next 3 seconds, the propeller turns through 60 rad. What is $\alpha$? Round your answer to the nearest rad/s$^2$.
(a) $-10 \text{ rad/s}^2$  (b) $-11 \text{ rad/s}^2$  
(c) $-12 \text{ rad/s}^2$  (d) $-13 \text{ rad/s}^2$  
(e) None of these

Problem 1099. A windmill is turning at 12 rad/s. When the wind abruptly dies, it slows down at a rate of 3 rad/s$^2$. How long does it take for the windmill to turn through 16 rad? Round your answer to two significant figures.
(a) 1.5 s  (b) 1.7 s  
(c) 1.9 s  (d) 2.1 s  
(e) None of these

Problem 1100. A turbine is turning at some constant angular velocity. The power to it is increased, producing an angular acceleration of 2.0 rad/s$^2$. Over the next 10 seconds, the turbine turns through 300 radians. What was its original angular velocity? Round your answer to two significant figures.
(a) 16 rad/s  (b) 18 rad/s  
(c) 20 rad/s  (d) 22 rad/s  
(e) None of these

Problem 1101. A winch is turning clockwise at 3 rad/s. The operator reverses the power to it, producing a counterclockwise angular acceleration of 0.50 rad/s$^2$ for 10 seconds. At the end of this time, what is the net angle through which the winch has turned?
(a) 5 rad clockwise  (b) 5 rad counterclockwise  
(c) 55 rad clockwise  (d) 55 rad counterclockwise  
(e) None of these

Problem 1102. A fan is turning at some constant angular velocity. It is subjected to a clockwise angular acceleration of 0.4 rad/s$^2$ for 6 seconds, during which it turns 8 radians clockwise. What was its initial angular velocity? Round your answer to two significant figures.
(a) 0.13 rad/s clockwise  (b) 0.50 rad/s clockwise  
(c) 0.13 rad/s counterclockwise  (d) 0.50 rad/s counterclockwise  
(e) None of these
Problem 1103. A windmill is initially turning at a rate of one revolution per 4.8 seconds. The wind strengthens, applying an angular acceleration of 0.2 rad/s\(^2\). Over the next 8 seconds, what angle does the windmill turn through? Round your answer to two significant figures.

(a) 6.4 rad  
(b) 10 rad  
(c) 17 rad  
(d) 27 rad  
(e) None of these

Problem 1104. A merry-go-round is turning at a rate of one revolution per 5.5 sec. The operator applies a brake, producing an angular acceleration \(\alpha\). In the next 4.0 seconds, the merry-go-round turns half a revolution. What is the magnitude of \(\alpha\)? Round your answer to two significant figures.

(a) 0.098 rad/s\(^2\)  
(b) 0.18 rad/s\(^2\)  
(c) 0.59 rad/s\(^2\)  
(d) 0.93 rad/s\(^2\)  
(e) None of these

Problem 1105. A water wheel is turning at a rate of one revolution per 7.5 seconds. The current to the wheel increases, applying an angular acceleration of 0.5 rad/s. How long does it take for the wheel to turn through two revolutions? Round your answer to two significant figures.

(a) 4.5 s  
(b) 5.0 s  
(c) 5.6 s  
(d) 6.2 s  
(e) None of these

Problem 1106. An evil physics instructor tries to fail as many students as possible by speeding up the clock during an exam. He secretly installs a device that applies a constant angular acceleration \(\alpha\) to the minute hand. At the beginning of the test, the hand is moving at the correct speed; because of the angular acceleration, it makes a complete circle in only 40 minutes. What is the value of \(\alpha\)? Round your answer to two significant figures.

(a) \(8.7 \times 10^{-8}\) rad/s\(^2\)  
(b) \(7.3 \times 10^{-7}\) rad/s\(^2\)  
(c) \(6.6 \times 10^{-5}\) rad/s\(^2\)  
(d) \(4.2 \times 10^{-4}\) rad/s\(^2\)  
(e) None of these

9 Two-dimensional kinematics

9.1 The basics of velocity and acceleration in two dimensions

9.1.1 The difference between average speed and average velocity in a plane

Problem 1107. (Speed vs. Velocity) A race car goes around a track at a constant speed of 200 mph (It has cruise control). What is the car's average speed and velocity over one lap on the race track?
Problem 1108. (Speed vs. Velocity) If we throw a ball straight up in the air and it comes back down to the same spot, what is the average velocity of the ball over one complete trip up and back down?

9.1.2 Average acceleration in a plane

Problem 1109. A circular room has a radius of 4 m and a small window facing north. A fly moving at 6 m/s flies directly south through the window and into the room. Over the next 20 s, he flies three times counterclockwise around the perimeter of the room, then flies directly north out the window at 6 m/s. What are the magnitude and direction of the fly’s average acceleration between the time he enters the room and the time he leaves?

9.1.3 Relative velocity in a plane

Problem 1110. A river has a current with a speed of 5 ft/s. You can swim at 4 ft/s in still water. At what angle to the cross-river direction do you need to swim to reach a point directly across the river from you?

Problem 1111. A train is going directly north at 70 mph. An action-movie star is atop the train, running southward at 12 mph. Relative to someone standing on the ground, how fast is the action-movie star moving, and in what direction?

9.1.4 Graphical interpretation of velocity and acceleration

Problem 1112. On the graph below, draw the velocity vectors at the points $P_1$ and $P_2$ on the curve. Assume the particle is moving with uniform speed from left to right.
**Problem 1113.** The graph at right shows the top view of the path of a particle moving along a horizontal surface. The particle is moving from the lower left to the upper right. Which of the labeled vectors is the velocity vector at point $P$?

(a) $\vec{A}$ 
(b) $\vec{B}$ 
(c) $\vec{C}$ 
(d) $\vec{D}$ 
(e) None of these

**Problem 1114.** The graph at right shows the path of a particle moving from the lower left to the upper right. Which of the labelled vectors is the acceleration vector at point $P$?

(a) $\vec{A}$ 
(b) $\vec{B}$ 
(c) $\vec{C}$ 
(d) $\vec{D}$ 
(e) None of these
9.2 Uniform circular motion

Acceleration in uniform circular motion:

\[ a_{\text{rad}} = \frac{v^2}{r}, \]

where \( v \) is the tangential speed of the object and \( r \) is the radius.

Speed in uniform circular motion:

\[ v = \frac{\text{circumference of circle}}{\text{total time}} = \frac{2\pi r}{T_{\text{period}}} = 2\pi r f, \]

where the frequency \( f \) is defined as \( f \equiv \frac{1}{T_{\text{period}}}. \)

Problem 1115. A freeway on-ramp has a curve with a radius of 59 m. You attempt to drive around the curve at 16 m/s. What is the centripetal acceleration on your car? Round your answer to the nearest 0.1 m/s².

(a) 4.3 m/s²  
(b) 4.8 m/s²  
(c) 5.3 m/s²  
(d) 5.8 m/s²  
(e) None of these

Problem 1116. A ceiling fan has blades 81 cm long. A spider is clinging to the tip of one of the blades, which are turning so that the spider moves at 10.2 m/s. What is the spider’s radial acceleration? Round your answer to the nearest m/s².

(a) 105 m/s²  
(b) 117 m/s²  
(c) 128 m/s²  
(d) 143 m/s²  
(e) None of these

Problem 1117. A toddler’s parents have placed him on a merry-go-round, 2.7 m from the center. If the toddler experiences a radial acceleration greater than 1.2 m/s², it will fall off, prompting the angry parents to sue the merry-go-round operator. How fast can the operator allow the toddler to go without fear of legal action? Round your answer to the nearest 0.1 m/s.

(a) 1.5 m/s  
(b) 1.6 m/s  
(c) 1.8 m/s  
(d) 2.0 m/s  
(e) None of these

Problem 1118. A curve in a highway has a radius of 93 m. A truck will roll over if it experiences a centripetal acceleration greater than 1.9 m/s². How fast can the truck drive around the curve? Round your answer to the nearest m/s.

(a) 11 m/s  
(b) 12 m/s  
(c) 13 m/s  
(d) 15 m/s  
(e) None of these
Problem 1119. You are designing a highway that will include a curve. Cars will skid off the curve if they experience a centripetal acceleration greater than 2.2 m/s$^2$. You want cars to be able to drive the curve safely at 20 m/s. What must the radius of the curve be? Round your answer to the nearest 10 m.

(a) 180 m  
(b) 200 m  
(c) 220 m  
(d) 240 m  
(e) None of these

Problem 1120. An airplane is moving at 330 m/s when the pilot realizes that he is going in the wrong direction and needs to make a U-turn. If the pilot experiences a radial acceleration greater than 49 m/s$^2$, he will lose consciousness and crash the plane. What is the minimum safe radius of the turn? Round your answer to the nearest 100 m.

(a) 1600 m  
(b) 1800 m  
(c) 2000 m  
(d) 2200 m  
(e) None of these

Problem 1121. A bug is standing atop the blade of a ceiling fan, which is turning at 1.7 revolutions per second. The bug will lose its grip on the fan and go sailing off into the room if it experiences a radial acceleration greater than 91 m/s$^2$. How far from the center of the fan can the bug go without losing its hold? Round your answer to the nearest centimeter.

(a) 58 cm  
(b) 65 cm  
(c) 72 cm  
(d) 80 cm  
(e) None of these

Problem 1122. A windmill has three blades, each 37 m long; the blades make a complete circle every 5.2 s. A bug is clinging to the tip of one of the blades. How much radial acceleration does the bug experience? Ignore the effect of gravity.

Problem 1123. A windmill has three blades, each 8.3 m long; the blades make a complete circle every 4.1 s. A bug is clinging to the tip of one of the blades. How much radial acceleration does the bug experience? Ignore the effect of gravity. Round your answer to the nearest m/s$^2$.

(a) 10 m/s$^2$  
(b) 0.1 m/s$^2$  
(c) 6 m/s$^2$  
(d) 19 m/s$^2$  
(e) None of these

Problem 1124. A ceiling fan is turning at a rate of 1.8 revolutions per second. A spider is clinging to the blade of the fan. If he experiences centripetal acceleration greater than 2.7 $g$, he will lose his grip on the blade and go sailing off. How far from the center of the fan can the spider safely go? Give your answer in centimeters.

Problem 1125. A highway has a curve with radius of curvature 72 m. A truck will skid off the highway if it experiences a radial acceleration of more than 2.0 m/s$^2$. How fast can the truck safely go on the curve?
9.2.1 Similarity problems

For all vehicles-going-around-circular-curve problems: Although it is not obvious at this point, the vehicle’s ability to stay on the road depends on the centripetal force provided by the friction and the tires. It turns out that there is a maximum centripetal force that the tires can supply to the road to keep the car from sliding. There is a direct correlation between force and acceleration, so it follows that there is a maximum centripetal acceleration for which the car can remain on the road. If this centripetal acceleration is exceeded, then

Problem 1126. (Similarity Problem) Two bugs are clinging to the second hand of a very large clock: one 3 ft from the center of the clock, the other 6 ft from the center. If the first bug experiences a radial acceleration of $a$, what radial acceleration does the second bug experience?

Problem 1127. (Similarity Problem) A highway has a curve with radius $r_1$. Large trucks must drive the curve at a speed of $v_1$ or less to keep from skidding off the road. What would the radius of the curve have to be in order for trucks to drive it safely at $\alpha v_1$, where $\alpha > 0$ is a real number?

(a) $\sqrt{\alpha r_1}$  (b) $\alpha^2 r_1$
(c) $\alpha \sqrt{r_1}$  (d) $\alpha r_1$
(e) None of these

Problem 1128. (Similarity Problem) A highway has a curve with a radius of $r_1$. Large trucks must drive the curve at a speed of $v_1$ or less to keep from skidding off the road. If the highway were rebuilt so that the curve had twice radius, how fast could trucks safely drive around the curve?

(a) $\sqrt{2} v_1$  (b) $2v_1$
(c) $2\sqrt{2} v_1$  (d) $4v_1$
(e) None of these

Problem 1129. (Similarity Problem) A highway has a curve with radius $r_1$. Large trucks must drive the curve at a speed of 20 mph or less to avoid skidding off the road. What would the radius of the curve have to be in order for trucks to drive it safely at 60 mph?

(a) $\sqrt{3} r_1$  (b) $3r_1$
(c) $3\sqrt{3} r_1$  (d) $9r_1$
(e) None of these

Problem 1130. (Similarity Problem) A highway has a quarter-circle curve with a radius of $r$. Large trucks must take the curve at a speed of 25 mph or less to avoid skidding off the road. What would the radius of the curve have to be in order for trucks to drive around it safely at 50 mph?
9.3 Projectile motion

If $\vec{A} = A_x \hat{i} + A_y \hat{j}$, then from the definition of the trig functions about the right triangle:

\[
\begin{align*}
A &= \|\vec{A}\| = \sqrt{A_x^2 + A_y^2} \\
\theta_0 &= \tan^{-1}\left(\frac{A_y}{A_x}\right)
\end{align*}
\]

9.3.1 General equations for projectile motion

Let the initial time be $t_0 = 0$, the initial position be $(x_0, y_0)$, and the initial velocity be $\vec{v}_0 = v_{0x} \hat{i} + v_{0y} \hat{j}$. Typically we take the initial position to be $(x_0, y_0) = (0, h)$, where $h$ is the height of the mechanism doing the launching.

The general equations governing motion in 2-D are the set of fundamental equations in 1-D repeated in each of the $x$ and $y$ directions. Substituting $(x_0, y_0) = (0, h)$ into the fundamental equations (1) and (3) in each coordinate direction, and taking $a_x = 0$ (since there is no force acting in the horizontal direction) yields

**velocity x-component:** $v_x = v_{0x}$  
**position x-component:** $x = v_{0x} t$  

If we take our $y$-axis pointing upward, then $a_y = -g$ and we have

**velocity y-component:** $v_y = v_{0y} - gt$  
**position y-component:** $y = h + v_{0y} t - \frac{1}{2} gt^2$

If the initial velocity is given in terms of magnitude $\|\vec{v}_0\| = v_0$, and direction $\theta_0$, as measured in the counterclockwise direction from the $x$-axis, then we must convert this information into information about the components of the vector in order to use the above formulas.

\[
\begin{align*}
v_{0x} &= v_0 \cos \theta_0 \quad \text{(x-component of initial velocity)} \\
v_{0y} &= v_0 \sin \theta_0 \quad \text{(y-component of initial velocity)}
\end{align*}
\]
9.3.2 Velocity and acceleration of a projectile

Problem 1131. The following four diagrams show the trajectory of a cannonball. Which diagram correctly shows the acceleration vectors at three points along the trajectory?

(a) None of these

Problem 1132. The following four diagrams show the trajectory of a cannonball. Which diagram correctly shows the velocity vectors at three points along the trajectory?

(e) None of these
Problem 1133. Draw the acceleration vector due to gravity at the points A, B, and C along the curve of the trajectory of the cannonball.
9.3.3 Special Case I: The Half-Parabola

Note: For the case of the half-parabola the initial height is the maximum height and the ball is fired horizontally. Setting \( h = y_{\text{max}} > 0 \) and \( \theta_0 = 0 \), which leads to \( v_{0y} = 0 \) and \( v_{0x} = v_0 \), in the general equations reduces them to

\[
\text{velocity x-component:} \quad v_x = v_0 \\
\text{position x-component:} \quad x = v_0 t
\]

If we take our \( y \)-axis pointing upward, then \( a_y = -g \) and we have

\[
\text{velocity y-component:} \quad v_y = -gt \\
\text{position y-component:} \quad y = y_{\text{max}} - \frac{1}{2} gt^2
\]

Note: For this case, it would also make sense to take the \( y \)-axis pointing downward. Then the time to fall is just a one-dimensional problem. The time to fall, denoted by \( t_{\text{fall}} \), is found from the fundamental equation 3, with the \( y \)-axis pointing downward so that \( a_y = g \). Once again \( v_{0y} = 0 \), so the equation becomes

\[
y_{\text{max}} = \frac{1}{2} gt_{\text{fall}}^2 \quad \Rightarrow \quad t_{\text{fall}} = \sqrt{\frac{2y_{\text{max}}}{g}}.
\]

Problem 1134. A skateboarder inadvertently rides off the edge of a flat roof at 8.2 m/s. The roof is 23 m high. How long does it take for the skateboarder to reach the ground? Round your answer to the nearest 0.1 s.

(a) 1.8 s \hspace{1cm} (b) 1.9 s \hspace{1cm} (c) 2.2 s \hspace{1cm} (d) 2.4 s \hspace{1cm} (e) None of these

Problem 1135. A bicyclist rides off a flat roof at 13.4 m/s. The roof is 3.6 m above the ground. How far from the edge of the building does the bicyclist land? Round your answer to the nearest meter.

(a) 11 m \hspace{1cm} (b) 12 m \hspace{1cm} (c) 14 m \hspace{1cm} (d) 15 m \hspace{1cm} (e) None of these

Problem 1136. You fire your potato gun horizontally from shoulder height, which is 140 cm above the ground. The potato strikes the ground 15 m away from you. What was the initial speed of the potato? Round your answer to the nearest m/s.

(a) 25 m/s \hspace{1cm} (b) 28 m/s \hspace{1cm} (c) 31 m/s \hspace{1cm} (d) 34 m/s \hspace{1cm} (e) None of these
Problem 1137. You fire your potato gun horizontally from shoulder height, which is 150 cm above the ground. The potato strikes the ground 11 m away from you. What was the initial speed of the potato? Round your answer to the nearest m/s.

(a) 20 m/s  (b) 22 m/s  
(c) 24 m/s  (d) 26 m/s 
(e) None of these

Problem 1138. Your potato gun has a muzzle velocity of 27 m/s. You shoot it horizontally from a window 12 m above the ground. How far from the building does the potato strike the ground? Round your answer to the nearest meter.

(a) 32 m  (b) 42 m  
(c) 52 m  (d) 62 m  
(e) None of these

Problem 1139. Your potato gun has a muzzle velocity of 10 m/s. You shoot it horizontally from a window 5 m above the ground. How far from the building does the potato strike the ground? Approximate gravity as $g = 10 \text{ m/s}^2$. Round your answer to the nearest meter.

(a) 10 m  (b) 12 m 
(c) 14 m  (d) 8 m 
(e) None of these

Problem 1140. (Similarity Problem) You shoot your potato gun horizontally from shoulder height, which is 150 cm. The potato lands 25 m away from you. You would like the potato to go 75 m, so you climb up a ladder and fire the gun horizontally from there. How high off the ground must the gun be?

Problem 1141. A movie stuntman wants to ride a motorcycle off a flat roof, sail across a gap 24 m wide, and land on another roof 3.6 m below the first one. How fast does he need to be moving when he leaves the first roof?

(a) 28 m/s  (b) 31 m/s 
(c) 34 m/s  (d) 37 m/s 
(e) None of these

Problem 1142. While an unmanned rover is exploring the surface of Planet X, it goes off the edge of a cliff at 11.2 m/s. The cliff is 3.1 m high; the rover strikes the ground 18.8 m from the base of the cliff. What is the surface gravity of Planet X? Round your answer to the nearest 0.1 m/s$^2$.

(a) 2.2 m/s$^2$  (b) 2.4 m/s$^2$  
(c) 2.6 m/s$^2$  (d) 2.9 m/s$^2$  
(e) None of these

Problem 1143. A ball rolls off the edge of a horizontal table 1.32 m high. It strikes the floor at a point 4.45 m horizontally away from the edge of the table. What was the ball’s speed at the moment that it left the tabletop? Round your answer to the nearest 0.1 m/s.
9.3.4 Special Case II: The Full Parabola

Note: For the case of the full-parabola the initial height is zero and the ball is fired at an angle of incline $0 < \theta_0 \leq \frac{\pi}{2}$. Setting $h = 0$ and $\theta_0 \neq 0$, which leads to $v_{0x} \neq 0$ (except if $\theta_0 = \frac{\pi}{2}$) and $v_{0y} \neq 0$, in the general equations reduces them to

velocity x-component: \[ v_x = v_{0x} \] (9.5a)
position x-component: \[ x = v_{0x}t \] (9.5b)

If we take our $y$-axis pointing upward, then $a_y = -g$ and we have

velocity y-component: \[ v_y = v_{0y} - gt \] (9.6a)
position y-component: \[ y = v_{0y}t - \frac{1}{2}gt^2 \] (9.6b)

Problem 1144. Your Civil War musket has a muzzle velocity of 290 m/s. If you fire it at an elevation of 6.2°, what is the maximum height reached by the bullet? Round your answer to the nearest meter.
(a) 36 m (b) 41 m (c) 45 m (d) 50 m (e) None of these

Problem 1145. A golfer hits a ball from ground level with an initial speed of 42 m/s at an elevation of 19°. How far from the initial location does the ball hit the ground? Round your answer to the nearest 10 m.
(a) 100 m (b) 110 m (c) 120 m (d) 130 m (e) None of these

Problem 1146. A golfer hits a ball from ground level with an initial speed of 57 m/s at an elevation of 34°. How far from the initial location does the ball hit the ground? Round your answer to the nearest 10 m.
(a) 310 m (b) 340 m (c) 370 m (d) 410 m (e) None of these

Problem 1147. You fire your potato gun from ground level at an angle of 36° above the horizontal. The potato strikes the ground 47 m away from you. What was the initial speed of the potato? Round your answer to the nearest m/s.
(a) 22 m/s (b) 24 m/s (c) 26 m/s (d) 29 m/s (e) None of these
Problem 1148. You are holding the neck of a champagne bottle at an elevation of 41° when you pop the cork. The highest point in the cork’s flight is 2.6 m above its initial height. What was the cork’s initial speed? Round your answer to the nearest m/s.

(a) 9 m/s  
(b) 10 m/s  
(c) 11 m/s  
(d) 12 m/s  
(e) None of these

Problem 1149. You are holding the neck of a champagne bottle at an elevation of 34° when you pop the cork. The highest point in the cork’s flight is 1.9 m above its initial height. What was the cork’s initial speed? Round your answer to the nearest m/s.

(a) 10 m/s  
(b) 11 m/s  
(c) 12 m/s  
(d) 13 m/s  
(e) None of these

Problem 1150. A motorcyclist would like to ride up a short ramp on a rooftop, fly across a gap between two buildings, and land on a second rooftop at the same height as the first. The ramp is inclined at 32°; the buildings are 18 m apart. How fast does the motorcyclist have to be going when he makes the jump? Round your answer to the nearest m/s.

(a) 13 m/s  
(b) 14 m/s  
(c) 15 m/s  
(d) 16 m/s  
(e) None of these

Problem 1151. A crossbow fires its arrow at 105 m/s. If the arrow is aimed at an elevation of 12° above the horizontal, what is the horizontal distance that it will travel before striking the ground? Round your answer to the nearest 10 m.

(a) 370 m  
(b) 410 m  
(c) 460 m  
(d) 500 m  
(e) None of these

Problem 1152. A Napoleon cannon has a muzzle velocity of 450 m/s. If the cannon is fired at an elevation of 5.1°, how much time elapses before the ball strikes the ground? Round your answer to the nearest 0.1 s.

(a) 8.2 s  
(b) 9.0 s  
(c) 9.8 s  
(d) 10.6 s  
(e) None of these
Problem 1153. Space pirates have kidnapped you from your physics lab and are holding you for ransom on another planet. Luckily, they have overlooked your cell phone and your spring cannon. The spring cannon has a muzzle velocity of 8.6 m/s; when you fire it at an elevation of 45°, the ball strikes the ground 41 m away. (The pirates are keeping you in a rather large room.) What is the surface gravity of the planet on which you’re being held? Round your answer to the nearest 0.1 m/s².

(a) 1.6 m/s² (b) 1.8 m/s²
(c) 2.0 m/s² (d) 2.2 m/s²
(e) None of these

Problem 1154. Your potato gun has a muzzle velocity of 23 m/s. You want to shoot a potato and have it strike the ground 3.5 seconds later. At what elevation do you have to fire the gun? Round your answer to the nearest degree.

(a) 43° (b) 48°
(c) 53° (d) 58°
(e) None of these

Problem 1155. You have designed a tennis-ball cannon with a muzzle velocity of 62 m/s. You want to land a tennis ball in a swimming pool 210 m away. At what angle to the horizontal do you need to aim the gun? Round your answer to the nearest degree.

(a) 11° (b) 13°
(c) 15° (d) 16°
(e) None of these

Problem 1156. You have designed a tennis-ball cannon with a muzzle velocity of 62 m/s. You want to land a tennis ball in a swimming pool 210 m away. At what angle to the horizontal do you need to aim the gun? Your answer should be less than π/4 radians.

(a) 1.08 rad (b) 0.13 rad
(c) 0.97 rad (d) 0.28 rad
(e) None of these

Problem 1157. A dietitian is standing at the edge of a cliff. He throws a piece of cheese upward and outward, so that it reaches a maximum height of 12 m above him, then falls down into the canyon 65 m below him and 40 m from the edge of the cliff. Between the time he releases the cheese and the time that it strikes the bottom, where is the acceleration at a minimum?

Problem 1158. The surface gravity on Mars is 0.4 times the surface gravity on the Earth. A golfer can hit a ball so that it strikes the ground 200 m from him on Earth. If he hits the ball with the same speed and at the same angle on Mars, how far will it go?

Problem 1159. Your physics teacher is trying to get a planeload of “prescription” drugs across the border. As an extra-credit project, he has assigned you and your lab partner to shoot down the radar blimp. The blimp flies at a height of 10,000 ft. You fire a cannon at an angle of θ to the horizontal, and the maximum height attained by the shell is 5000 ft. Your lab partner says that if you increase the angle to 2θ, the shell will go high enough to hit the blimp. Is he correct? Why or why not?
Problem 1160. You are trying to put out a fire 150 m away with a water cannon. The manual informs you that if you aim the water cannon at an elevation of 23° above the horizontal, it will have a range of 150 m. Unfortunately, there is a building between you and the fire, and at an elevation of 23° the water will hit it. Is there another angle of elevation $\theta$ that you could use to get a range of 150 m? Give $\theta$, or explain why it doesn’t exist.

Problem 1161. If a champagne bottle is held with its neck elevated 78° above the horizontal, the popped cork will reach a maximum altitude of 4.8 m. Is there another angle $\theta$ between 0° and 90° for which the cork will reach the same maximum altitude? Give $\theta$, or explain why it doesn’t exist.

Problem 1162. The Paris Gun had a muzzle velocity of 1600 m/s. If the gun was fired at an elevation of 45° above the horizontal, what was the maximum altitude reached by the shell? Give your answer in kilometers.

Problem 1163. A crossbow fires its arrow at 345 ft/s. If the arrow is aimed at an elevation of 12° above the horizontal, what is the horizontal distance that it will travel before striking the ground?

Problem 1164. Barbora Špotáková won a gold medal in the 2008 Olympics by throwing a javelin 71.42 m. Assuming that Špotáková threw the javelin at an angle of $\pi/4$ radians to the horizontal, what was its speed when initially thrown?

Problem 1165. The Napoleon cannon of the Civil War had a muzzle velocity of 1485 ft/s. If the cannon was fired at an elevation of 5.0° above the horizontal, how much time would elapse between the firing and the ball’s striking the ground?

Problem 1166. Using surgical tubing and ingenuity, you have constructed a water-balloon launcher in a window 22 m above the ground. Your device launches a water balloon at a speed of 19 m/s, and at an elevation of 0.25 radians above the horizontal. At what horizontal distance from the building will the water balloon strike the ground?

Problem 1167. A bicyclist unfamiliar with the terrain is riding along a level sidewalk when he goes over the edge of a staircase. He hits the staircase 3.7 m below the edge, and 3.7 m horizontally from the edge. How fast was he going when he went over the edge?

Problem 1168. You fire a cannon at an angle of $\theta$ to the horizontal, with a muzzle velocity of $v_0$. The ball lands at a horizontal distance of $x$ from the cannon. For your second shot, you increase the powder charge in the cannon so that the muzzle velocity will be $2v_0$. How far from the cannon will the ball land?
9.4 Lab application problems: Two-Dimensional Kinematics

This is a mixture of problem types based on actual equipment found in instructional physic labs everywhere.

9.4.1 Spring-cannon apparatus

Problem 1169. (Lab Problem) You fire a spring cannon horizontally from a fixed height \( h_{\text{cannon}} = y_{\text{max}} \). It strikes the ground at a distance \( x_{\text{max}} \) from the base of the mouth of the muzzle. Find a formula for the ball’s initial velocity as a function of the initial height \( y_{\text{max}} \) and range \( x_{\text{max}} \). The initial velocity is taken to be the velocity of the ball as it leaves the mouth of the cannon.

(a) \( x_{\text{max}} \sqrt{2gy_{\text{max}}} \)  
(b) \( x_{\text{max}} \sqrt{\frac{2y_{\text{max}}}{g}} \)  
(c) \( x_{\text{max}} \sqrt{\frac{2g}{y_{\text{max}}}} \)  
(d) \( x_{\text{max}} \sqrt{\frac{g}{2y_{\text{max}}}} \)  
(e) None of these

\(^a\)This is a common technique for determining initial velocity, based on measured lengths rather than on measured time. It is good for estimating the initial velocity when the spring cannon is shot at small to moderate angles of incidence (\( \theta_0 \leq 45^\circ \)).

Problem 1170. (Lab Similarity Problem) You fire a spring cannon horizontally from a fixed height. The initial velocity of the ball is \( v_{0,1} \); it strikes the ground at a distance \( x_{\text{max},1} \) from the base of the mouth of the muzzle. If you want the ball to travel twice the horizontal distance, what must the initial velocity be?

(a) \( \sqrt{2}v_{0,1} \)  
(b) \( 2v_{0,1} \)  
(c) \( 2\sqrt{2}v_{0,1} \)  
(d) \( 4v_{0,1} \)  
(e) None of these

Problem 1171. (Lab Similarity Problem) If you fire a spring cannon horizontally from a height of \( h_{\text{cannon},1} \), the ball travels a horizontal distance of \( x_{\text{max},1} \) before hitting the floor. How high must the spring cannon be to get the ball to travel twice the horizontal distance before hitting the ground?

(a) \( \sqrt{2}h_{\text{cannon},1} \)  
(b) \( 2h_{\text{cannon},1} \)  
(c) \( 2\sqrt{2}h_{\text{cannon},1} \)  
(d) \( 4h_{\text{cannon},1} \)  
(e) None of these

Problem 1172. (Lab Problem) You fire a spring cannon from ground level at an elevation of 23\(^\circ\). The ball emerges from the cannon at a speed of 14.2 m/s. How long does it take before the ball strikes the ground? Round your answer to the nearest 0.1 s.

(a) 1.0 s  
(b) 1.1 s  
(c) 1.2 s  
(d) 1.4 s  
(e) None of these
Problem 1173. (Lab Problem) A high-powered spring cannon discharges its ball at 16.6 m/s. If the cannon is fired at an elevation of 77°, what is the maximum height reached by the ball? Round your answer to the nearest meter.

(a) 12 m  (b) 13 m  (c) 15 m  (d) 16 m  (e) None of these

Problem 1174. (Lab Problem) In your physics lab, you are using a new high-powered spring cannon with a muzzle velocity of 19 m/s. The ceiling of the lab is 2.8 m above the level of the cannon. What is the maximum elevation at which you can fire the cannon without hitting the ceiling? Round your answer to the nearest degree.

(a) 16°  (b) 18°  (c) 21°  (d) 23°  (e) None of these

Problem 1175. (Lab Similarity Problem) You fire a spring cannon at a fixed elevation θ₀ ≠ 0. The spring cannon uses a compressed spring to launch the ball. The cannon has three settings with the first being the lowest and the third being the strongest. Using the first setting (one click) you find that the ball emerges from the muzzle with a speed of v₀,1; it strikes the ground at a distance xₘₐₓ,1 from the cannon. You then increase the energy stored in the spring by using the second setting (two clicks). The muzzle velocity of the ball is now measured to be v₀,2 = αv₀,1, where α is a positive constant. What is the new range of the ball?

(a) \sqrt{\alpha}xₘₐₓ,1  (b) αxₘₐₓ,1  (c) α\sqrt{\alpha}xₘₐₓ,1  (d) α²xₘₐₓ,1  (e) None of these

Problem 1176. (Lab Similarity Problem) You fire a spring cannon at a fixed elevation θ₀ ≠ 0. The ball emerges from the muzzle with a speed of v₀,1; it reaches a maximum height of yₘₐₓ. You would like to increase the muzzle velocity so that the ball can double its height. What must the new muzzle velocity be?

(a) \sqrt{2}v₀,1  (b) 2v₀,1  (c) 2\sqrt{2}v₀,1  (d) 4v₀,1  (e) None of these

Problem 1177. (Derive Problem) A ball rolls off the edge of a horizontal table of height H. It strikes the floor at a horizontal distance X away from the edge of the table. What was the ball’s speed at the moment that it left the tabletop? Derive a formula that involves the given data: H and X.

Problem 1178. (Derive Problem) A child whirls a stone in a horizontal circle at a height of H above the ground, on the end of a string with length L. The string breaks and the stone flies off horizontally, striking the ground at a horizontal distance of X from where it was when the string broke. What was the centripetal acceleration of the stone before the string broke? Your answer should be a formula of the form aₐ = aₐ(H, L, X, g), where g is the acceleration due to gravity.
Problem 1179. (Lab Derive Problem) A ball is fired horizontally out of a spring cannon with an initial velocity of $v_0$; the cannon is a height $h$ above the ground. The ball strikes the ground at a horizontal distance $x_{\text{max}}$ from the muzzle of the cannon. Find $v_0$ as a function of $h$, $x_{\text{max}}$, and $g$.

**Note:** The purpose of this problem is to find the initial velocity of the ball as it leaves the muzzle of the spring cannon. Using this initial velocity one can then point the cannon in any direction and predict the maximum height and the range of the projectile.
Problem 1180. (Lab Derive Problem) Consider the situation shown in the figure 3 below. Take the mouth of the cannon to be at the point \((x_0, y_0) = (0, h)\). Assume that you know the speed \(v_0\) and direction \(\theta\) of the initial velocity \(\vec{v}_0 = v_0 (\cos \theta, \sin \theta)\). Derive a formula for the vertex of the trajectory \((x_{ymax}, y_{ymax})\) and the range point \((x_{max}, 0)\) in terms of the given information.

Note: The purpose of this problem is to find the vertex and the range of a projectile.

Figure 3: Spring cannon
Problem 1181. In one of your physics labs you are given the challenge of shooting a ball from a spring cannon into a bucket. Your job is to figure out where to place the center of the bucket in order to launch the ball into the bucket! Also, you are only allowed one attempt to put the ball in the bucket.

You are given the following information (see figure 5):

- The ball is released from the mouth of the cannon and is located at the initial position \((x_0, y_0) = (0, H_{\text{cannon}})\).
- The bucket stands \(H_{\text{bucket}}\) units above the table with the center of the bucket located \(x_{\text{max}}\) units in the horizontal direction from the mouth of the cannon. Also, as the picture suggests, \(H_{\text{bucket}} < H_{\text{cannon}}\).
- The mouth of the bucket is just large enough to allow the ball to be captured in the bucket provided that the ball lands close enough to the point \((x_{\text{max}}, H_{\text{bucket}})\).
- The initial speed \(v_0 = \|\vec{v}_0\|\) and direction \(\theta_0\) that the ball leaves the mouth of the spring cannon.

Figure 4: Spring cannon and bucket
Problem 1182. (Lab Derive Problem [Put the ball in the bucket])
In one of your physics labs you are given the challenge of shooting a ball from a spring cannon into a bucket. Specifically, you need to work out the angle the cannon should be aimed in order to launch the ball into the bucket! You are only allowed one attempt to put the ball in the bucket and you cannot move the bucket. You are also told that the cannon has plenty enough power to launch the ball into the bucket.

You are given the following information (see figure 5):

- The bucket stands $H_{\text{bucket}}$ units above the table with the center of the bucket located $x_{\text{max}}$ units in the horizontal direction from the mouth of the cannon.
- The mouth of the bucket is just large enough to allow the ball to be captured in the bucket provided that the ball lands close enough to the point $(x_{\text{max}}, H_{\text{bucket}})$.
- You are instructed to take the mouth of the cannon to be at the point $(x_0, y_0) = (0, H_{\text{cannon}})$. By using a swivel mount at the mouth of the cannon, the device is designed in such a way as to keep the height of the ball the same regardless of the direction $\theta$ that the cannon is aimed. Thus $H_{\text{cannon}}$ is independent of $\theta$.
- The initial speed $v_0 = \|\vec{v}_0\|$ that the ball leaves the cannon.

Derive a formula for $\theta$ in terms of the given information.

Figure 5: Spring cannon and bucket
Problem 1183. (Lab Derive Problem [Put the ball in the moving bucket])

The Idea: In one of your physics labs you are given the challenge of shooting a ball from a spring cannon into a bucket mounted on top of a glider moving at a constant velocity along an air track that is aligned in the direction of the cannon.

The Apparatus: A horizontal air track is sitting atop a table. The track passes under a spring cannon that is attached to a mount that holds the cannon directly above the air track. From a top view the cannon is aimed along the track in the direction of motion of the glider. From a side view, the cannon makes an angle $0 \leq \theta_0 \leq \pi/2$ with respect to the horizontal air track (see figure 6 below).

A bucket sits atop a glider moving down the air track at constant speed $V_{\text{glider}}$. When the bucket passes directly below the the cannon, the cannon is fired and the ball emerges at the muzzle of the cannon with a velocity of $V_0$. The apparatus is designed so that the center of mass of the ball, denoted by $x$, is directly over the center of the bucket $x_{\text{center}}$ as the ball leaves the muzzle. Take this time to be the initial time $t_0 = 0$ and take the location along the air track to be $x_0 = 0$.

Figure 6: Side view of the spring cannon firing ball into a bucket mounted on top of a glider

The Setup: You are given the following information

- You are instructed to take the mouth of the cannon to be at the point $(x_0, y_0) = (0, H_{\text{cannon}})$. By using a swivel mount at the mouth of the cannon, the device is designed in such a way as to keep the height of the ball the same regardless of the direction $\theta_0$ that the cannon is aimed. Thus $H_{\text{cannon}}$ is independent of $\theta_0$. 
The bucket stands $H_{\text{bucket}}$ units above the table with the center of the bucket located a distance $x_{\text{center}} = V_{\text{glider}}t$ from the mouth of the cannon in the horizontal direction along the track. Thus the ordinates of the center of the bucket, the target, are $(x_{\text{center}}, H_{\text{bucket}})$.

The mouth of the bucket is large enough to offset the small effects of air drag so that the ball can be captured in the bucket provided that the ball lands close enough to the point $(x_{\text{center}}, H_{\text{bucket}})$.

Assume $H_{\text{bucket}} < H_{\text{cannon}}$.

The initial speed of the ball as it leaves the mouth of the cannon is $\|\vec{v}_0\| = V_0 > 0$.

Assume that you know the speed of the glider $V_{\text{glider}}$ and the initial speed of the ball as it leaves the mouth of the cannon $V_0$.

The Questions:

(a) (2 points) Starting from the general equations for projectile motion with $x_0 = 0$:

$$\begin{align*}
x(t) &= v_{0x}t \quad \text{(the ball’s x-component:)} \\
y(t) &= h + v_{0y}t - \frac{1}{2}gt^2 \quad \text{(the ball’s y-component:)}
\end{align*}$$

substitute the values for the initial position $(0, h)$, velocity $\vec{v}_0 = V_0(\cos(\theta_0), \sin(\theta_0))$, etc. That is, find the set of equations that are particular to this problem.

(b) (2 points) Taking the initial speed of the ball as it leaves the mouth of the cannon $V_0$ to be known from a previous experiment, what is the range of speeds for the glider $V_{\text{glider}} > 0$ for which there is a solution. That is, for what range of speeds will the ball land in the bucket, provided it is aimed properly.

(c) (2 points) In order for the ball to land in the bucket, what must the cannon’s angle of elevation $\theta_0$ be?

(d) (4 points) At what horizontal distance $x_{\text{max}}$ from the mouth of the cannon will the ball land in the bucket?

Potential future problem: What would happen if the air track were not level?
Part IV

Newton’s Laws
10 Introduction to Newton’s Laws

10.1 Concept questions involving Newton’s three laws

Problem 1184. (Derived quantities) Determine the expression for the unit of force (the Newton N) in terms of the fundamental dimensions mass, length, and time from the equation expressing Newton’s 2nd Law \( F = ma \).

Problem 1185. An object is subjected to a nonzero constant net force \( \vec{F} \). Which of the following properties of the object will be constant?

(a) Position
(b) Speed
(c) Velocity
(d) Acceleration
(e) None of these

Problem 1186. (Similarity Problem) Consider the following two experiments. In the first experiment you push a block along a level surface at a constant velocity \( V \). To overcome the friction between the block and the surface, you must apply a constant horizontal force \( F_{\text{push},1} \). In the second experiment you push the block at a constant speed \( 2V \) across the same surface. Assuming that the friction is the same as it was the first experiment, which of the following choices best describes the new push force \( F_{\text{push},2} \) in terms of the first push force?

(a) A constant force \( F_{\text{push},1} \)
(b) A constant force \( 2F_{\text{push},1} \)
(c) A force that increases from \( F_{\text{push},1} \) to \( 2F_{\text{push},1} \)
(d) A force of zero, since the block does not accelerate.

Problem 1187. You are standing on a spring scale in an elevator that is moving at a constant speed \( V \). Before the elevator started moving, the scale registered your weight as \( W \). It now registers a weight of \( W_{\text{app}} \). What is the relationship between \( W_{\text{app}} \) and \( W \)?

(a) \( W_{\text{app}} < W \)
(b) \( W_{\text{app}} = W \)
(c) \( W_{\text{app}} > W \)
(d) Not enough information; it depends on the direction of \( \vec{V} \).

Problem 1188. You come into your physics lab and find three books stacked on a table, as shown at right. What is the net force on the middle book?

(a) 10 N downward
(b) 20 N upward
(c) 15 N downward
(d) 0 N
(e) None of these
**Problem 1189.** You are piloting your spaceship through interstellar space where there is no gravity and no air, when your timing belt breaks and your engine abruptly quits. This occurs at time \( T \), when you are travelling at speed \( V \). Which of the graphs below describes your motion starting at time \( T \)?

![Graphs](image)

**Problem 1190.** A piano with a weight of 4500 N is resting on a frictionless horizontal ice-covered parking lot. You push on the piano with a constant horizontal force of \( F \). Which of the following statements is true?

(a) The piano will accelerate, no matter how small \( F \) is.
(b) The piano will not accelerate unless \( F > 4500 \text{ N} \).
(c) The piano will move with constant velocity, because \( F \) is constant.
(d) The piano will move as long as you apply \( F \); once the force is removed, it will stop.

**Problem 1191.** You come into your physics lab and find three weights suspended by thin strings from the ceiling, as shown at right. Which of the following statements is true about the tensions \( T_1 \), \( T_2 \), and \( T_3 \) in the strings?

(a) \( T_1 = T_2 = T_3 \)
(b) \( T_1 > T_2 > T_3 \)
(c) \( T_3 > T_2 > T_1 \)
(d) \( T_3 > T_1 > T_2 \)

**Problem 1192.** Your physics instructor asks you to help him push an atomic bomb into the classroom. The bomb weighs 8000 N. You push on it with a horizontal force of 400 N. Which of the following statements is true?

(a) If the atomic bomb moves across the floor, then you feel it pushing back on you with a force of less than 400 N.
(b) You feel the atomic bomb pushing back on you with a force of 400 N, whether it moves or not.
(c) If the atomic bomb does not move, you feel it pushing back on you with a force of 8000 N.
(d) You feel the atomic bomb pushing back on you with a force of 8000 N, whether it moves or not.
Problem 1193. You are pushing two crates across a level frictionless floor, as shown at right. You apply a horizontal force of 300 N to crate A, which is heavier than crate B. Which of the following statements is true?

(a) Crate A pushes on crate B with a force of 300 N; crate B pushes on crate A with a force of 300 N.
(b) Crate A pushes on crate B with more force than crate B pushes on crate A.
(c) Crate A and crate B push on one another with forces that are equal to one another and less than 300 N.
(d) The two crates will not move if their weight is greater than 300 N.

Problem 1194. A spaceship is very far away from stars, planets, or other sources of gravitational force. It is moving at high speed through the intergalactic void when its rockets stop firing. Which of the following describes the spaceship’s future course?

(a) It will immediately stop, throwing the cargo and crew violently forward.
(b) It will begin slowing down, eventually coming to a complete stop in the vacuum of space.
(c) It will move at constant speed for a while, but will then start slowing down.
(d) It will keep on going forever at constant speed.
(e) None of these

Problem 1195. You are idly playing with a ball on a level floor. You give the ball a light push, and it rolls about halfway across the floor before slowing down and coming to a stop. Why does the ball slow and stop?

(a) Because you weren’t pushing it.
(b) Because speed is proportional to force.
(c) Because there must have been some force on it opposing the direction of its motion.
(d) Because the net force on the ball was zero.
(e) None of these

Problem 1196. Two objects with masses $M$ and $m$, where $M > m$, are on a level frictionless surface. If a force $F$ is applied to the smaller object, it will experience an acceleration of $a$. If that same force $F$ is applied to the larger object, what will happen to that object?

(a) It will experience an acceleration greater than $a$.
(b) It will experience an acceleration of $a$.
(c) It will experience an acceleration less than $a$.
(d) It will move only if the force $F$ is greater than some minimum value.
(e) None of these
Problem 1197. An object is moving northward with increasing speed. What do we know about the forces on the object?

(a) There is a single force on the object, directed northward.
(b) There is a net force on the object, directed northward.
(c) There may be several forces on the object, but the largest is directed northward.
(d) We know nothing about the forces on the object.
(e) None of these

Problem 1198. An object is moving northward. What do we know about the forces on the object?

(a) There is a single force on the object, directed northward.
(b) There is a net force on the object, directed northward.
(c) There may be several forces on the object, but the largest is directed northward.
(d) We know nothing about the forces on the object.
(e) None of these

Problem 1199. Which of the following objects is not experiencing a net force directed northward?

(a) An object moving southward with decreasing speed.
(b) An object moving northward with increasing speed.
(c) An object that is instantaneously at rest, and that then begins moving northward.
(d) An object moving northward with constant speed.
(e) None of these

Problem 1200. A rock is at rest on a level surface. The force that the surface exerts on the rock has magnitude $F_{SR}$; the force that the rock exerts on the surface has magnitude $F_{RS}$. What do we know about the two forces?

(a) $F_{SR} < F_{RS}$   (b) $F_{SR} = F_{RS}$   (c) $F_{SR} > F_{RS}$
(d) We don’t know anything about the relative sizes of the forces.
(e) None of these

Problem 1201. An object with a mass of 20 kg is suspended from two ropes, as shown at right. The magnitude of the force exerted by each rope on the object is 139 N; the magnitude of the force of gravity on the object is 196 N. What is the magnitude of the net force on the object?

(a) 47.4 N   (b) 33.5 N
(c) 13.9 N   (d) 0 N
(e) None of these
Problem 1202. An astronaut with a mass of 100 kg is trying to move a very small asteroid with a mass of 400 kg. The astronaut pushes on the asteroid with a force of 120 N. What force does the asteroid exert on the astronaut? Round your answer to the nearest newton.

(a) 30 N  
(b) 96 N  
(c) 120 N  
(d) 480 N  
(e) None of these

Problem 1203. You are pushing a crate with mass \( m \) across a floor. You are pushing forward with a force whose magnitude is \( F_{\text{push}} \); the force of friction is directed backward, with magnitude \( f \). If the crate is moving at a constant velocity, which of the following is true?

(a) \( F_{\text{push}} = f \)  
(b) \( F_{\text{push}} > f \)  
(c) \( F_{\text{push}} = mg + f \)  
(d) \( F_{\text{push}} > mg + f \)  
(e) None of these

Problem 1204. An object with a mass of 12 kg is suspended from the ceiling by two ropes of equal length, each making an angle of 74° to the horizontal. Each rope has a tension of 61 N. What is the magnitude of the net force on the object? Round your answer to the nearest newton.

(a) 0 N  
(b) 59 N  
(c) 118 N  
(d) 122 N  
(e) None of these

10.2 Free-body diagrams

Problem 1205. A book is given a push along a frictionless horizontal tabletop. After it has been pushed, it slides along the tabletop and off the edge.

(a) What are the forces acting on the book while it is being pushed along the table? What are the reaction forces to those forces?
(b) What are the forces acting on the book while it is sliding along the table after being pushed? What are the reaction forces to those forces?
(c) What are the forces acting on the book while it is falling from the table to the floor? What are the reaction forces to those forces?
Problem 1206. Two crates are connected by a thin horizontal rope and sitting on a horizontal floor. You are pulling them by a second horizontal rope attached to one of the crates. There is friction between the crates and the floor. Draw a free-body diagram for each of the crates.

10.3 Pushing and pulling objects on horizontal frictionless surfaces

Problem 1207. Two blocks are connected by a thin rope and sitting on a horizontal frictionless surface. You are pulling them by a second rope attached to the smaller block. You pull on your rope with a horizontal force of \( P \); the tension in the rope connecting the blocks is \( T \). Which of the following is true of \( P \) and \( T \)?

(a) \( P < T \)  
(b) \( P = T \)  
(c) \( P > T \)  
(d) Not enough information; it could be any of these

Problem 1208. You are pushing two blocks across a horizontal frictionless surface, as shown at right. You apply a horizontal force of 240 N to the smaller block. How much force does the smaller block exert on the larger one?

(a) 60 N  
(b) 120 N  
(c) 180 N  
(d) 240 N  
(e) None of these
11 Applications of Newton’s Laws to Linear Motion

11.1 Newton’s laws in one-dimension

11.1.1 One-dimensional kinematics and Newton’s second law

The idea behind these problems is add one more equation to our three fundamental kinematic equations:

\[
\{ F_{\text{net}} = ma \} + \begin{cases} 
\text{Fund. Eq}^n 1: \ v = v_0 + at \\
\text{Fund. Eq}^n 2: \ v^2 = v_0^2 + 2a(x - x_0) \\
\text{Fund. Eq}^n 3: \ x = x_0 + v_0 t + \frac{1}{2}at^2
\end{cases}
\]

This builds our system of possible fundamental equations to 4 equations:

\[
\begin{cases} 
\text{Fund. Eq}^n 1: \ v = v_0 + at \\
\text{Fund. Eq}^n 2: \ v^2 = v_0^2 + 2a(x - x_0) \\
\text{Fund. Eq}^n 3: \ x = x_0 + v_0 t + \frac{1}{2}at^2 \\
\text{Fund. Eq}^n 4: \ F_{\text{net}} = ma
\end{cases}
\]

with the catch that when mass or force is involved in a problem then fundamental equation 4 is always used together with one of the other three equations. That is, we are always solving a system of equations: \( F = ma \) plus one of the fundamental kinematic equations.
Problem 1209. A block with a mass of 132 kg is pulled with a horizontal force of \( \vec{F} \) across a rough floor. The coefficient of friction between the floor and the block is 0.51. If the block is moving at a constant velocity, what is the magnitude of \( \vec{F} \)? Round your answer to the nearest newton.

(a) 67 N  
(b) 259 N  
(c) 660 N  
(d) 2536 N  
(e) None of these

Problem 1210. A 5.5-kg block is initially at rest on a frictionless horizontal surface. It is pulled with with a constant horizontal force of 3.8 N. How long must it be pulled before its speed is 5.2 m/s?

(a) 2.5 s  
(b) 5.0 s  
(c) 7.5 s  
(d) 10.0 s  
(e) None of these

Problem 1211. A 523-kg experimental rocket sled can be accelerated from rest to 1620 km/h in 1.82 s. What is the net force required to produce this motion? Assume the applied force is constant.

(a) \( 1.29 \times 10^5 \) N  
(b) \( 6.34 \times 10^3 \) N  
(c) \( 2.74 \times 10^6 \) N  
(d) \( 1.29 \times 10^4 \) N  
(e) None of these

Problem 1212. A box has a mass of 160 kg. It is sitting on an icy sidewalk, which constitutes a horizontal frictionless surface. A horizontal force of 32 N is applied to the box. What is the box’s acceleration?

(a) 5 m/s\(^2\)  
(b) 0.2 m/s\(^2\)  
(c) 6.4 m/s\(^2\)  
(d) 0.16 m/s\(^2\)  
(e) None of these

Problem 1213. A box has a mass of 60 kg. It is sitting on an icy sidewalk, which constitutes a horizontal frictionless surface. A horizontal force of 30 N is applied to the box. What is the box’s acceleration?

(a) 15 m/s\(^2\)  
(b) 5 m/s\(^2\)  
(c) 2 m/s\(^2\)  
(d) 0.5 m/s\(^2\)  
(e) None of these

Problem 1214. You are trying to move a 150-lb crate across a floor by pulling on a rope inclined at 17\(^\circ\) to the horizontal. The coefficient of static friction is 0.52; the coefficient of kinetic friction is 0.35. You pull on the rope until the crate starts moving. What is the crate’s initial acceleration?

Problem 1215. A 10-kg block is initially at rest on a frictionless horizontal surface. It is pulled with with a constant horizontal force of 10 N until it reaches a final velocity of 10 m/s. How far does it move in this time?

(a) 5 m  
(b) 10 m  
(c) 100 m  
(d) 50 m  
(e) None of these
Problem 1216. (The problem of 0’s and 1’s) A 1.0-kg block is initially at rest on a frictionless horizontal surface. It is pulled with with a constant horizontal force of 1.0 N. How long must it be pulled before its speed is 1.0 m/s?
(a) 0.25 s  (b) 0.5 s  (c) .75 s  (d) 1.0 s  (e) None of these

Problem 1217. A crate of auto parts is sitting on a horizontal frictionless frozen lake. A worker applies a constant horizontal force of 750 N to the crate. Starting from rest, the crate moves 20 m in the first 4 s. What is the mass of the crate?
(a) 250 kg  (b) 300 kg  (c) 350 kg  (d) 400 kg  (e) None of these

Problem 1218. A ball with a mass of 0.4 kg is at rest on top of a tee when it is hit with a bat moving horizontally. The bat is in contact with the ball for 20 ms; when the ball leaves the bat, it is moving horizontally at 50 m/s. If the bat applies a constant force $F_{\text{net}}$ to the ball during the whole time that they are in contact, what is the magnitude of $F_{\text{net}}$?
(a) 400 N  (b) 1000 N  (c) 2500 N  (d) 4000 N  (e) None of these

Problem 1219. A 10.0-kg block is initially at rest on a frictionless horizontal surface. It is pulled with with a constant horizontal force of 10.0 N.
(a) How long must it be pulled before its speed is 100 m/s? Round your answer to the nearest second
(b) How far does it move in this time? Round your answer to the nearest meter.

Problem 1220. A 1-kg ketchup bottle is slid across a smooth counter at a local diner. You notice that the bottle slid 1.5 meters and took 2 seconds to come to rest. Assuming a constant net force owing to the friction on the table, what was the initial velocity of the bottle? Round your answer to the nearest 0.1 m/s.
(a) 5 m/s  (b) 0.5 m/s  (c) 1.5 m/s  (d) 0.15 m/s  (e) None of these

Problem 1221. A block of mass $M$ is pulled along a horizontal frictionless surface by a bar of mass $m$ that is attached to the block and horizontal to the surface. A horizontal force $\vec{F}$ (P for pull) is applied to the free end of the bar (see figure below).
(a) Find the acceleration of the bar-block system (i.e., the bar and the block taken together).
(b) What is the force that the bar exerts on the block?
11.1.2 Weight

Remember weight is defined as \( w = mg \), where \( m \) is mass and \( g \) is the magnitude of the acceleration due to gravity. \( g \) is always positive.

Problem 1222. The acceleration due to gravity at the surface of Neptune’s moon Triton is 2.41 m/s\(^2\). If a rock weighs 106 N on Triton, what is its mass? Round your answer to the nearest kilogram.

(a) 26 kg  (b) 44 kg
(c) 255 kg (d) 431 kg
(e) None of these

Problem 1223. Triton’s gravity is 2.41 m/s\(^2\). If a rock weighs 482 N on Triton, what is its mass? Round your answer to the nearest kilogram.

(a) 206 kg  (b) 200 kg
(c) 225 kg  (d) 22 kg
(e) None of these

Problem 1224. The acceleration due to gravity at the surface of Neptune is 11.15 m/s\(^2\). If a rock weighs 266 N on Neptune, what is its mass? Round your answer to the nearest 0.1 kg.

(a) 27.1 kg  (b) 2965.9 kg
(c) 0.4 kg   (d) 23.9 kg
(e) None of these

Problem 1225. The acceleration due to gravity at the surface of Planet X is 12.2 m/s\(^2\). If a rock weighs 307 N on Planet X, what is its mass? Round your answer to the nearest kilogram.

(a) 25 kg  (b) 247 kg
(c) 382 kg (d) 3745 kg
(e) None of these


(a) 81 kg  (b) 89 kg
(c) 98 kg  (d) 107 kg
(e) None of these

Problem 1227. An astronaut weighs 790 N on Earth. On Planet X, he weighs 640 N. What is the astronaut’s mass on Earth? Round your answer to the nearest kilogram.

(a) 73 kg  (b) 81 kg
(c) 89 kg  (d) 98 kg
(e) None of these
Problem 1228. (Similarity Problem) An astronaut weighs 790 N on Earth. On Planet X, he weighs 640 N. What is the gravitational acceleration on Planet X? Round your answer to the nearest 0.1 m/s$^2$.

(a) 5.8 m/s$^2$  (b) 6.4 m/s$^2$
(c) 7.1 m/s$^2$  (d) 7.9 m/s$^2$
(e) None of these

Problem 1229. A uniform chain is 2.0 m long and has a mass of 20 kg. It hangs from the ceiling and supports a lamp with a mass of 60 kg. What is the tension in the top link of the chain? For ease of calculation, assume that $g = 10$ m/s$^2$.

(a) 200 N  (b) 600 N
(c) 700 N  (d) 800 N
(e) None of these

Problem 1230. A uniform chain is 2.0 m long and has a mass of 20 kg. It hangs from the ceiling and supports a lamp with a mass of 60 kg. What is the tension in the middle link of the chain? For ease of calculation, assume that $g = 10$ m/s$^2$.

(a) 100 N  (b) 600 N
(c) 700 N  (d) 800 N
(e) None of these

Problem 1231. A uniform chain is 2.0 m long and has a mass of 20 kg. It hangs from the ceiling and supports a lamp with a mass of 60 kg. What is the tension in the bottom link of the chain? For ease of calculation, assume that $g = 10$ m/s$^2$.

(a) 200 N  (b) 600 N
(c) 700 N  (d) 800 N
(e) None of these

Problem 1232. (Force-to-weight ratio) A softball is at rest on top of a tee when it is hit with a bat moving horizontally. The bat is in contact with the ball for 20 ms; when the ball leaves the bat, it is moving horizontally at 50 m/s. If the bat applies a constant force $F_{\text{net}}$ to the ball during the whole time that they are in contact, what is the force-to-weight ratio $F_{\text{net}}/w_{\text{ball}}$ for the ball? For ease of calculation, assume that $g = 10$ m/s$^2$.

(a) 250  (b) 500
(c) 1250  (d) 2500
(e) None of these
Problem 1233. (Determining the weight of an object without a scale) You have just landed on Planet X and realized that you’ve left your scale behind. However, you have a 5.0 kg ball, and a spring cannon that can launch the ball straight upward at a speed of 15 m/s. You launch the ball upward from ground level; it strikes the ground again 3.0 s after launching. What is the magnitude of gravity on Planet X? Use this to determine the weight of the object. Round your answer to the nearest newton.

(a) 40 N (b) 45 N
(c) 50 N (d) 55 N
(e) None of these

Problem 1234. You are standing on a 100 m tall cliff on Planet X. You drop a 5 kg stone off the cliff, and notice that it takes 10 seconds for it to hit the ground below. What is the magnitude of gravity $g_X$ on planet X? You may assume that you can ignore air drag.

(a) 1.0 m/s$^2$ (b) 2.0 m/s$^2$
(c) 5.0 m/s$^2$ (d) 10.0 m/s$^2$
(e) None of these
11.1.3 Apparent weight problems (The Elevator Equation)

The apparent weight of a mass that is accelerating (upward or downward) with the $y$-axis taken pointing upward is

$$w_{app} = w \left(1 + \frac{a_y}{g}\right)$$

Problem 1235. A block is hanging from a spring scale that is attached to the ceiling of an elevator. Under which of the following circumstances will the reading on the scale be less than the weight of the block?

(a) The elevator is moving downward and slowing down.
(b) The elevator is moving upward and slowing down.
(c) The elevator is moving downward at constant speed.
(d) The elevator is moving upward at constant speed.

Problem 1236. A block is hanging from a spring scale that is attached to the ceiling of an elevator. What will the reading on the scale be if the elevator cable breaks?

Problem 1237. An object is hung from a spring scale attached to the ceiling of an elevator. The scale reads 100 N when the elevator is sitting still. What is the reading on the scale if the elevator is moving downward with a constant speed of 10 m/s? Approximate $g$ as 10 m/s$^2$.

(a) 0 N  
(b) 50 N
(c) 100 N  
(d) 200 N
(e) None of these

Problem 1238. What is the apparent weight of a 150 N block if it is accelerating upward at 1 m/s$^2$? For ease of calculation, assume that $g = 10$ m/s$^2$.

(a) 20 N  
(b) 135 N
(c) 150 N  
(d) 165 N
(e) None of these

Problem 1239. What is the apparent weight of a 150 N block if it is accelerating downward at 1 m/s$^2$? For ease of calculation, assume that $g = 10$ m/s$^2$.

(a) 20 N  
(b) 135 N
(c) 150 N  
(d) 165 N
(e) None of these

Problem 1240. What is the apparent weight of a 150 N block if it is moving upward at a constant velocity of 1 m/s? For ease of calculation, assume that $g = 10$ m/s$^2$.

(a) 20 N  
(b) 135 N
(c) 150 N  
(d) 165 N
(e) None of these
Problem 1241. What is the weight of a 150 N block if it is accelerating downward at 1 m/s²? For ease of calculation, assume that \( g = 10 \text{ m/s}^2 \).

(a) 20 N  
(b) 135 N  
(c) 150 N  
(d) 165 N  
(e) None of these

Problem 1242. An elevator weighing 6200 lb is pulled upward by a cable, with an upward acceleration of 3.2 ft/s². What is the tension in the cable? Round your answer to the nearest 100 lb.

(a) 218,200 lb  
(b) 19,800 lb  
(c) 6800 lb  
(d) 600 lb  
(e) None of these

Problem 1243. A bucket weighing 500 N is lifted with an upward acceleration of 2 m/s² by a uniform chain weighing 200 N. What is the tension in the top link of the chain? For ease of calculation, assume that \( g = 10 \text{ m/s}^2 \).

(a) 360 N  
(b) 720 N  
(c) 840 N  
(d) 1080 N  
(e) None of these

Problem 1244. A bucket weighing 500 N is lifted with an upward acceleration of 2 m/s² by a uniform chain weighing 200 N. What is the tension in the middle link of the chain? For ease of calculation, assume that \( g = 10 \text{ m/s}^2 \).

(a) 600 N  
(b) 720 N  
(c) 840 N  
(d) 960 N  
(e) None of these

Problem 1245. A bucket weighing 500 N is lifted with an upward acceleration of 2 m/s² by a uniform chain weighing 200 N. What is the tension in the bottom link of the chain? For ease of calculation, assume that \( g = 10 \text{ m/s}^2 \).

(a) 600 N  
(b) 660 N  
(c) 720 N  
(d) 840 N  
(e) None of these

Problem 1246. You are standing on a spring scale on an elevator. When the elevator is not moving, the scale registers a weight of 600 N. When the elevator starts moving, it has an upward acceleration of 2 m/s². What weight does the scale register while the elevator is accelerating? For ease of calculation, assume that \( g = 10 \text{ m/s}^2 \).

(a) 360 N  
(b) 500 N  
(c) 720 N  
(d) 1200 N  
(e) None of these
Problem 1247. You are standing on a spring scale on an elevator. When the elevator is not moving, the scale registers a weight of 730 N. When the elevator starts moving, it has an upward acceleration of 1.6 m/s². What weight does the scale register while the elevator is accelerating? Round your answer to the nearest 10 N.

(a) 760 N  (b) 850 N  
(c) 930 N  (d) 1030 N  
(e) None of these

Problem 1248. Someone has left a spring scale on an elevator. While the elevator is not moving, you get on the scale and see that your weight is 810 N. When the elevator starts moving, you see that the scale registers 890 N. What is the acceleration of the elevator? Round your answer to the nearest 0.1 m/s²; use a positive sign if the acceleration is upward, a negative sign if the acceleration is downward.

(a) 1.0 m/s²  (b) 10.8 m/s²  
(c) −1.0 m/s²  (d) −10.8 m/s²  
(e) None of these

Problem 1249. Someone has left a spring scale in an express elevator. While the elevator is standing still, you get on the scale and see that you weigh 142 lb. You ride the elevator nonstop to the top floor of the building. In the middle of the ride, you are going upward at a constant speed of 44 ft/s. What weight does the scale register at this point? Round your answer to the nearest pound.

(a) 103 lb  (b) 142 lb  
(c) 154 lb  (d) 195 lb  
(e) None of these

Problem 1250. A lamp with a mass of 7.0 kg is hanging from a thin cord attached to the ceiling of an elevator. When the elevator is accelerating upward at 3 m/s², what is the tension in the cord? For ease of calculation, assume that $g = 10$ m/s².

(a) 21 N  (b) 49 N  
(c) 70 N  (d) 91 N  
(e) None of these

Problem 1251. A block with weight $w$ is hanging from a spring scale that is attached to the ceiling of an elevator. If the elevator has an upward acceleration of $a > 0$ and the reading on the scale is $T$, what is $w$ as a function of $a$ and $T$?

(a) $w = \frac{Tg}{g - a}$  (b) $w = \frac{T(g - a)}{g}$  
(c) $w = \frac{Tg}{g + a}$  (d) $w = \frac{T(g + a)}{g}$  
(e) None of these
Problem 1252. A block with weight $w$ is hanging from a spring scale that is attached to the ceiling of an elevator. If the elevator has an upward acceleration of $a > 0$, what is the reading on the scale $T$?

(a) $T = w \frac{a}{g}$  
(b) $T = w \frac{g}{a}$

(c) $T = w \left(1 + \frac{g}{a}\right)$  
(d) $T = w \left(1 + \frac{a}{g}\right)$

(e) None of these
11.1.4 Spring Problems

Problem 1253. (Derived quantities) Determine the units of the spring constant $k$ from the equation for the ideal spring force: $F_{spring} = -kx$, where $F$ is force, $k$ is the spring constant, and $x$ is the displacement from the equilibrium position. Write the answer in terms of Newtons.

Problem 1254. A rubber band has an unstretched length of 8 cm and a spring constant of 4 N/cm. What is the force required to stretch it to a length of 12 cm?

- (a) 3 N
- (b) 16 N
- (c) 32 N
- (d) 48 N
- (e) None of these

Problem 1255. A spring is hanging from the ceiling. Its unstretched length is 12 cm. When a block with a mass of 6 kg is hanging from the end, it stretches to a length of 15 cm. What is the spring constant of the spring? For ease of calculation, assume that $g = 10 m/s^2$.

- (a) 4 N/m
- (b) 20 N/m
- (c) 400 N/m
- (d) 2000 N/m
- (e) None of these

Problem 1256. A spring is hanging from the ceiling. Its unstretched length is 10 cm. When a block with a mass of 8 kg is attached to the end, it stretches to a length of 20 cm. If you want to stretch the spring to a length of 30 cm, how much mass must you hang from it?

- (a) 8 kg
- (b) 12 kg
- (c) 16 kg
- (d) 24 kg
- (e) None of these

Problem 1257. A horizontal spring with a spring constant of 500 N/m is used to pull a sled across a frictionless frozen lake. The sled has a mass of 40 kg, and is undergoing an acceleration of 0.5 m/s$^2$. How much does the spring stretch while pulling the sled? For ease of calculation, use $g = 10 m/s^2$.

- (a) 2.5 cm
- (b) 4 cm
- (c) 10 cm
- (d) 25 cm
- (e) None of these

Problem 1258. An unstretched spring has a length of 22.8 cm. When an object with a mass of 12.0 kg is hung from it, it stretches to a length of 35.5 cm. What is the spring constant of the spring? Round your answer to the nearest N/m.

- (a) 34 N/m
- (b) 94 N/m
- (c) 331 N/m
- (d) 926 N/m
- (e) None of these
Problem 1259. An unstretched spring is 62 cm long. When you hang an object with a mass of 4.6 kg from it, it stretches to a length of 71 cm. What is the mass of the object you would have to hang from the spring to get it to stretch to a length of 90 cm? Round your answer to the nearest 0.1 kg.

(a) 1.8 kg  (b) 5.8 kg
(c) 14.3 kg  (d) 450.8 kg
(e) None of these

Problem 1260. A bungee cord obeys Hooke’s law, with a spring constant of 850 N/m. It is attached to a crate with a mass of 170 kg and pulled at an angle of 19° to the horizontal, in order to move the crate across a horizontal frictionless floor. If the crate accelerates at 0.81 m/s², how much does the bungee cord stretch? Round your answer to the nearest centimeter.

(a) 16 cm  (b) 17 cm
(c) 159 cm  (d) 168 cm
(e) None of these

Problem 1261. A bungee cord obeys Hooke’s law, with a spring constant of 620 N/m. It is attached to a crate with a mass of 220 kg and pulled at an angle of 24° to the horizontal, in order to move the crate across a horizontal frictionless floor. If the crate accelerates at 0.72 m/s², how much does the bungee cord stretch? Round your answer to the nearest centimeter.

(a) 13 cm  (b) 14 cm
(c) 26 cm  (d) 28 cm
(e) None of these

Problem 1262. (Lab Problem) A spring is hanging from the ceiling in the physics lab. When there is no weight hanging from it, the spring is 42 cm long; it has a spring constant of 8300 N/m. You hang a lead block with a mass of 87 kg from the spring, stretching it. What is the length of the spring with the weight hanging from it? Round your answer to the nearest centimeter.

(a) 10 cm  (b) 21 cm
(c) 32 cm  (d) 52 cm
(e) None of these

Problem 1263. (Lab Problem) An unstretched spring is 62 cm long. When you hang an object with a mass of 4.6 kg from it, it stretches to a length of 71 cm. What is the mass of the object you would have to hang from the spring to get it to stretch to a length of 90 cm? Round your answer to the nearest 0.1 kg.

(a) 1.8 kg  (b) 5.8 kg
(c) 14.3 kg  (d) 450.8 kg
(e) None of these
Problem 1264. (Derive Problem) Two air-track gliders of mass $m_1$ and $m_2$ with $m_1 < m_2$ are connected by a spring whose mass is negligible compared to the mass of the gliders. The gliders sit on a horizontal frictionless air-track and are constrained to one-dimensional motion. Treat the problem as a one-dimensional problem by assuming that the force of gravity is exactly cancelled by the upward blowing air of the air track.

The system: Take the pair of gliders together with the spring to be our system under investigation. There are no external forces acting on the system in the direction of the track.

The initial condition: The system is initially at rest with the spring compressed and held in place by a thin massless string connecting the two gliders. At time $t = 0$, the string is cut setting the mass-spring system in motion.

The final condition:
(a) If at time $t_f > 0$ the acceleration of mass $m_2$ is $a_2$, then what is the force on $m_2$ by the spring, denoted $F_{m_2s}$?
(b) What is the acceleration of mass $m_1$ at time $t_f$?

Comment: After the string is cut, the energy stored in the spring sets the system in motion. The motion comes from internal forces, there are no external forces acting on the mass-spring system.
11.2  Newton’s laws in two-dimensions

11.2.1  Static equilibrium problems

**Problem 1265.** Two nails are driven into a wall at different heights, and a weight is hung from two thin strings fastened to the nails, as shown at right. Both strings make the same angle $\theta$ to the horizontal, but the first string is longer than the second. If $T_1$ is the tension in the longer string and $T_2$ is the tension in the shorter, what is the relationship between $T_1$ and $T_2$?

(a) $T_1 < T_2$  (b) $T_1 = T_2$
(c) $T_1 > T_2$  (d) Not enough information: could be any of these

**Problem 1266.** A rope is stretched between two poles. The rope is initially horizontal. After an 82 N block is hung from the middle of the rope each side of the rope makes an angle of $9.5^\circ$ with the horizontal. What is the tension in the rope? Round your answer to the nearest 10 N.

(a) 40 N  (b) 50 N
(c) 80 N  (d) 250 N
(e) None of these

**Problem 1267.** In an attempt to keep bears from eating his food, a hiker stretches a rope between two trees and hangs the food from the middle of the rope. When the food is hung from the rope, each side of the rope makes an angle of $11.4^\circ$ with the horizontal. The rope will break if its tension exceeds 220 N. What is the maximum weight of food $w_{\text{max}}$ that the hiker can hang without breaking the rope? Round your answer to the nearest newton.

(a) 43 N  (b) 87 N
(c) 216 N  (d) 432 N
(e) None of these
Problem 1268. In an attempt to keep bears from eating his food, a hiker stretches a rope between two trees and hangs the food from the rope. The food weighs 82 N; each side of the rope makes an angle of 9.5° with the horizontal. What is the tension in the rope? Round your answer to the nearest 10 N.

(a) 40 N  
(b) 50 N  
(c) 80 N  
(d) 250 N  
(e) None of these

Problem 1269. Two ropes are attached to the ceiling, and their free ends are tied together. A block with weight \( w \) is hung from the point where the ropes are joined. Rope A makes an angle of 51° to the horizontal; rope B makes an angle of 68° to the horizontal. The tension in rope B is \( T_B = 138 \) N. What is the weight \( w \) of the block? Round your answer to the nearest newton.

Problem 1270. A hanging mass with weight \( w \) is suspended by two thin ropes, labelled A and B. Rope A makes an angle of \( \theta_A \) with the horizontal; rope B makes an angle of \( \theta_B \) with the horizontal. The tension in rope A is \( T_A \); the tension in rope B is \( T_B \). Which of the following equations represents the condition for static equilibrium of the hanging-mass system in the horizontal direction?

(a) \( T_A \cos \theta_A = T_B \cos \theta_B \)  
(b) \( T_A \sin \theta_A = T_B \sin \theta_B \)  
(c) \( T_A \cos \theta_B = T_B \cos \theta_A \)  
(d) \( T_A \sin \theta_B = T_B \sin \theta_A \)  
(e) None of these
**Problem 1271.** A hanging mass with weight $w$ is suspended by two thin ropes, labelled $A$ and $B$. Rope $A$ makes an angle of $\theta_A$ with the horizontal; rope $B$ makes an angle of $\theta_B$ with the horizontal. The tension in rope $A$ is $T_A$; the tension in rope $B$ is $T_B$. Which of the following equations represents the condition for static equilibrium of the hanging-mass system in the vertical direction?

(a) $w = T_A \cos \theta_A + T_B \cos \theta_B$  
(b) $w = \frac{T_A}{\cos \theta_A} + \frac{T_B}{\cos \theta_B}$

(c) $w = T_A \sin \theta_A + T_B \sin \theta_B$  
(d) $w = \frac{T_A}{\sin \theta_A} + \frac{T_B}{\sin \theta_B}$

(e) None of these

**Problem 1272.** A block with weight $w = 130$ N is suspended by two thin ropes, labelled $A$ and $B$. Rope $A$ makes an angle of $\theta_A = 37^\circ$ with the horizontal; rope $B$ makes an angle of $\theta_B = 54^\circ$ with the horizontal. The tension in rope $A$ is $T_A$; the tension in rope $B$ is $T_B$. Find $T_A$; round your answer to the nearest newton.

(a) 62 N  
(b) 69 N

(c) 76 N  
(d) 85 N

(e) None of these

**Problem 1273.** A block with weight $w = 130$ N is suspended by two thin ropes, labelled $A$ and $B$. Rope $A$ makes an angle of $\theta_A = 37^\circ$ with the horizontal; rope $B$ makes an angle of $\theta_B = 54^\circ$ with the horizontal. The tension in rope $A$ is $T_A$; the tension in rope $B$ is $T_B$. Find $T_B$; round your answer to the nearest newton.

(a) 93 N  
(b) 104 N

(c) 115 N  
(d) 128 N

(e) None of these
Problem 1274. (Lab Derive Problem) You walk into the lab and find a block hanging from the ceiling suspended by two threads as shown in the figure to the right. Using a scale, you find the weight of the block $w_{\text{block}}$. Using a protractor, you find the angle between the ceiling and the left-hand string to be $\theta_L$. Similarly, $\theta_R$ is the angle between the ceiling and the right-hand thread. From this information determine the tensions in the threads: $T_L$ and $T_R$.

Problem 1275. Two blocks with weights $w_A$ and $w_B$ are held in place on a frictionless inclined plane by two thin ropes, as shown at right. One rope connects block A to a wall at the top of the plane; a second rope connects block B to block A. Both ropes are parallel to the inclined plane, which makes an angle of $\alpha$ with the horizontal. What is the tension $T_{AB}$ in the rope that connects blocks A and B?

(a) $w_B \cos \alpha$   (b) $w_B \sin \alpha$
(c) $(w_A + w_B) \cos \alpha$   (d) $(w_A + w_B) \sin \alpha$
(e) None of these

Problem 1276. Two blocks with weights $w_A$ and $w_B$ are held in place on a frictionless inclined plane by two thin ropes, as shown at right. One rope connects block A to a wall at the top of the plane; a second rope connects block B to block A. Both ropes are parallel to the inclined plane, which makes an angle of $\alpha$ with the horizontal. What is the tension $T_{Awall}$ in the rope that connects block A to the wall?

(a) $w_A \cos \alpha$   (b) $w_A \sin \alpha$
(c) $(w_A + w_B) \cos \alpha$   (d) $(w_A + w_B) \sin \alpha$
(e) None of these
Problem 1277. Two blocks with weights $w_A$ and $w_B$ are held in place on a frictionless inclined plane by two thin ropes, as shown at right. One rope connects block A to a wall at the top of the plane; a second rope connects block B to block A. Both ropes are parallel to the inclined plane, which makes an angle of $\alpha$ with the horizontal. What is the magnitude of the force that the plane exerts on block B?

(a) $w_B \cos \alpha$
(b) $w_B \sin \alpha$
(c) $(w_A + w_B) \cos \alpha$
(d) $(w_A + w_B) \sin \alpha$
(e) None of these

Problem 1278. Two blocks with weights $w_A$ and $w_B$ are held in place on a frictionless inclined plane by two thin ropes, as shown at right. One rope connects block A to a wall at the top of the plane; a second rope connects block B to block A. Both ropes are parallel to the inclined plane, which makes an angle of $\alpha$ with the horizontal. What is magnitude of the force that the plane exerts on block A?

(a) $w_A \cos \alpha$
(b) $w_A \sin \alpha$
(c) $(w_A + w_B) \cos \alpha$
(d) $(w_A + w_B) \sin \alpha$
(e) None of these

Problem 1279. Two forces with magnitudes $F_1$ and $F_2$ are acting on an object. Which of the following inequalities always hold for the magnitude of the net force $F_{\text{net}}$ on the object?

(a) $F_1 \leq F_{\text{net}} \leq F_2$
(b) $(F_1 - F_2)/2 \leq F_{\text{net}} \leq (F_1 + F_2)/2$
(c) $|F_1 - F_2| \leq F_{\text{net}} \leq F_1 + F_2$
(d) $F_1^2 - F_2^2 \leq F_{\text{net}}^2 \leq F_1^2 + F_2^2$
(e) None of these

Problem 1280. An object with a mass of 20 kg is suspended from two ropes, as shown at right. The magnitude of the force exerted by each rope on the object is 139 N; the magnitude of the force of gravity on the object is 196 N. What is the magnitude of the net force on the object?

(a) 47.4 N
(b) 33.5 N
(c) 13.9 N
(d) 0 N
(e) None of these
11.2.2 Block and pulley frictionless systems

**Key Idea:** Frictionless block-and-pulley systems are just one-dimensional systems in disguise! The system sits in two dimensions, but can be converted to a one-dimensional system. The background space is known as the ambient space and the system can be shown to be one dimensional by mathematical arguments from a field known as Topology. We will just take these topological arguments as fact.

**Problem 1281.** A rope, one end of which is attached to a ceiling, passes through a pulley and then goes straight upward where it is held in place by a man that is standing on the ground. A weight of 400 N is attached to the pulley. Assuming that the weight of the pulley can be neglected in comparison to the 400 N weight and that there is no friction between the pulley and the rope, what upward force must the man exert to hold the weight in place?

(a) 800 N  (b) 640 N  
(c) 400 N  (d) 200 N  
(e) None of these

**Problem 1282.** Two crates are connected by a thin rope and sitting on a frictionless horizontal floor. You are pulling them by a second rope attached to the smaller crate. You pull on your rope with a horizontal force of $P$; the tension in the rope connecting the crates is $T$. Which of the following is true of $P$ and $T$?

(a) $P < T$  (b) $P = T$  
(c) $P > T$  (d) Not enough information; it could be any of these

**Problem 1283.** In your physics lab, you find the apparatus shown at right. A block with a mass of $m_1$ is resting on a horizontal table. A light string runs horizontally from the block, over a massless frictionless pulley, and down to a hanging bucket. The coefficient of static friction between the block and the table is $\mu_s$, and the coefficient of kinetic friction between the block and the table is $\mu_k$. Initially, the system does not move. You slowly add water to the bucket until the system starts moving. At this point, what is the weight $w$ of the bucket?

(a) $m_1$  (b) $m_1 g$  
(c) $\mu_s m_1 g$  (d) $\mu_k m_1 g$
**Problem 1284.** In your physics lab, you find the apparatus shown at right. A block with a weight of 60 N is resting on a horizontal table. A light string runs horizontally from the block, over a massless frictionless pulley, and down to a hanging bucket. The coefficient of static friction between the block and the table is $\mu_s = 0.6$, and the coefficient of kinetic friction between the block and the table is $\mu_k = 0.2$. Initially, the system does not move. You slowly add water to the bucket until the system starts moving. At this point, what is the weight $w$ of the bucket?

(a) 12 N  (b) 36 N  
(c) 48 N  (d) 60 N

**Problem 1285.** In your physics lab, you find the apparatus shown at right. A glider with a weight of $m_1$ is resting on a horizontal air-track with the air off. A light string runs horizontally from the glider, over a massless frictionless pulley, and down to a hanging block with a weight of $m_2$. The coefficient of static friction between the glider and the air-track is $\mu_s$, and the coefficient of kinetic friction between the glider and the air-track is $\mu_k$. The coefficients of friction are assumed known. After the glider-string-block system have been released to move freely, and before the hanging block has hit the floor, what is the tension $T$ in the string?

(a) $\left( \frac{m_1}{m_1 + m_2} \right)(1 + \mu_k)g$  
(b) $\left( \frac{m_1 m_2}{m_1 + m_2} \right)(1 + \mu_k)g$

(c) $\left( \frac{m_1 m_2}{m_1 + m_2} \right)(1 - \mu_k)g$  
(d) $\left( \frac{m_1 + m_2}{m_1 m_2} \right)(1 + \mu_k)g$
Problem 1286. (Lab Problem) This problem has a fundamental mistake in it, can you find it?
A glider is sitting at rest on a horizontal frictionless air track. A light string is attached to a light string that runs over a massless frictionless pulley and attaches to a hanging mass of 1 kg. The system is held in place and released from rest. Using photo gates that are placed 1 m apart the time is measured to be 2 s. Assuming that the string remains horizontal and taut throughout the entire run, what is the mass of the glider? For ease of computation take $g$ to be 10 m/s$^2$. Round your answer to the nearest kilogram.

(a) 1 kg  
(b) 2 kg  
(c) 4 kg  
(d) 19 kg  
(e) None of these

**Solution:** The massless, frictionless pulley justifies the assumption that the tension is the same throughout the string, from which it follows that there is a constant horizontal force on the glider of $mg = 10$ N from the hanging weight. This force has been transmitted through the string. Recall that a pulley is a machine that redirects force. The light string means that we can ignore the mass of the string.

If you sketch a free-body diagram of the glider, you’ll see that the only horizontal force is the constant 1 N. There’s no net vertical force.

**Given:** $F_{net} = 1$ N; $v_i = 0$ m/s; $\Delta x = 1$ m; and $t = 2$ s;
**Want:** $m = ?$

**Note:** We don’t know, nor do we want $v_f$.

If we try to solve for $m$ directly using Newton’s second law, we get stuck:

$$F_{net} = ma \quad \rightarrow \quad m = \frac{F_{net}}{a} = \frac{1 \text{ N}}{a} \quad (11.1)$$

We need to use one of our three fundamental kinetic equations to find $a$. Since we know $\Delta x$, $v_i = 0$, and $t$, we can use

$$\Delta x = v_i t + \frac{1}{2}at^2 = \frac{at^2}{2} \quad \rightarrow \quad a = \frac{2\Delta x}{t^2} \quad (11.2)$$

Substituting equation (11.2) into equation (11.1) gives us

$$m = \frac{F_{net}t^2}{2\Delta x} = \frac{(1 \text{ N})(2 \text{ s})^2}{2(1 \text{ m})} = 2 \text{ kg}$$
Problem 1287. In your physics lab, you find the apparatus shown at right. A tank of water is resting on a horizontal frictionless table. A light string runs horizontally from the tank, over a massless frictionless pulley, and down to a hanging block with a mass of $m_1$. You add increasing amounts of water to the tank, each time releasing the system and observing its motion or lack thereof. How does the system behave as you increase the mass $m_2$ of the tank?

(a) The acceleration does not change, since there is no friction and $m_1$ remains constant.
(b) The system accelerates as long as $m_2 < m_1$; when $m_2 = m_1$, the system no longer accelerates.
(c) The system moves as long as $m_2 < m_1$; when $m_2 = m_1$, the system no longer moves.
(d) The system always accelerates; but the acceleration gets smaller as $m_2$ becomes larger.
Problem 1288. (Derive Problem) In your physics lab, you find the apparatus shown at right. A block with mass \( m_1 \) is resting on a horizontal table. A light string runs horizontally from the block, over a massless frictionless pulley, and down to a hanging bucket. The coefficient of static friction between the block and the table is \( \mu_s \), and the coefficient of kinetic friction between the block and the table is \( \mu_k < \mu_s \). Initially, the system does not move. You slowly add water to the bucket until the system just starts to move. Call this moment in time \( t = t_0 = 0 \); at this time, call the mass of the bucket of water \( m_2 \).

**Setup:** For the block, take the \( x_1 \)-axis in the direction of motion along the table and the \( y_1 \)-axis going upward. Take the \( y_2 \)-axis pointing downward, in the direction of motion. Let the origin along the \( x_1 \)-axis be the initial position of the block with mass \( m_1 \) \( (x_{1,i} = 0) \). The initial velocity of mass \( m_1 \) is zero (at \( t = 0 \), \( v_{1,i} = 0 \)).

(a) Find the value of \( m_2 \): that is, find \( m_2 \) such that the force of gravity on the hanging bucket is just enough to overcome the force of static friction on the block with mass \( m_1 \).

(b) Find a formula for the initial acceleration of the system, denoted \( a_{sys} \).

(c) Assuming that the initial position and velocity of the block with mass \( m_1 \) is zero, find an equation for its position as a function of time: \( x_1 = x_1(t) \). Ignore the fact that the block will eventually crash into the pulley.
**Problem 1289. (Derive Problem)** In your physics lab, you find the apparatus shown below. A glider with mass $m_1$ is resting on a horizontal air track. A light string runs horizontally from the glider, over a massless frictionless pulley, and down to a hanging bucket.

The air track is specially designed so that the coefficient of static friction between the glider and the air track is $\mu_s > 0$ (a constant), and the coefficient of kinetic friction between the block and the table is $\mu_k = \mu_s e^{-x}$ (a function of position) provided that the front of the glider is initially at rest at a location marked as the origin on the air track. The coefficient of static friction between the block and the table is $\mu_s$ (a constant), and the coefficient of kinetic friction between the block and the table is $\mu_k = \mu_s e^{-x}$ (a function of position). Initially, the system does not move. You slowly add water to the bucket until the system just starts to move. Call this moment the initial time $t_0 = 0$; at this time, call the mass of the bucket of water $m_2$. Define the initial position of the center of mass for block $m_1$ to be the origin and take the initial velocity to be zero, since the block starts from rest.

![Diagram of the apparatus](image)

**The Coordinate System:** For the glider, take the $x_1$-axis in the direction of motion along the table and the $y_1$-axis going upward. Take the $y_2$-axis pointing downward, in the direction of motion. Let the origin along the $x_1$-axis be the initial position of the glider with mass $m_1$ ($x_{1,i} = 0$). Since the direction of motion is one dimensional, you can simplify things by letting $x = x_1 = y_2$.

**The Setup and Initial Conditions:** Take the initial position of the glider to be at the origin defined above (i.e., $x_{1,i} = 0$) and the initial velocity of the two-mass-plus-string system to be zero at time $t_0 = 0$. Take the point of zero potential to be $y_2 = \text{initial height of the hanging mass } m_2$. This is equivalent to taking $x = 0$ as the point of zero potential.

**The Problem:** Find an equation for the mechanical energy of the system as a function of time and position:

$$K.E_{\text{sys}}(x, t) + P.E_{\text{sys}}(x, t) = D(x, t),$$
where $D(x,t)$ is a dissipation term. Ignore the fact that the glider will eventually crash into the pulley, or the hanging mass will hit the ground. Your model is only designed to describe the motion in the interim time.

Comments, Advice, and Hints:

Comment (1): The first step is to determine if the system will even move! To do this you will need to find the force exerted by the hanging weight $m_2$ that overcomes the force of static friction. That is, find $m_2$ such that the force of gravity on the hanging bucket plus water is just enough to overcome the force of static friction $f_{s,\text{max}}$ on the block with mass $m_1$.

Comment (2): Notice that the coefficient of kinetic friction $\mu_k$ is a function of position! We have never solved such a problem in class! However, we have solved a similar equation in the new lab 6.

Comment (3): Be sure to clearly label all of the intermediate steps in comments (1)-(3). If I have to go searching for them, I will take points off.

Advice: To find the kinetic and potential energy, and any possible dissipation term for the system, as a function of position and time, you will need to first find a formula for the initial acceleration of the system $a_{\text{sys}}$ as a function of time $t$ and position $x$ and integrate the system using the initial conditions. When you get your result, ask yourself what is the source of the kinetic energy of the system, and what is the source of the potential energy of the system. Also, ask yourself: is the system mechanically conservative? If not, then what are the sources of energy loss? Can you identify these terms in the equation?

Hint: To perform one integration on a differential equation of the form:

$$\frac{d^2x}{dt^2} + \frac{dF(x(t))}{dx} = \text{constant},$$

The standard trick is to multiply the equation by the velocity term $v = \dot{x}$ and integrate the equation using the chain rule together with the Fundamental Theorem of Calculus (FTC). With this hint, you can solve the resulting equation with the techniques that you learn in Calculus 1 (a prerequisite for the course).
Problem 1290. Two identical blocks, each with mass $m$, are attached to opposite ends of a thin rope that passes over a massless frictionless pulley. The pulley is suspended from the ceiling by a thin chain. Draw a free-body diagram for each block, and one for the block-pulley system.

Problem 1291. Two identical blocks, each with mass $m$, are attached to opposite ends of a thin rope that passes over a massless frictionless pulley. The pulley is suspended from the ceiling by a thin chain. What is the tension in the chain holding up the pulley-block system?
(a) 0  (b) $mg/2$
(c) $mg$  (d) $2mg$

Problem 1292. Two identical blocks, each with mass $m$, are attached to opposite ends of a thin rope that passes over a massless frictionless pulley. The pulley is suspended from the ceiling by a thin chain. What is the tension in the rope?
(a) 0  (b) $mg/2$
(c) $mg$  (d) $2mg$

Problem 1293. In your physics lab, you find the apparatus at right. A block with mass $M_1$ is on a frictionless slope. A thin string from the block runs parallel to the slope, up to and over a massless frictionless pulley, then down to a second block with mass $M_2$. Initially, the system is in static equilibrium. If it is shaken slightly, what will the system do?
(a) The first block will accelerate up the slope; the second will accelerate downward.
(b) The first block will accelerate down the slope; the second will accelerate upward.
(c) The blocks will not accelerate.
Problem 1294. A block with mass $m_1$ is on a frictionless ramp that makes an angle of $\theta$ to the horizontal. A thin string runs from the block, parallel to the ramp, up and over a massless frictionless pulley, and down to a hanging block with mass $m_2$. The system is held in place, then released. If the blocks do not move after the release, which of the following must be true?

(a) $m_1 \sin \theta = m_2$  
(b) $m_2 \sin \theta = m_1$  
(c) $m_1 \cos \theta = m_2$  
(d) $m_2 \cos \theta = m_1$  
(e) None of these

Problem 1295. A block with mass $m_1$ is on a frictionless ramp that makes an angle of $\theta$ to the horizontal. A thin string runs from the block, parallel to the ramp, up and over a massless frictionless pulley, and down to a hanging block with mass $m_2$. The system is held in place, then released. What is the acceleration of the hanging block? Use downward as the positive direction.

(a) $\frac{(m_2 - m_1 \cos \theta)g}{m_1 + m_2}$  
(b) $\frac{(m_2 - m_1 \sin \theta)g}{m_1 + m_2}$  
(c) $\frac{(m_2 - m_1 \cos \theta)g}{m_2}$  
(d) $\frac{(m_2 - m_1 \sin \theta)g}{m_2}$  
(e) None of these

Problem 1296. A block with a mass of 50 kg is on a frictionless ramp that makes an angle of $30^\circ$ to the horizontal. A thin string runs parallel to the ramp from the block to a massless frictionless pulley, then down to a second block with a mass of 30 kg. The system is initially held in place, then released. What is the acceleration of the hanging (30 kg) block? Round your answer to the nearest 0.1 m/s$^2$. [The picture is not drawn to scale.]

(a) 0.6 m/s$^2$ upward  
(b) 0.6 m/s$^2$ downward  
(c) 1.6 m/s$^2$ upward  
(d) 1.6 m/s$^2$ downward  
(e) None of these
11.2.3 Accelerometers

Problem 1297. (Accelerometer) A thin string is attached to the ceiling of a car, and a lead sinker with mass $m$ is attached to the free end of the string. The car is going at highway speed when the driver hits the brakes. While the car is slowing down, the string makes an angle of $\theta$ to the vertical. What is the magnitude of the car’s acceleration? Take the direction of motion to be the positive $x$-axis and the $y$-axis to point upward.

(a) $-g \tan \theta$ \hspace{1cm} (b) $-mg \tan \theta$

(c) $\frac{-g}{\tan \theta}$ \hspace{1cm} (d) $\frac{mg}{\tan \theta}$

(e) None of these

Problem 1298. (Accelerometer) A thin string is attached to the ceiling of a car, and a weight is attached to the free end of the string. The car is going at 43 m/s when the driver hits the brakes. The car’s acceleration has a magnitude of 2.3 m/s$^2$. While the car is slowing down, what angle $\theta$ does the string make to the vertical? Take the direction of motion to be the positive $x$-axis and the $y$-axis to point upward. Round your answer to the nearest degree.

(a) 11° \hspace{1cm} (b) 12°

(c) 13° \hspace{1cm} (d) 15°

(e) None of these
Problem 1299. (Accelerometer) A thin string is attached to the ceiling of a car, and a lead sinker with mass $m$ is attached to the free end of the string. The car is going at highway speed when the driver hits the brakes, producing an acceleration with magnitude $a_x$. While the car is slowing down, the string makes an angle of $\theta$ to the vertical. What is $\theta$? Take the direction of motion to be the positive $x$-axis and the $y$-axis to point upward.

(a) $\theta = \tan \left( \frac{g}{-a_x} \right)$  
(b) $\theta = \tan^{-1} \left( \frac{g}{-a_x} \right)$

(c) $\theta = \tan \left( \frac{-a_x}{g} \right)$  
(d) $\theta = \tan^{-1} \left( \frac{-a_x}{g} \right)$

(e) None of these
11.2.4 Static and kinetic friction problems

Problem 1300. (Similarity Problem) You are pushing a crate of physics books across a concrete floor, with friction present. If you push with a horizontal force of $F_1$, the crate experiences an acceleration $a_1$. If you increase the horizontal force to $F_2 = 2F_1$, the acceleration is $a_2$. What is the relationship between $a_2$ and $a_1$?

(a) $a_2 = a_1$  
(b) $a_1 < a_2 < 2a_1$  
(c) $a_2 = 2a_1$  
(d) $a_2 > 2a_1$

Problem 1301. Two blocks, one with mass $m_1$ and one with mass $m_2$, are connected by a thin horizontal rope. A second horizontal rope is attached to the block with mass $m_1$ and pulled to the left with a force whose magnitude is $P$. The coefficient of kinetic friction between each block and the floor is $\mu_k$. If the blocks move at a constant velocity, what is the tension $T$ in the rope connecting the two blocks?

(a) $m_1g\mu_k$  
(b) $(m_2 + m_1)g\mu_k$  
(c) $m_2g\mu_k$  
(d) $(m_2 - m_1)\mu_k$  
(e) None of these

Problem 1302. Two blocks, one with mass $m_1$ and one with mass $m_2$, are connected by a thin horizontal rope. A second horizontal rope is attached to the block with mass $m_1$ and pulled to the left with a force whose magnitude is $P$. The coefficient of kinetic friction between each block and the floor is $\mu_k$. If the blocks move at a constant velocity, what is $P$?

(a) 0  
(b) $(m_1 + m_2)g\mu_k$  
(c) $m_1\mu_k$  
(d) $(m_2 - m_1)\mu_k$  
(e) None of these
Problem 1303. Two blocks, one with mass $m_1 = 3$ kg and one with mass $m_2 = 7$ kg, are connected by a thin horizontal rope. A second horizontal rope is attached to the block with mass $m_1$ and pulled to the left with a force whose magnitude is $P$. The coefficient of kinetic friction between each block and the floor is $\mu_k = 0.4$. If the blocks move at a constant velocity, what is $P$? For ease of calculation, assume that $g = 10 \text{ m/s}^2$.

(a) 12 N  
(b) 16 N  
(c) 28 N  
(d) 40 N  
(e) None of these

Problem 1304. Two blocks, one with mass $m_1 = 3$ kg and one with mass $m_2 = 5$ kg, are connected by a thin horizontal rope. A second horizontal rope is attached to the block with mass $m_1$ and pulled to the left with a force whose magnitude is $P$. The coefficient of kinetic friction between each block and the floor is $\mu_k = 0.2$. If the blocks move at a constant velocity, what is the tension $T$ in the rope connecting the two blocks? For ease of calculation, assume that $g = 10 \text{ m/s}^2$.

(a) 6 N  
(b) 8 N  
(c) 10 N  
(d) 16 N  
(e) None of these

Problem 1305. You are towing a block on a horizontal surface, using a rope that makes an angle of $\theta$ to the horizontal. The block has a mass of $m$; the coefficient of kinetic friction between the block and the floor is $\mu_k$. As you pull the block, you are giving the block a forward acceleration of $a$. What is the tension $T$ in the rope, expressed in terms of $m$, $a$, $\theta$, $\mu_k$, and the gravitation $g$?

(a) $(mg\mu_k + a) \cos \theta$  
(b) $\frac{m(g\mu_k + a)}{\cos \theta + \mu_k \sin \theta}$  
(c) $\frac{m(g\mu_k + a)}{\cos \theta}$  
(d) $mg\mu_k + a$  
(e) None of these
Problem 1306. A large stone block has been loaded in the bed of a dump truck. The bed is slowly raised until it makes an angle of $\theta_s$ with the horizontal; at this point, the block starts to slide downward. What is the coefficient of static friction $\mu_s$ between the block and the truck bed?

(a) $\sin \theta_s$  
(b) $\tan \theta_s$  
(c) $\cos \theta_s$  
(d) $\theta_s$  
(e) None of these

Problem 1307. A brick is sitting on a roof, which makes an angle of $\theta_k$ with the horizontal. If the brick is given a slight nudge, it slides down the rooftop at a constant speed. What is the coefficient of kinetic friction $\mu_k$ between the brick and the rooftop?

(a) $\sin \theta_k$  
(b) $\tan \theta_k$  
(c) $\cos \theta_k$  
(d) $\theta_k$  
(e) None of these

Problem 1308. A crate whose weight is $w$ is being pulled up a ramp that makes an angle of $\theta$ with the horizontal, using a thin rope that runs parallel to the ramp. The coefficient of kinetic friction between the crate and the ramp is $\mu_k$. If the crate is moving at a constant speed of $v_0$, what is the tension in the rope?

(a) $w(\sin \theta + \mu_k \cos \theta)$  
(b) $w(\sin \theta - \mu_k \cos \theta)$  
(c) $w(\cos \theta + \mu_k \sin \theta)$  
(d) $w(\cos \theta - \mu_k \sin \theta)$  
(e) None of these

Problem 1309. A crate whose mass is $m_c$ is sliding down a ramp that makes an angle of $\theta$ with the horizontal. The coefficient of kinetic friction between the crate and the ramp is $\mu_k$. What is the magnitude of the crate’s acceleration?

(a) $g(\sin \theta + \mu_k \cos \theta)$  
(b) $g(\sin \theta - \mu_k \cos \theta)$  
(c) $g(\cos \theta + \mu_k \sin \theta)$  
(d) $g(\cos \theta - \mu_k \sin \theta)$  
(e) None of these
Problem 1310. Two crates, one full of avocados and the other full of beets, are connected by a light horizontal rope. A second horizontal rope is attached to the crate of avocados and used to pull both crates across the floor at a constant speed of 1.4 m/s. The avocados have a mass of 72 kg; the beets have a mass of 93 kg. The coefficient of kinetic friction between the crates and the floor is 0.21. What is the tension \( T_{AB} \) on the rope connecting the two crates? Round your answer to the nearest newton. (See the illustration following the answers.)

(a) 148 N  
(b) 191 N  
(c) 219 N  
(d) 340 N  
(e) None of these

Problem 1311. A block with a mass of 9.7 kg is sliding down a plane at a constant speed of 4.4 m/s. The plane is angled at 22.4° to the horizontal. What is the coefficient of kinetic friction \( \mu_k \) of the block on the plane? Round your answer to the nearest 0.01.

(a) 0.37  
(b) 0.39  
(c) 0.41  
(d) 0.43  
(e) None of these

Problem 1312. A block with a mass of 132 kg is pulled with a horizontal force of \( \vec{F} \) across a rough floor. The coefficient of friction between the floor and the block is 0.51. If the block is moving at a constant velocity, what is the magnitude of \( \vec{F} \)? Round your answer to the nearest newton.

(a) 67 N  
(b) 259 N  
(c) 660 N  
(d) 2536 N  
(e) None of these

Problem 1313. A block with a mass of 84 kg is pulled with a horizontal force of \( \vec{F} \) across a rough floor. The coefficient of friction between the floor and the block is 0.64. If the block is moving at a constant velocity, what is the magnitude of \( \vec{F} \)? Round your answer to the nearest newton.

(a) 54 N  
(b) 131 N  
(c) 527 N  
(d) 1286 N  
(e) None of these

Problem 1314. You trying to move a crate across a floor by pulling horizontally on a rope. The crate has a mass of 114 kg. The coefficient of static friction between the crate and the floor is 0.76; the coefficient of kinetic friction is 0.51. You gradually increase your pull on the rope until the crate starts to move. What is the crate’s initial acceleration?
Problem 1315. A block has mass $m$. It is sliding at a constant speed $v_0$ down a plane that makes an angle of $\theta_k$ to the horizontal. If the coefficient of kinetic friction of the block on the plane is $\mu_k$, which equation gives the value of $\mu_k$?

(a) $\mu_k = mg \sin \theta_k$
(b) $\mu_k = \mu_s - \frac{1}{g \cos \theta_s} \frac{2 \Delta x}{(\Delta t)^2}$
(c) $\mu_k = \frac{mg}{\sin \theta_k}$
(d) $\mu_k = \tan \theta_k$
(e) None of these
11.2.5 Stopping distance problems

Problem 1316. The coefficient of kinetic friction between a set of tires and the road surface is $\mu_k$. If you are driving on level pavement at a speed of $v_0$, and you lock up the brakes and skid to a halt, how far do you travel before you stop?

- $v_0^2 / 2\mu_k g$ (a)
- $v_0 / \mu_k g$ (b)
- $v_0^2 / \mu_k g$ (c)
- $v_0 g^2 / \mu_k^2$ (d)
- None of these (e)

Problem 1317. The coefficient of kinetic friction between a set of tires and the road surface is $\mu_{k,\text{dry}}$ when the road is dry, and $\mu_{k,\text{wet}}$ when the road is wet, with $2\mu_{k,\text{wet}} = \mu_{k,\text{dry}}$. If you are driving on level wet pavement at a speed of $v_{\text{wet}}$ and hit the brakes, you will travel a distance of $\Delta x$ before you come to a stop. How fast can you drive on dry pavement and be able to stop in the same distance?

- $v_{\text{wet}}$ (a)
- $\sqrt{2} v_{\text{wet}}$ (b)
- $2v_{\text{wet}}$ (c)
- $4v_{\text{wet}}$ (d)
- None of these (e)

Problem 1318. The coefficient of kinetic friction between a set of tires and the road surface is $\mu_{k,\text{dry}}$ when the road is dry, and $\mu_{k,\text{ice}} = \mu_{k,\text{dry}} / 4$ when the road is icy. If you are driving on level dry pavement at a speed of $v_0$ and hit the brakes, you will travel a distance of $\Delta x_{\text{dry}}$ before you come to a stop. What will your stopping distance be if you are driving at $v_0$ on an icy road?

- $\Delta x_{\text{dry}}$ (a)
- $2\Delta x_{\text{dry}}$ (b)
- $4\Delta x_{\text{dry}}$ (c)
- $16\Delta x_{\text{dry}}$ (d)
- None of these (e)
12 Applications of Newton’s Laws to Uniform Circular Motion

12.1 Centripetal force

12.1.1 Conceptual questions: centripetal force

Problem 1319. A satellite is in circular orbit around the Earth. What is the source of the centripetal force?
(a) Momentum  (b) Kinetic energy
(c) Gravity  (d) There is no centripetal force
(e) None of these

Problem 1320. A car goes around a banked curve on an icy highway. What is the source of the centripetal force?
(a) Kinetic energy  (b) The car’s engine
(c) Normal force  (d) There is no centripetal force
(e) None of these

Problem 1321. A car goes around an unbanked curve. What is the source of the centripetal force?
(a) Momentum  (b) Normal force
(c) Friction  (d) There is no centripetal force
(e) None of these

Problem 1322. A model airplane is attached to a wire so that it flies in horizontal circles around a central pylon. What is the source of the centripetal force?
(a) Tension in the wire  (b) Air resistance
(c) Inertia  (d) There is no centripetal force
(e) None of these

Problem 1323. A bullet is fired straight up into the air; it slows down, comes to a momentary halt, then falls straight down again. What is the source of the centripetal force?
(a) Inertia  (b) Air resistance
(c) Normal force  (d) There is no centripetal force
(e) None of these

Problem 1324. You have left your coffee mug on the flat roof of your car. As you drive around an unbanked curve, what is the source of the centripetal force on the mug?
(a) Normal force  (b) Kinetic energy
(c) Static friction  (d) There is no centripetal force
(e) None of these
Problem 1325. Your keys are attached to the end of a thin chain. You are holding the end of the chain and twirling the keys in a vertical circle. What is the source of the centripetal force on the keys at the top of the circle?

(a) Gravity  (b) Tension in the chain  (c) Both gravity and tension  (d) There is no centripetal force  (e) None of these

Problem 1326. A car is moving down a straight and level road. The driver comes to a stop sign, at which he slows down but does not stop completely, then speeds up again. What is the source of the centripetal force?

(a) Kinetic friction  (b) Air resistance  (c) Normal force  (d) There is no centripetal force  (e) None of these

Problem 1327. A dead cockroach is lying on its back on the flat blade of a ceiling fan, which is slowly turning in a horizontal circle. What is the source of the centripetal force on the roach?

(a) Static friction  (b) Kinetic friction  (c) Gravity  (d) There is no centripetal force  (e) None of these

Problem 1328. Which of the following statements is correct?

(a) Uniform circular motion causes a constant force toward the center.  
(b) Uniform circular motion is caused by a constant force toward the center.  
(c) Uniform circular motion is caused by a net force toward the center with a constant magnitude.  
(d) Uniform circular motion is caused by a net force away from the center with a constant magnitude.  
(e) None of these

Problem 1329. Let \( \vec{F}_{net} \) be the net force on an object moving in a circle (not necessarily with constant speed). Which of the following statements best describes the component of the net force in the radial direction?

(a) The radial component of the net force is the centripetal force.  
(b) The radial component of the net force is the centrifugal force.  
(c) The radial component of the net force causes the object to spiral inwards towards the center of the circular path.  
(d) The radial component of the net force causes the object to spiral outwards away from the center of the circular path.  
(e) None of these
Problem 1330. Determine whether the two claims are true or false.

Claim 1: Centripetal force is a real force and its existence is brought about when objects move in circles.

Claim 2: Centrifugal force is a real force that points radially outwards when objects move in circles.

(a) Claim 1 is true; Claim 2 is true
(b) Claim 1 is true; Claim 2 is false
(c) Claim 1 is false; Claim 2 is true
(d) Claim 1 is false; Claim 2 is false

12.1.2 Horizontal Motion

Problem 1331. A car is driving at a constant speed counter-clockwise around the horizontal track shown at right. At each of the labelled points on the track, sketch a vector representing the net force on the car. Ignore gravity, which points downward into the page. Remember to make the lengths of your vectors roughly proportional to the magnitude of the force: greater forces should be indicated by longer vectors.

Problem 1332. A car is travelling at a constant speed clockwise around a horizontal track. Which of the diagrams below best shows the force vectors on the car at two points on the track?
Problem 1333. A model gas airplane is flying in a horizontal circle, attached to the central point by a string 5 m long. The airplane has a mass of 1 kg. The string will break if subjected to a tension of more than 100 N. How fast can the airplane fly without breaking the string? Round your answer to the nearest 0.1 m/s.

Problem 1334. A circular curve on a highway is designed for traffic moving at 15 m/s. The radius of the unbanked curve is 100 m. What is the minimum coefficient of friction between tires and the highway necessary to keep cars from sliding off the curve? Round your answer to the nearest 0.01.

(a) 0.15 (b) 0.19
(c) 0.23 (d) 2.25
(e) None of these

Problem 1335. You are designing a highway interchange that includes an unbanked circular curve. Assuming that the coefficient of static friction between tires and pavement is $\mu_s = 0.51$ (pointing in the direction transverse to the motion), and that cars will need to negotiate the curve at 20 m/s, what must the radius of the curve be? Round your answer to the nearest meter.

(a) 39 m (b) 80 m
(c) 204 m (d) 784 m
(e) None of these

Problem 1336. A highway curve in northern Minnesota has a radius of 140 m. The curve is banked so that a car travelling at 25 m/s will not skid sideways, even if the curve is coated with a frictionless glaze of ice. Sketch a free-body diagram of a car under these circumstances.

Problem 1337. A highway curve in northern Minnesota has a radius of 160 m. The curve is banked so that a car travelling at 25 m/s will not skid sideways, even if the curve is coated with a frictionless glaze of ice. At what angle to the horizontal is the curve banked? Round your answer to the nearest degree.

(a) 18° (b) 20°
(c) 22° (d) 24°
(e) None of these
Problem 1338. (Derive Problem) You are a civil engineer working in the wilds of North Dakota. You are designing a highway with a circular curve of radius $r$. You know that at times, the highway will be glazed with ice so that the coefficient of friction between the highway and a car’s tires is essentially zero. In order for the cars to safely negotiate the icy curve at a speed of $v$, the road must be banked at an angle of $\theta_{\text{bank}}$ above the horizontal. Find a formula for $\theta_{\text{bank}}$ in terms of $r$, $v$, and $g$.

(a) $\tan^{-1}\left(\frac{rg}{v^2}\right)$  
(b) $\tan^{-1}\left(\frac{v^2rg}{g}\right)$

(c) $\tan^{-1}\left(\frac{v^2}{rg}\right)$  
(d) $\tan^{-1}\left(\frac{v}{r^2g}\right)$

(e) None of these

Problem 1339. A highway curve in northern Minnesota has a radius of 90 m. The curve is banked so that a car travelling at 15 m/s will not skid sideways, even if the curve is coated with a frictionless glaze of ice. At what angle to the horizontal is the curve banked? Round your answer to the nearest degree.

(a) 14°  
(b) 16°

(c) 17°  
(d) 19°

(e) None of these

Problem 1340. A circular curve on a highway is designed for traffic moving at 15 m/s. The radius of the curve is 100 m. What is the correct angle of banking for the road? Round your answer to the nearest degree.

(a) 13°  
(b) 11°

(c) 9°  
(d) 7°

(e) None of these

Problem 1341. (Similarity Problem) A car travelling at a speed of $V$ goes around an unbanked curve with a radius of $R$. The centripetal force that keeps it on the road is the frictional force $F$ between the road and the tires. A few miles later, the car comes to another curve, this one with radius $R/2$, and tries to go around the curve at the same speed. What must the frictional force $f_s$ be in order to keep the car on the road?

(a) $F/2$  
(b) $F/\sqrt{2}$

(c) $\sqrt{2}F$  
(d) $2F$

(e) None of these

Problem 1342. (Similarity Problem) A car with mass $M$ and a pickup with mass $3M$ both go around an unbanked curve with radius $R$. Each vehicle is moving at a constant speed $V$. While the car is going around the curve, it experiences a centripetal force of $F$. What is the centripetal force $f_s$ experienced by the pickup on the curve?

(a) $F$  
(b) $\sqrt{3}F$

(c) $3F$  
(d) $9F$

(e) None of these
Problem 1343. (Similarity Problem) A car with mass $M$ and a pickup with mass $2M$ both go around a curve with radius $R$. Each vehicle is moving at a constant speed $V$. While the car is going around the curve, it experiences a centripetal force of $F$. What is the centripetal force $F_{\text{truck}}$ experienced by the pickup on the curve?

(a) $F/2$  
(b) $F$  
(c) $2F$  
(d) $4F$  
(e) None of these

Problem 1344. (Similarity Problem) An object with a mass of $m$ is on a frictionless horizontal surface. The object is attached to a horizontal string with length $r$ whose other end is attached to a pivot fixed in the surface; the object is then set to moving in a horizontal circle at a constant speed $v_1$. Under these circumstances, the tension in the string is $T_1$. If the speed of the object were doubled to a constant $v_2 = 2v_1$, what would be the tension in the string $T_2$?

(a) $T_1$  
(b) $\sqrt{2}T_1$  
(c) $2T_1$  
(d) $4T_1$  
(e) None of these

Problem 1345. (Similarity Problem) An object with a mass of $m$ is on a frictionless horizontal surface. The object is attached to a horizontal string with length $r_1$ whose other end is attached to a pivot fixed in the surface; the object is then set to moving in a horizontal circle at a constant speed $v_1$. Under these circumstances, the tension in the string is $T_1$. If the speed of the object is doubled to a constant $v_2 = 2v_1$, and the length of the string is increased to $r_2 = 4r_1$, what will be the tension in the string $T_2$?

(a) $T_1$  
(b) $\sqrt{2}T_1$  
(c) $2T_1$  
(d) $4T_1$  
(e) None of these
12.1.3 Vertical Motion

Problem 1346. A block with mass $M$ is whirled on the end of a thin rigid rod that is attached to the axel of a motor so that it moves at a constant speed in a vertical circle with radius $R$. At the top of the circle, the tension in the rod is twice the weight of the block. What is the speed of the block?

(a) $\sqrt{2gR}$  
(b) $\sqrt{3gR}$  
(c) $2\sqrt{gR}$  
(d) $4\sqrt{gR}$  
(e) None of these

Problem 1347. A brass sphere with weight $W$ is attached to the end of a thin rigid rod; the other end of the rod is attached to the axle of a motor, which is set to turning so that the sphere moves in a vertical circle at a constant speed. When the sphere is at the lowest point on the circle, the tension in the rod is $2W$. What is the tension in the rod when the sphere is at the top of the circle?

(a) 0  
(b) $W/2$  
(c) $W$  
(d) $2W$  
(e) None of these

Problem 1348. A brass sphere with weight $W$ is attached to the end of a thin rigid rod; the other end of the rod is attached to an axle, which is set to turning so that the sphere moves in a vertical circle at a constant speed. When the sphere is at the lowest point on the circle, the tension in the rod is $3W$. What is the tension in the rod when the sphere is at the top of the circle?

(a) 0  
(b) $W$  
(c) $2W$  
(d) $4W$  
(e) None of these

Problem 1349. A pendulum in the physics lab consists of a metal sphere with a mass of 220 g attached to the end of a thin string 104 cm long. The sphere is pulled away from the center with the string taut and then released to swing freely. When the sphere reaches the lowest point in its arc, it is moving at 83 cm/s. At that point, what is the magnitude of the sphere’s acceleration? Round your answer to the nearest 0.01 m/s$^2$.

(a) 0.66 m/s$^2$  
(b) 0.73 m/s$^2$  
(c) 0.80 m/s$^2$  
(d) 0.88 m/s$^2$  
(e) None of these

Problem 1350. A Ferris wheel has a radius of 79 m. At its maximum speed, it makes one revolution every 38 s. If you are riding the Ferris wheel at this speed, how fast are you moving? Round your answer to the nearest m/s.

(a) 13 m/s  
(b) 14 m/s  
(c) 16 m/s  
(d) 17 m/s  
(e) None of these
Problem 1351. A Ferris wheel has a radius of 65 m. At its maximum speed, it makes one revolution every 31 s. A dog is sitting on a spring scale on one of the seats on the wheel. When the wheel is not moving, the scale registers 124 N. When the wheel is turning at its maximum speed, what weight does the scale register at the highest point in the circle? Round your answer to the nearest newton.

(a) 81 N  (b) 90 N  
(c) 99 N  (d) 109 N  
(e) None of these

Problem 1352. A Ferris wheel has a radius of 79 m. At its maximum speed, it makes one revolution every 44 s. A dog is sitting on a spring scale on one of the seats on the wheel. When the wheel is not moving, the scale registers 227 N. When the wheel is turning at its maximum speed, what weight does the scale register at the lowest point in the circle? Round your answer to the nearest newton.

(a) 193 N  (b) 214 N  
(c) 238 N  (d) 264 N  
(e) None of these

Problem 1353. A Ferris wheel has a radius of 88 m. A dog is sitting on a spring scale on one of the seats on the wheel. When the wheel is not moving, the scale registers 265 N. When the wheel is turning at its maximum speed, the scale registers zero at the top of the circle. At this speed, how long does it take for the wheel to make one revolution? Round your answer to the nearest second.

(a) 14 s  (b) 15 s  
(c) 17 s  (d) 19 s  
(e) None of these

Problem 1354. A Ferris wheel has a radius of 81 m. At its maximum speed, it makes one revolution every 56 s. A dog is sitting on a spring scale on one of the seats on the wheel. When the wheel is not moving, the scale registers 119 N. When the wheel is turning at its maximum speed, what weight does the scale register at the lowest point in the circle? Round your answer to the nearest newton.

(a) 96 N  (b) 107 N  
(c) 131 N  (d) 145 N  
(e) None of these

Problem 1355. A Ferris wheel has a radius of 57 m. At its maximum speed, the seats are moving at 12.2 m/s. A dog is sitting on a spring scale on one of the seats on the wheel. When the wheel is not moving, the scale registers 303 N. When the wheel is turning at its maximum speed, what weight does the scale register at the lowest point in the circle? Round your answer to the nearest newton.

(a) 345 N  (b) 384 N  
(c) 422 N  (d) 464 N  
(e) None of these
Problem 1356. An airplane pilot is flying downward. He wants to pull out of the dive and start flying upward again by making a vertical semicircle. At the bottom of the semicircle, his speed will be 125 m/s. If he experiences more than 8.2 g’s, he will lose consciousness and crash the plane. What is the minimum radius of the semicircle? Round your answer to two significant figures.

(a) 200 m  (b) 220 m  
(c) 2100 m  (d) 2200 m  
(e) None of these

Problem 1357. A skateboard park includes a U-shaped half-pipe with a semicircular cross section whose radius is 3.4 m. A skater who starts at the top will be moving at 7.4 m/s at the lowest point. If a skater weighs 160 lb, what is his apparent weight at the lowest point? Round your answer to the nearest 10 lb.

(a) 260 lb  (b) 330 lb  
(c) 420 lb  (d) 540 lb  
(e) None of these

Problem 1358. You have tied a short length of rope to the handle of a bucket full of water, and are swinging the bucket in a vertical circle with radius 83 cm. What is the minimum speed that the bucket must be moving in order for no water to spill out at the top of the circle? Round your answer to two significant figures.

(a) 2.6 m/s  (b) 2.9 m/s  
(c) 26 m/s  (d) 29 m/s  
(e) None of these

Problem 1359. A block with a mass of $M$ slides down a frictionless loop-the-loop track. The loop has a radius of $R$. At the top of the loop, the block is moving at a speed of $V$. What is the net force acting on the block at that point?

Problem 1360. A sack of potatoes has a mass of 4.4 kg. It is hanging from a spring scale that is hanging from the ceiling of a car. The car’s is driven around a vertical loop-the-loop track with a radius of 7.9 m. At the top of the track, the car is going at a speed of 9.6 m/s. At this point, what apparent weight does the scale show for the potatoes? Round your answer to the nearest 0.1 N.

(a) 8.2 N  (b) 9.0 N  
(c) 9.9 N  (d) 10.9 N  
(e) None of these
Problem 1361. The college’s Wellness Committee has decreed that overweight faculty members will not receive pay raises. To get his raise, a certain physics instructor must get his weight from 200 lbs down to 180 lbs. Rather than giving up pizza and beer, he decides to employ trickery. He stands on a spring scale in the back of a moving van and has it driven around a vertical loop-the-loop track with a radius of 8.7 m. At the top of the loop, he has a friend take a picture with the scale showing an apparent weight of 180 lbs. At what speed does the van need to be going at the top of the loop? Round your answer to the nearest 0.1 m/s.

(a) 12.7 m/s  (b) 14.0 m/s
(c) 15.4 m/s  (d) 16.9 m/s
(e) None of these

12.2 Gravitation and circular orbits

12.2.1 Gravation

Problem 1362. A wrench has a mass of 1.43 kg. It has been left outside of a spaceship at a distance of $1.61 \times 10^7$ m from the center of a planet whose mass is $5.98 \times 10^{24}$ kg. What is the gravitational force on the wrench? Round your answer to the nearest 0.01 N.

(a) 1.78 N  (b) 1.98 N
(c) 2.20 N  (d) 2.42 N
(e) None of these

Problem 1363. A very small asteroid with a mass of 32.5 kg is at a distance of $7.71 \times 10^6$ m from the center of a planet whose mass is $2.12 \times 10^{21}$ kg. What is the gravitational force on the asteroid? Round your answer to three significant figures.

(a) $6.96 \times 10^{-2}$ N  (b) $7.74 \times 10^{-2}$ N
(c) $8.51 \times 10^{-2}$ N  (d) $9.36 \times 10^{-2}$ N
(e) None of these

Problem 1364. An astronaut with a mass of 73.3 kg is standing on the surface of a planet whose radius is $1.15 \times 10^6$ m and whose mass is $1.31 \times 10^{22}$ kg. What is the astronaut’s weight on the planet? Round your answer to the nearest 0.1 N.

(a) 39.3 N  (b) 43.6 N
(c) 48.5 N  (d) 53.3 N
(e) None of these

Problem 1365. An unmanned space probe has a mass of 129 kg. It is sent to the surface of a planet with a radius of $2.48 \times 10^7$ m and a mass of $1.02 \times 10^{26}$ kg. What is the probe’s weight on the planet? Round your answer to the nearest 10 N.

(a) 1040 N  (b) 1160 N
(c) 1290 N  (d) 1430 N
(e) None of these
Problem 1366. A space probe has a mass of 31.2 kg. It is sent to the surface of Planet X, whose radius is \(2.11 \times 10^6\) m. On the planet’s surface, the probe’s weight is 109 N. What is the mass of Planet X? Round your answer to three significant figures.

(a) \(1.70 \times 10^{23}\) kg  (b) \(1.89 \times 10^{23}\) kg  
(c) \(2.10 \times 10^{23}\) kg  (d) \(2.33 \times 10^{23}\) kg  
(e) None of these

Problem 1367. An astronaut has a mass of 67.3 kg. On the surface of Planet X, his weight is 418 N. The radius of Planet X is \(4.87 \times 10^6\) m. What is the mass of Planet X? Round your answer to three significant figures.

(a) \(1.99 \times 10^{24}\) kg  (b) \(2.21 \times 10^{24}\) kg  
(c) \(2.43 \times 10^{24}\) kg  (d) \(2.67 \times 10^{24}\) kg  
(e) None of these

12.2.2 Orbits

Problem 1368. You want to calculate the period of a satellite in a circular orbit around a planet. Which of the following do you not need to know for your calculation?

(a) The mass of the planet  (b) The radius of the planet  
(c) The radius of the orbit  (d) The gravitational constant \(G\)  
(e) None of these

Problem 1369. Astronomers discover a small planet in a circular orbit around a distant star. They are able to determine the distance of the planet from the star, and the period of the planet’s orbit. Which of the following can they calculate based on these?

(a) The mass of the planet  (b) The radius of the planet  
(c) The mass of the star  (d) The radius of the star  
(e) None of these

Problem 1370. A satellite in circular orbit around the Earth has a period of 90 minutes. A second satellite in circular orbit around Planet X has a period of 180 minutes. If the mass of the Earth is \(M_E\) and the mass of Planet X is \(M_X\), which planet’s mass is greater?

(a) \(M_E < M_X\)  (b) Not enough information: need radii of orbits  
(c) \(M_E > M_X\)  (d) Not enough information: need masses of satellites  
(e) None of these
Problem 1371. A satellite with a mass of 24 kg in circular orbit at a distance of 10,000 km from the center of Planet X has a period of 2 hours. Which of the following satellites circling Planet X would also have a period of 2 hours?

(a) A satellite with a mass of 24 kg at a distance of 20,000 km
(b) A satellite with a mass of 96 kg at a distance of 10,000 km
(c) A satellite with a mass of 3 kg at a distance of 20,000 km
(d) A satellite with a mass of 3 kg at a distance of 5,000 km
(e) None of these

Problem 1372. Russell’s celestial teapot was observed to be in a circular orbit around the Earth at a distance of $1.48 \times 10^7$ m from the center of the planet. The mass of the Earth is $5.97 \times 10^{24}$ kg. What is the teapot’s speed? Round your answer to the nearest 10 m/s.

(a) 3780 m/s   (b) 4200 m/s
(c) 4670 m/s   (d) 5190 m/s
(e) None of these

Problem 1373. An asteroid is in circular orbit around the sun, at a distance of $1.50 \times 10^{11}$ m from the sun’s center. The mass of the sun is $1.99 \times 10^{30}$ kg. What is the asteroid’s speed? Round your answer to three significant figures.

(a) $2.41 \times 10^4$ m/s   (b) $2.68 \times 10^4$ m/s
(c) $2.98 \times 10^4$ m/s   (d) $3.27 \times 10^4$ m/s
(e) None of these

Problem 1374. A small moon is in circular orbit around Planet X, at a distance of $1.68 \times 10^7$ m from the planet’s center. The moon’s orbital speed is 1120 m/s. What is the mass of Planet X? Round your answer to three significant figures.

(a) $2.84 \times 10^{23}$ kg   (b) $3.16 \times 10^{23}$ kg
(c) $3.47 \times 10^{23}$ kg   (d) $3.82 \times 10^{23}$ kg
(e) None of these

Problem 1375. A small planet is in circular orbit around a star, at a distance of $3.78 \times 10^{11}$ m from the star’s center. The orbital speed of the planet is $3.44 \times 10^4$ m/s. What is the star’s mass? Round your answer to three significant figures.

(a) $5.43 \times 10^{30}$ kg   (b) $6.03 \times 10^{30}$ kg
(c) $6.70 \times 10^{30}$ kg   (d) $7.37 \times 10^{30}$ kg
(e) None of these

Problem 1376. A satellite has a mass of 6 kg. It is in circular orbit around a planet with a mass of $6 \times 10^{24}$ kg. The radius of the planet is $10^6$ m; the radius of the orbit is $10^8$ m. What is the satellite’s orbital period? For ease of calculation, assume that $G = \frac{2}{3} \times 10^{-10}$ N m$^2$/kg$^2$.

(a) $\pi \times 10^2$ s   (b) $\pi \times 10^5$ s
(c) $\pi \times 10^{14}$ s   (d) $\pi \times 10^{17}$ s
(e) None of these
Problem 1377. A satellite with a mass of 150 kg is in circular orbit around a planet with a mass of $1.5 \times 10^{26}$ kg. The radius of the planet is $10^8$ m; the radius of the orbit is $10^{10}$ m. What is the satellite’s orbital period? For ease of calculation, assume that $G = \frac{2}{3} \times 10^{-10}$ N m$^2$/kg$^2$.

(a) $2\pi \times 10^4$ s  
(b) $2\pi \times 10^5$ s  
(c) $2\pi \times 10^6$ s  
(d) $2\pi \times 10^7$ s  
(e) None of these

Problem 1378. A star has a mass of $2.4 \times 10^{31}$ kg and a radius of $10^9$ m. A planet in circular orbit around the star has a mass of $2.4 \times 10^{25}$ kg and a radius of $8 \times 10^6$ m. The radius of the planet’s orbit is $4 \times 10^{12}$ m. What is the planet’s orbital period? For ease of calculation, assume that $G = \frac{2}{3} \times 10^{-10}$ N m$^2$/kg$^2$.

(a) $4\pi \times 10^8$ s  
(b) $4\pi \times 10^9$ s  
(c) $4\pi \times 10^{10}$ s  
(d) $4\pi \times 10^{11}$ s  
(e) None of these

Problem 1379. (Similarity Problem) A satellite with mass $m_1$ is in circular orbit at a distance of $r_0$ from the center of a planet with mass $M_p$. A second satellite with mass $m_2 = 2m_1$ is in circular orbit around the same planet, at the same distance from the center. If the period of the first satellite is $T_1$, what is the period of the second satellite?

(a) $T_1/4$  
(b) $T_1/2$  
(c) $T_1$  
(d) $\sqrt{2} T_1$  
(e) None of these

Problem 1380. (Similarity Problem) A satellite with mass $m_1$ is in circular orbit around the Earth at a distance of $r_1$ from the center of the planet; it has a period of $T_1$. Scientists would like to launch a second satellite into circular orbit with a period of $T_2 = 2T_1$. What should they do to give the second satellite this period?

(a) Orbit the second satellite at a distance of $r_2 = 64r_1$  
(b) Orbit the second satellite at a distance of $r_2 = 4^{1/3} \cdot r_1$  
(c) Give the second satellite a mass of $m_2 = 4m_1$  
(d) Give the second satellite a mass of $m_2 = m_1/4$  
(e) None of these

Problem 1381. (Similarity Problem) Planet X has mass $M_X$. A satellite with mass $m_X$ is in circular orbit around it at a distance of $r_X$ from the center of the planet. Planet Y has mass $M_Y$; a satellite with mass $m_Y$ is in circular orbit around it at a distance of $r_Y = 2r_X$ from the planet’s center. If the two satellites have the same period, which of the following is true?

(a) $M_Y = M_X/8$  
(b) $M_Y = 8M_X$  
(c) $m_Y = m_X/8$  
(d) $m_Y = 8m_x$  
(e) None of these
13 Applications of Newton’s Laws to Rotating Bodies

13.1 Torque

Problem 1382. A diving board extends 2 m horizontally from the side of a pool. A swimmer weighing 600 N stands on the very end of the board. What torque does the swimmer exert about the base of the board? Round your answer to two significant figures.

- (a) 300 N·m
- (b) 600 N·m
- (c) 1200 N·m
- (d) 2400 N·m
- (e) None of these

Problem 1383. A fisherman is holding a fishing rod horizontally over the edge of a bridge. The rod is 150 cm long; from the end of it hangs a catfish with a mass of 6.0 kg. What torque does the fish exert about the base of the road? Round your answer to two significant figures.

- (a) 0.92 N·m
- (b) 9.0 N·m
- (c) 88 N·m
- (d) 900 N·m
- (e) None of these

Problem 1384. A cord wraps around a pulley with a radius of 8 cm. The pulley is rusty, and requires a torque of 120 N·m to make it turn. How much force do you have to pull on the cord with in order to make the pulley turn? Round your answer to two significant figures.

- (a) 15 N
- (b) 96 N
- (c) 1500 N
- (d) 9600 N
- (e) None of these

Problem 1385. On a construction site, a plank is fastened at one end and projects 2.0 m horizontally from the building. The fastening will break if subjected to a torque of more than 1500 N·m. What is the mass of the heaviest piece of equipment that can safely be placed on the end of the plank? Round your answer to two significant figures.

- (a) 77 kg
- (b) 310 kg
- (c) 750 kg
- (d) 3000 kg
- (e) None of these

Problem 1386. You are out in the middle of the desert with a flat tire. Your lug nuts are rusty, and require a torque of 300 N·m to loosen them. You are strong enough to exert a force of 500 N on the end of a wrench. How long does the wrench have to be in order for you to get the lug nuts off? Round your answer to two significant figures.

- (a) 15 cm
- (b) 17 cm
- (c) 60 cm
- (d) 67 cm
- (e) None of these
Problem 1387. An action-movie hero who weighs 70 kg is hanging from a flagpole that projects horizontally from the side of a building. The flagpole will break off if subjected to a torque of more than 1200 N·m. How far out on the flagpole can the hero go? Round your answer to two significant figures.

(a) 0.84 m  (b) 1.1 m  
(c) 1.4 m  (d) 1.7 m  
(e) None of these

Problem 1388. You are trying to loosen a rusty nut with a wrench 30 cm long. Because the nut is in an awkward place, you can only pull at an angle of 70° to the wrench handle. If you can pull with a force of 600 N, how much torque do you exert on the nut? Round your answer to two significant figures.

(a) 62 N·m  (b) 100 N·m  
(c) 170 N·m  (d) 280 N·m  
(e) None of these

Problem 1389. A fisherman has a bass on the end of his line. His rod is 1.2 m long; the line makes an angle of 50° with the rod; and the bass is pulling on the line with a force of 40 N. How much torque does the bass produce about the handle of the rod? Round your answer to two significant figures.

(a) 31 N·m  (b) 33 N·m  
(c) 35 N·m  (d) 37 N·m  
(e) None of these

Problem 1390. A telephone pole is supported by a guy wire that is attached 3.0 m up on the pole and makes an angle of 60° with the ground. If the tension in the wire is 6000 N, how much torque does it exert about the base of the pole? Round your answer to two significant figures.

(a) 9000 N·m  (b) 11,000 N·m  
(c) 13,000 N·m  (d) 16,000 N·m  
(e) None of these

Problem 1391. A fisherman is holding a rod 120 cm long at an angle of 55° above the horizontal. A carp weighing 60 N is hanging from the end of the rod. How much torque does the fish exert about the handle of the rod? Round your answer to two significant figures.

(a) 29 N·m  (b) 41 N·m  
(c) 59 N·m  (d) 72 N·m  
(e) None of these
Problem 1392. A flagpole with a length of 40 m has been installed carelessly, and makes an angle of 12° to the vertical. The wind exerts a horizontal force of 300 N on the flag at the top of the pole. How much torque does this produce about the base of the pole? Round your answer to two significant figures.

(a) 2500 N·m  
(b) 4200 N·m  
(c) 7000 N·m  
(d) 12,000 N·m  
(e) None of these

Problem 1393. A bird with a mass of 4.2 kg lands on top of a pole 3.5 m long that makes an angle of 15° to the vertical. How much torque does the bird exert about the base of the pole? Round your answer to two significant figures.

(a) 19 N·m  
(b) 37 N·m  
(c) 72 N·m  
(d) 140 N·m  
(e) None of these

Problem 1394. A fisherman has a marlin on the end of his line. The line makes an angle of 70° with the rod, which is 2.2 m long. The fisherman will lose his grip on the rod if it is subjected to a torque of more than 1200 N·m. How much force must the marlin exert on the line in order to break the fisherman’s grip? Round your answer to two significant figures.

(a) 580 N  
(b) 900 N  
(c) 1600 N  
(d) 2500 N  
(e) None of these

Problem 1395. A flagpole 2.5 m long projects from the side of a building, at an angle of 35° to the vertical. A bird lands on the end of the flagpole, producing a torque of 80 N·m about the flagpole’s base. What is the bird’s mass? Round your answer to two significant figures.

(a) 4.2 kg  
(b) 4.6 kg  
(c) 5.1 kg  
(d) 5.7 kg  
(e) None of these

Problem 1396. Rebels have overthrown the President-for-life of Tyrannia, and are now attempting to pull down a large statue of him. They have tied a cable around the neck of the statue, 12 m above the ground, and attached the other end to a pickup. The rope makes an angle of 40° to the vertical. In order to topple the statue, it is necessary to produce a torque of 80,000 N·m about its base. How much force does the pickup need to apply to the rope? Round your answer to two significant figures.

(a) 8700 N  
(b) 10,000 N  
(c) 12,000 N  
(d) 15,000 N  
(e) None of these
**Problem 1397.** A flagpole is supported by a guy wire that is attached to the pole 5.0 m above the ground, and that makes an angle of 65° to the horizontal. To keep the flagpole from blowing over in high winds, the wire must be able to produce a torque of 6000 N·m about the base of the pole. How much tension must the wire be able to withstand? Round your answer to two significant figures.

(a) 1300 N  (b) 2800 N  
(c) 6100 N  (d) 13,000 N  
(e) None of these

**Problem 1398.** A branch projects diagonally upward from the trunk of a tree, making an angle of 70° with the vertical trunk. A monkey weighing 240 N climbs out along the branch. The branch will break off if it is subjected to a torque of more than 300 N·m about its base. How far along the branch can the monkey go without breaking it? Round your answer to two significant figures.

(a) 1.3 m  (b) 1.9 m  
(c) 2.6 m  (d) 3.7 m  
(e) None of these

**Problem 1399.** A partly open drawbridge makes an angle of 15° to the horizontal. A truck weighing $1.2 \times 10^5$ N drives slowly up the slope. The drawbridge will collapse if subjected to a torque of more than $2.0 \times 10^6$ N. How far along the drawbridge can the truck go? Round your answer to two significant figures.

(a) 11 m  (b) 17 m  
(c) 33 m  (d) 64 m  
(e) None of these

**Problem 1400.** The boom of a crane is 20 m long. A cement truck weighing 300,000 N is hanging from the end of the crane. The crane will topple over if it is subjected to a torque about its base greater than $5.0 \times 10^6$ N·m. What is the largest angle with the vertical that the boom can safely make? Round your answer to the nearest degree.

(a) 34°  (b) 41°  
(c) 49°  (d) 56°  
(e) None of these

**Problem 1401.** Rebels have overthrown the Dear Leader of Annexia, and are attempting to pull down his statue. They have attached a cable around the statue’s neck, 11 m above ground level, and have attached the other end to a truck. If the truck can pull with a force of 50,000 N, and if it will require a torque of 400,000 N·m about the statue’s base to pull it over, what angle must the cable make with the horizontal? Round your answer to the nearest degree.

(a) 43°  (b) 47°  
(c) 50°  (d) 53°  
(e) None of these
Problem 1402. Two people are standing on a horizontal diving board. One person weighs 400 N, and is standing 3 m from the base of the board. The other weighs 800 N, and is standing 2 m from the base. What is the magnitude of the net torque that the two people produce about the base of the board? Round your answer to two significant figures.

(a) 400 N·m  (b) 1200 N·m  (c) 2800 N·m  (d) 3000 N·m  (e) None of these

Problem 1403. A teeter-totter consists of a massless horizontal pivoting on a fulcrum. With the board held horizontal, a physics instructor weighing \( w_1 = 800 \) N sits 1 m from the fulcrum. A physics student weighing \( w_2 = 500 \) N sits on the opposite side, 2 m from the fulcrum. What is the net torque about the fulcrum? Assume that the instructor produces a positive torque. Round your answer to two significant figures.

(a) 200 N·m  (b) −200 N·m  (c) 300 N·m  (d) −300 N·m  (e) None of these

Problem 1404. Two pulleys are fixed on a shaft, as shown at right. One pulley has a radius of 15 cm; a weight of \( w_1 = 6 \) N hangs from that pulley on the left side of the shaft. The second pulley has a radius of 10 cm; a weight of \( w_2 = 4 \) N hangs from it, on the right side of the shaft. What is the net torque on the shaft? Assume that \( w_1 \) produces a positive torque. Round your answer to two significant figures.

(a) 0.50 N·m  (b) −0.50 N·m  (c) 1.3 N·m  (d) −1.3 N·m  (e) None of these

Problem 1405. You are trying to close a door; your dog is on the opposite side, trying to push it open. You apply a force of 200 N at a distance of 80 cm from the hinges. The dog applies a force of 400 N at a distance of 60 cm from the hinges. What is the net torque on the door? Assume that you are applying a positive torque. Round your answer to two significant figures.

(a) −160 N·m  (b) −80 N·m  (c) 120 N·m  (d) 400 N·m  (e) None of these
Problem 1406. Two civil engineering students are attempting to loosen a nut on a bridge, using a very large wrench. Both students are pulling in the same direction. One student is applying a force of 400 N at an angle of 90° to the handle of the wrench, 1.2 m from the nut; the other student is applying a force of 500 N at an angle of 80° to the handle, 1.6 m from the nut. What is the net torque applied to the nut? Round your answer to two significant figures.

(a) 310 N·m  (b) 900 N·m
(c) 1300 N·m  (d) 1400 N·m
(e) None of these

Problem 1407. Rebels have overthrown the Tsar of Fanatistan, and are now trying to pull down his statue. However, the two rebel organizations are in disagreement on how to do it. The People’s Front has attached a rope 6.0 m up on the statue, and is pulling northward with a force of 20,000 N at an angle of 40° to the horizontal. The Popular Front has attached their rope 7.5 m up on the statue, and is pulling southward with a force of 15,000 N at an angle of 35° to the horizontal. What is the net torque about the base of the statue? Assume that the People’s Front is applying positive torque. Round your answer to the nearest 1000 N·m.

(a) 0 N·m  (b) −13,000 N·m
(c) 38,000 N·m  (d) 180,000 N·m
(e) None of these

Problem 1408. A massless plank is lying on the ground and extending partly out over the edge of the Grand Canyon. A tourist weighing 800 N wants to walk to the end of the plank, 2.5 m beyond the cliff edge, and have his wife take his picture. If the wife weighs 500 N, how far back from the cliff edge must she stand on the plank to keep her husband from falling into the canyon? Round your answer to two significant figures.

(a) 1.6 m  (b) 2.5 m
(c) 4.0 m  (d) 6.4 m
(e) None of these

Problem 1409. A teeter-totter consists of a massless board pivoting on a fulcrum. A physics instructor weighing $w_1 = 800$ N sits 1.2 m from the fulcrum. A physics student weighing $w_2 = 500$ N sits on the opposite side. How far from the fulcrum must the student sit to balance the instructor’s torque? Round your answer to two significant figures.

(a) 0.75 m  (b) 1.4 m
(c) 1.6 m  (d) 1.9 m
(e) None of these
Problem 1410. A vertical pulley has a radius of 5 cm. A thin string is wound around the outside of the pulley, and a weight of 8 N is suspended from it. The pulley has a brake, located 3 cm from the pulley’s axle. How much force must the brake exert to keep the pulley from turning? Round your answer to two significant figures.

(a) 1.9 N  
(b) 4.8 N  
(c) 13 N  
(d) 120 N  
(e) None of these

Problem 1411. A massless horizontal pole projects 1.6 m from the side of a building. It is held up by a cable running from the side of the building to the end of the pole, making an angle of 35° to the horizontal. A sign weighing \( w = 30 \) N hangs from the pole 1.5 m out from the building. What is the tension in the cable? Round your answer to two significant figures.

(a) 28 N  
(b) 34 N  
(c) 39 N  
(d) 49 N  
(e) None of these

Problem 1412. A flagpole 2.4 m long projects from the side of a building, making an angle of 40° to the vertical. The flagpole has a mass of 80 kg; its center of mass is at its midpoint. It is supported by a horizontal cable running from the side of the building to the end of the pole. What is the tension in the cable? Round your answer to two significant figures.

(a) 34 N  
(b) 40 N  
(c) 330 N  
(d) 390 N  
(e) None of these

Problem 1413. A flagpole 7 m high is supported by a guy wire attached 5 m high on the north side of the pole and meeting the ground at an angle of \( \theta = 50^\circ \). The wind blows southward, exerting a force of 200 N on the flag at the top of the pole. What is the tension in the wire? Round your answer to two significant figures.

(a) 180 N  
(b) 210 N  
(c) 370 N  
(d) 440 N  
(e) None of these

Problem 1414. A pendulum consists of a lead sinker with weight \( W \) attached to the end of a thin string with length \( L \); the other end of the string is attached to a hook in the ceiling. The pendulum is pulled back and released. When the string makes an angle of \( \theta \) with the vertical, it has a tension of \( T \). At that point, what is the net torque about the hook in the ceiling?

(a) 0  
(b) \( (T + W) \sin \theta \)  
(c) \( WL \sin \theta \)  
(d) \( (T - W)L \cos \theta \)  
(e) None of these
13.2 Angular acceleration

**Problem 1415.** A windmill has a moment of inertia of 5.0 kg·m². When the wind begins to blow, it accelerates at 0.40 rad/s². What is the torque on the windmill? Round your answer to two significant figures.

(a) 0.80 N·m  
(b) 2.0 N·m  
(c) 5.0 N·m  
(d) 13 N·m  
(e) None of these

**Problem 1416.** A shaft’s moment of inertia is 24 kg·m². How much torque does it take to accelerate the shaft at 3.0 rad/s²? Round your answer to two significant figures.

(a) 8.0 N·m  
(b) 17 N·m  
(c) 35 N·m  
(d) 72 N·m  
(e) None of these

**Problem 1417.** A flywheel has a mass of 1200 kg, a radius of 0.75 m, and a moment of inertia of 480 kg·m². When a motor connected to the flywheel is switched on, the flywheel accelerates at 0.80 rad/s². How much torque does the motor apply to the flywheel? Round your answer to two significant figures.

(a) 290 N·m  
(b) 340 N·m  
(c) 380 N·m  
(d) 900 N·m  
(e) None of these

**Problem 1418.** A bicycle wheel’s moment of inertia is 0.30 kg·m². If you apply a torque of 20 N·m to it, what is its angular acceleration? Round your answer to two significant figures.

(a) 1.8 rad/s²  
(b) 6.0 rad/s²  
(c) 20 rad/s²  
(d) 67 rad/s²  
(e) None of these

**Problem 1419.** A revolving door has moment of inertia 120 kg·m². You apply a torque of 50 N·m to it. What is its angular acceleration? Round your answer to two significant figures.

(a) 0.12 rad/s²  
(b) 0.42 rad/s²  
(c) 1.4 rad/s²  
(d) 2.4 rad/s²  
(e) None of these
Problem 1420. You are trying to determine the moment of inertia of a roast turkey on a massless turntable. Unfortunately, your physics book does not include a formula for the moment of inertia of a turkey, roasted or otherwise. You apply a torque of 0.060 N·m to the turntable, and it accelerates at 1.4 rad/s². What is the turkey’s moment of inertia? Round your answer to two significant figures.

(a) 0.043 kg·m²  
(b) 0.054 kg·m²  
(c) 0.067 kg·m²  
(d) 0.084 kg·m²  
(e) None of these

Problem 1421. A motor supplies a torque of 160,000 N·m to a merry-go-round, which accelerates at 0.3 rad/s². What is the merry-go-round’s moment of inertia? Round your answer to two significant figures.

(a) 48,000 kg·m²  
(b) 110,000 kg·m²  
(c) 240,000 kg·m²  
(d) 530,000 kg·m²  
(e) None of these

Problem 1422. A flywheel has a radius of 60 cm and a moment of inertia of 150 kg·m². A belt around the rim of the wheel imparts a force of 80 N. What is the flywheel’s angular acceleration? Round your answer to two significant figures.

(a) 0.29 rad/s²  
(b) 0.32 rad/s²  
(c) 0.35 rad/s²  
(d) 0.39 rad/s²  
(e) None of these

Problem 1423. A gate is 2 m long, and has a moment of inertia of 200 kg·m². A cow pushes on the gate 1.5 m from the hinges, with a force of 400 N. What is the gate’s angular acceleration? Round your answer to two significant figures.

(a) 1.5 rad/s²  
(b) 2.3 rad/s²  
(c) 3.0 rad/s²  
(d) 4.0 rad/s²  
(e) None of these

Problem 1424. A shaft is 10 m long, with a radius of 10 cm and a moment of inertia of 12 kg·m². A belt around the shaft imparts a force of 50 N. What is the shaft’s angular acceleration? Round your answer to two significant figures.

(a) 0.30 rad/s²  
(b) 0.34 rad/s²  
(c) 0.38 rad/s²  
(d) 0.42 rad/s²  
(e) None of these

Problem 1425. A large gear has a radius of 1.2 m, a mass of 4100 kg, and a moment of inertia of 4000 kg·m². A second gear engages the teeth on the outer edge of the first one and imparts a force of 200 N. What is the angular acceleration of the first gear? Round your answer to two significant figures.

(a) 0.050 rad/s²  
(b) 0.060 rad/s²  
(c) 0.067 rad/s²  
(d) 0.075 rad/s²  
(e) None of these
Problem 1426. A merry-go-round has a radius of 2.0 m. A physics professor pushes with a force of 120 N tangent to the outer edge of the merry-go-round, giving it an angular acceleration of 1.8 rad/s². What is the merry-go-round’s moment of inertia? Round your answer to two significant figures.

(a) 110 kg·m²  (b) 120 kg·m²  
(c) 130 kg·m²  (d) 150 kg·m²  
(e) None of these

Problem 1427. A belt runs over a pulley with a radius of 12 cm. When the belt is pulled with a force of 60 N, the pulley experiences a radial acceleration of 2.5 m/s². What is the pulley’s moment of inertia? Round your answer to two significant figures.

(a) 2.1 kg·m²  (b) 2.3 kg·m²  
(c) 2.6 kg·m²  (d) 2.9 kg·m²  
(e) None of these

Problem 1428. A fountain features a stone sphere with a radius of 2.8 m, suspended frictionlessly on a film of water in a hemispherical basin. The sphere has a moment of inertia of 2800 kg·m². How much force do you have to apply along the sphere’s equator in order to produce an acceleration of 0.04 m/s²? Round your answer to two significant figures.

(a) 40 N  (b) 44 N  
(c) 48 N  (d) 53 N  
(e) None of these

Problem 1429. A gate is 2.1 m long, and has a moment of inertia of 260 kg·m². How hard do you have to push on the outer edge of the gate to induce an acceleration of 1.5 rad/s²? Round your answer to two significant figures.

(a) 140 N  (b) 150 N  
(c) 170 N  (d) 190 N  
(e) None of these

Problem 1430. A windmill’s moment of inertia is 20 kg·m². It is initially at rest; when the wind starts to blow, it begins turning, reaching a speed of 4.5 rad/s in 8 seconds. What is the magnitude of the torque that the wind applied to the windmill? Round your answer to two significant figures.

(a) 10 N·m  (b) 11 N·m  
(c) 12 N·m  (d) 14 N·m  
(e) None of these
Problem 1431. A shaft has a radius of 8 cm and a moment of inertia of 15 kg·m². It is driven by a belt wrapped around it. If the shaft is initially at rest and the belt is pulled with a force of 20 N, how long does it take for the shaft to turn 10 radians? Round your answer to two significant figures.

(a) 12 s  (b) 14 s
(c) 15 s  (d) 17 s
(e) None of these

Problem 1432. A wheel’s moment of inertia is 30 kg·m². It is spinning at 25 rad/s when a brake is applied at a distance of 30 cm from the wheel’s center. The wheel turns through 15 radians before coming to a halt. How much force was produced by the brake? Round your answer to two significant figures.

(a) 2100 N  (b) 2300 N
(c) 2500 N  (d) 2800 N
(e) None of these

Problem 1433. An evil physics professor decides to stop the rotation of the Earth. He embeds a large rocket in a mountain on the equator, pointing westward. The Earth has a radius of $6.4 \times 10^6$ m and a moment of inertia of $9.8 \times 10^{37}$ kg·m²; it is initially turning at $7.3 \times 10^{-5}$ rad/s. If the professor wants to stop the rotation in $3.2 \times 10^7$ s (approximately one year), how much thrust must his rocket produce? Round your answer to two significant figures.

(a) $3.2 \times 10^{18}$ N  (b) $3.5 \times 10^{19}$ N
(c) $3.9 \times 10^{20}$ N  (d) $4.2 \times 10^{21}$ N
(e) None of these

Problem 1434. A large gear’s radius is 30 cm, and its moment of inertia is 40 kg·m². A second gear engages the teeth on the outer edge of the first one and applies a force of 20 N. The gears are initially at rest. After the first gear has turned through 5 rad, how fast will it be turning? Round your answer to two significant figures.

(a) 1.0 rad/s  (b) 1.1 rad/s
(c) 1.2 rad/s  (d) 1.3 rad/s
(e) None of these

Problem 1435. A flywheel with a radius of 0.50 m and a moment of inertia of 120 kg·m² is driven by a belt around the rim. The flywheel is initially at rest. You would like it to turn through 20 rad in 6 seconds. What force do you need to apply to the belt? Round your answer to two significant figures.

(a) 190 N  (b) 220 N
(c) 240 N  (d) 270 N
(e) None of these
13.3 Moment of Inertia

**Problem 1436.** A physics instructor has a mass of 90 kg. His wife has a mass of 60 kg. The two are riding together on a merry-go-round with two concentric circles of wooden horses. In which of the following situations is the system’s moment of inertia the smallest?

(a) Both sit together one one of the outer horses  
(b) Both sit on outer horses, on opposite sides of the merry-go-round  
(c) She sits on one of the outer horses; he sits on the inner horse next to her  
(d) He sits on one of the outer horses; she sits on an inner horse on the opposite side

**Problem 1437.** A flywheel consists of a solid iron disc with two holes drilled through it, turning on a shaft through its center. In which of the following arrangements of holes is the moment of inertia the largest?

(a) ![Diagram](a)  
(b) ![Diagram](b)  
(c) ![Diagram](c)  
(d) ![Diagram](d)

**Problem 1438.** A lead block with mass \(m_b\) is attached to a massless turntable, at a distance of \(r_1\) from the center. The system’s moment of inertia is \(I_1\). The block is then moved to a distance of \(2r_1\) from the center. What is the new moment of inertia?

(a) \(I_1\)  
(b) \(\sqrt{2}I_1\)  
(c) \(2I_1\)  
(d) \(4I_1\)  
(e) None of these

**Problem 1439.** A lead block with mass \(m_b\) is attached to a massless turntable, at a distance of \(r_1\) from the center. The system’s moment of inertia is \(I_1\). A second block with the same mass is then attached at the same distance from the center, on the opposite side. What is the new moment of inertia?

(a) \(I_1\)  
(b) \(\sqrt{2}I_1\)  
(c) \(2I_1\)  
(d) \(4I_1\)  
(e) None of these

**Problem 1440.** A system consists of a massless turntable with a lead block with mass \(m_1\) attached at a distance of \(r_1\) from the center. A second system has the same moment of inertia, but uses a block with mass \(m_2 = 2m_1\). At what distance \(r_2\) is this block attached?

(a) \(r_1/4\)  
(b) \(r_1/2\)  
(c) \(r_1/\sqrt{2}\)  
(d) \(r_1\)  
(e) None of these
Problem 1441. A gate can be regarded as a thin rectangular plate hinged along one side, with a mass of 80 kg and a length of 3.0 m. The gate is pushed with a force of 50 N, 2.0 m from its hinges. What angular acceleration does it experience? Round your answer to two significant figures.

(a) 0.30 rad/s²  (b) 0.34 rad/s²
(c) 0.38 rad/s²  (d) 0.42 rad/s²
(e) None of these

Problem 1442. A wind turbine has three blades. Each blade has a mass of 1200 kg and a length of 20 m. Each blade can be regarded as a thin rod connected at one end to the axis. How much torque must be applied to the turbine to make it accelerate at 0.10 rad/s²? Round your answer to two significant figures.

(a) 35,000 N·m  (b) 39,000 N·m
(c) 43,000 N·m  (d) 48,000 N·m
(e) None of these

Problem 1443. A flywheel consists of a solid cylinder with a radius of 70 cm and a thickness of \( z = 8 \) cm; it is made of steel with a density of \( \rho = 7.5 \) g/cm³. The flywheel is driven by a belt that wraps around its rim. If you want it to go from rest to a speed of 12 rad/s in 5 seconds, how much force do you need to apply to the belt? Round your answer to two significant figures.

(a) 780 N  (b) 850 N
(c) 940 N  (d) 1000 N
(e) None of these

Problem 1444. A flywheel can be regarded as a cylindrical shell with an inside radius of 50 cm, an outside radius of 60 cm, a thickness (length) of \( z = 7.5 \) cm, and a density of \( \rho = 7.5 \) g/cm³. It is mounted on a shaft with radius 1.5 cm. (The moment of inertia of the shaft, and of the wheel’s spokes, can be ignored.) A belt around the shaft drives the flywheel. How much force do you need to apply to the belt if you want the wheel, starting from rest, to reach a speed of 1.2 rad/s by the time it has made one complete revolution? Round your answer to two significant figures.

(a) 450 N  (b) 500 N
(c) 550 N  (d) 600 N
(e) None of these

Problem 1445. (Similarity Problem) A solid sphere, solid cylinder, thin-walled hollow cylinder, and thin-walled hollow sphere all having the same mass \( M \) and radius \( R \) are released from rest from the same height \( H \) on a ramp. Assuming that the ramp is smooth and each of the four objects rolls without slipping, which of these objects will be the first one to reach the bottom of the hill?

(a) solid sphere  (b) solid cylinder
(c) thin-walled hollow cylinder  (d) thin-walled hollow sphere
(e) None of these
Part V

Work, Energy, and Momentum
14 Work and mechanical energy

14.1 Work

Problem 1446. A rocket is fired upward. As it rises, the work done by gravity on the rocket is:

(a) positive  (b) negative  (c) zero

Problem 1447. A piano is dropped from a high window. As it falls, the work done by gravity on the piano is:

(a) positive  (b) negative  (c) zero

Problem 1448. An ice skater is gliding across the frictionless surface of a frozen pond. As he crosses the pond, the work done by gravity on the skater is:

(a) positive  (b) negative  (c) zero

Problem 1449. A pickup is towing a car along a level road. The two are connected by a horizontal tow-strap, which has a tension of 4000 N. How much work does the pickup do in towing the car a distance of 100 m in a time of 10 s?

(a) $2 \times 10^4$ J  (b) $2 \times 10^5$ J
(c) $4 \times 10^5$ J  (d) Not enough information: need car’s mass
(e) None of these

Problem 1450. You are pushing a shopping cart across a level parking lot. The cart has a mass of 10 kg. You push it with a horizontal force of 100 N to move it at a constant speed of 4 m/s. How much work do you do on the cart?

(a) 400 J  (b) 800 J
(c) 1000 J  (d) Not enough information: need the distance
(e) None of these

Problem 1451. A filing cabinet has a mass of 69 kg. You push on it horizontally in order to move it at a constant velocity. The coefficient of kinetic friction between the cabinet and the floor is 0.77. How much work do you do in moving the cabinet 0.85 m? Round your answer to two significant figures.

(a) 45 J  (b) 59 J
(c) 440 J  (d) 520 J
(e) None of these

Problem 1452. A crate full of pineapples has a mass of 170 kg. A worker pushes horizontally on the crate to move it 12 m across the floor at a constant speed. The coefficient of kinetic friction between the sliding crate and the floor is 1.12. How much work does the worker do on the crate? Round your answer to the nearest 100 J.

(a) 900 J  (b) 2300 J
(c) 20,000 J  (d) 22,400 J
(e) None of these
Problem 1453. A sidewalk runs parallel to a street. A skateboarder is being towed down the sidewalk by a rope attached to the back of a car on the street. The rope makes an angle of 23° to the direction in which the car and the skateboarder are moving. The tension in the rope is 185 N. How much work does the car do in towing the skateboarder 78 m down the sidewalk? Round your answer to the nearest 100 J.

(a) 5600 J  (b) 13,300 J  
(c) 14,400 J  (d) 15,700 J  
(e) None of these

Problem 1454. A road runs beside and parallel to a straight stretch of railroad track. A train robber attaches his pickup truck to a boxcar, using a horizontal rope that makes an angle of 17° to the tracks, and tows the boxcar 1200 m down the tracks. If the tension in the rope was 7400 N, how much work was done on the boxcar? Give your answer in scientific notation, rounded to two significant figures.

(a) $2.4 \times 10^6$ J  (b) $2.6 \times 10^6$ J  
(c) $8.5 \times 10^6$ J  (d) $9.3 \times 10^6$ J  
(e) None of these

Problem 1455. You are taking your German shepherd for a walk in Reid Park when she decides to chase a Black-crowned Night-Heron by the pond. You pull back with a force of $T$ on her leash, which makes an angle of $\theta$ with the horizontal. However, the dog is stronger than you are, and pulls you forward a distance of $x$. How much work do you do on the dog?

(a) $Tx \cos \theta$  (b) $-Tx \cos \theta$  
(c) $Tx \sin \theta$  (d) $-Tx \sin \theta$  
(e) None of these

Problem 1456. A case of ball bearings has a mass of 145 kg. A worker pushes horizontally on the case to move it 4.2 m across the floor at a constant speed. The coefficient of friction between the sliding case and the floor is 0.58. How much work does the worker do on the case? Round your answer to two significant figures.

(a) 180 J  (b) 350 J  
(c) 1700 J  (d) 3500 J  
(e) None of these
14.2 Power

Problem 1457. You are pushing a shopping cart with a mass of 23 kg across a parking lot. The forces of resistance total 52 N. How much power is necessary to push the cart at 1.8 m/s? Round your answer to two significant figures.

(a) 47 W  
(b) 94 W  
(c) 460 W  
(d) 920 W  
(e) None of these

Problem 1458. (Power) How much power, in kilowatts, must be developed by the electric motor of a 1600-kg car moving at 26 m/s on a level road if the forces of resistance total 720 N? Round your answer to the nearest tenth of a kilowatt.

(a) 95.2 kW  
(b) 36.3 kW  
(c) 133.2 kW  
(d) 18.7 kW  
(e) None of these
14.3 Kinetic energy (K.E.)

**Problem 1459.** A model airplane with a mass of 6 kg is flying at a speed of 10 m/s. What is the airplane’s kinetic energy?

- (a) 30 J
- (b) 60 J
- (c) 300 J
- (d) 600 J
- (e) None of these

**Problem 1460.** A skier with a mass of 80 kg is sliding down a slope at 20 m/s. What is the skier’s kinetic energy?

- (a) $3.2 \times 10^3$ J
- (b) $6.4 \times 10^3$ J
- (c) $1.6 \times 10^4$ J
- (d) $3.2 \times 10^4$ J
- (e) None of these

**Problem 1461.** A suitcase with a mass of 20 kg falls out of an airplane flying at a height of 500 m. Just before it hits the ground, the suitcase is falling with a speed of 80 m/s. What is the suitcase’s kinetic energy at this point?

- (a) $1.6 \times 10^3$ J
- (b) $5 \times 10^4$ J
- (c) $6.4 \times 10^4$ J
- (d) $1 \times 10^5$ J
- (e) None of these

**Problem 1462. (Similarity Problem)** A car with a mass of $M$ and a truck with a mass of $9M$ are moving at the same speed $V$. If $K_1$ is the kinetic energy of the car and $K_2$ is the kinetic energy of the truck, which of the following equations is true?

- (a) $K_2 = K_1$
- (b) $K_2 = 3K_1$
- (c) $K_2 = 9K_1$
- (d) $K_2 = 81K_1$
- (e) None of these

**Problem 1463. (Similarity Problem)** When a car is moving at speed $v_1$, its kinetic energy is $K_1$. When it is moving at speed $v_2$, its kinetic energy is $K_2 = 2K_1$. What is the relationship between $v_2$ and $v_1$?

- (a) $v_2 = \sqrt{2} v_1$
- (b) $v_2 = 2v_1$
- (c) $v_2 = 2\sqrt{2} v_1$
- (d) $v_2 = 4v_1$
- (e) None of these

**Problem 1464. (Similarity Problem)** When a crate is pushed at speed $v_c$ across a smooth floor with a low coefficient of friction, it has kinetic energy $K_1$. When the same crate is pushed at the same speed across a rough floor with a higher coefficient of friction, its kinetic energy is $K_2$. What is the relationship between $K_1$ and $K_2$?

- (a) $K_1 > K_2$
- (b) $K_1 = K_2$
- (c) $K_1 < K_2$
- (d) Not enough information: need mass of crate
- (e) None of these
Problem 1465. You are testing your new potato gun at the ballistics lab. A potato fired at a speed of \( v \) has kinetic energy of \( KE \). What is the mass of the potato?

\[
\begin{align*}
(\text{a}) \quad & \sqrt{\frac{2KE}{m}} \\
(\text{b}) \quad & \sqrt{\frac{KE}{2m}} \\
(\text{c}) \quad & \sqrt{\frac{m}{2KE}} \\
(\text{d}) \quad & \sqrt{\frac{KE}{2}} \\
(\text{e}) \quad & \text{None of these}
\end{align*}
\]

Problem 1466. You have just acquired a Civil War cannon, and you want to know its muzzle velocity. You fire a cannonball with a mass of 7.3 kg into a ballistic pendulum, and discover that the kinetic energy of the cannonball was 370,000 J. What was the speed of the cannonball? Round your answer to the nearest 10 m/s.

\[
\begin{align*}
(\text{a}) \quad & 70 \text{ m/s} \\
(\text{b}) \quad & 230 \text{ m/s} \\
(\text{c}) \quad & 320 \text{ m/s} \\
(\text{d}) \quad & 5170 \text{ m/s} \\
(\text{e}) \quad & \text{None of these}
\end{align*}
\]

Problem 1467. (Impulse and K.E.) Two railroad cars are at rest on two horizontal frictionless tracks. One car is three times as heavy as the other. Each car is pushed with the same horizontal force of \( F \) for the same time \( T \). At the end of this time, the kinetic energy of the lighter car is \( K \). What is the kinetic energy of the heavier car?

\[
\begin{align*}
(\text{a}) \quad & 3K \\
(\text{b}) \quad & \sqrt{3K} \\
(\text{c}) \quad & K/\sqrt{3} \\
(\text{d}) \quad & K/3 \\
(\text{e}) \quad & \text{None of these}
\end{align*}
\]
14.4 Potential energy (P.E.)

Problem 1468. Which of the following statements is true?
(a) If a force is directed downward, it does negative work.
(b) The work done by a force on an object equals the increase in the object’s potential energy.
(c) If a force has moved an object around a circle and back to its starting point, then the force has done no work on the object.
(d) If an object is moving in the positive direction, and if a force is applied to it in the negative direction, then the force does negative work on the object.

Problem 1469. You move 56 kg of potatoes from the floor to a shelf 1.9 m high. By how much did you increase the gravitational potential energy of the potatoes? Round your answer to the nearest joule.
(a) 101 J  (b) 106 J  (c) 202 J  (d) 1043 J  (e) None of these

Problem 1470. A case of rat food has a mass of 10.3 kg. To keep rats from eating it, you move it from the floor to a shelf 2.2 m high. By how much did you increase the potential energy of the case? Round your answer to the nearest 10 J.
(a) 160 J  (b) 180 J  (c) 200 J  (d) 220 J  (e) None of these

Problem 1471. A cat has a mass of 4.3 kg. It jumps up from the floor onto a countertop 1.2 m high. How much did the cat’s gravitational potential energy increase in going from the floor to the countertop? Round your answer to the nearest joule.
(a) 5 J  (b) 25 J  (c) 30 J  (d) 51 J  (e) None of these

Problem 1472. A spring can store 36 J of energy when compressed by 0.12 m. What is the spring constant of the spring?
(a) 300 N/m  (b) 600 N/m  (c) 2500 N/m  (d) 5000 N/m  (e) None of these

Problem 1473. You move 41 kg of physics books from the floor to a shelf 2.1 m high. By how much did you increase the gravitational potential energy of the books? Round your answer to the nearest joule.
(a) 86 J  (b) 181 J  (c) 422 J  (d) 844 J  (e) None of these
14.5 Work-Mechanical-Energy Theorem

Work-Mechanical-Energy Theorem: \( W_{ext} = \Delta ME \).

14.5.1 Work-Kinetic-Energy Theorem

Special case 1: Work-Kinetic-Energy Theorem: \( W_{ext} = \Delta KE \).

Problem 1474. A bartender slides a glass of beer with a mass of 480 g along the top of a level bar. The coefficient of kinetic friction between the glass and the bar is \( \mu_k = 0.13 \). When the glass leaves the bartender’s hand, it is moving at a speed of \( v_0 = 2.1 \) m/s. How far does the glass travel before coming to a stop? Round your answer to the nearest 10 cm.

(a) 140 cm (b) 160 cm
(c) 170 cm (d) 190 cm
(e) None of these

Problem 1475. A skier slides down a slope and onto a frozen lake, where he slows down and comes to a halt. When he first reaches the lake, he is moving at 24 m/s. The coefficient of kinetic friction between his skis and the ice is 0.13. How far does he travel across the lake before stopping? Round your answer to the nearest 10 m.

(a) 230 m (b) 250 m
(c) 270 m (d) 300 m
(e) None of these

Problem 1476. A skier slides down a small slope and onto a frozen lake, where he slows down and comes to a halt. When he first reaches the lake, he is moving at 10 m/s. He travels 50 meters across the lake before coming to a stop. What is the coefficient of kinetic friction between his skis and the ice? Round your answer to the nearest hundredth and approximate \( g \) by 10 m/s\(^2\).

(a) .15 (b) .10
(c) .29 (d) .19
(e) None of these

Problem 1477. A car has a weight of 13,500 N. The coefficient of kinetic friction between its wheels and the wet highway is 0.47. The car is travelling at 18.4 m/s when the driver locks up the brakes. How far does the car travel before it comes to a complete stop? Round your answer to the nearest meter.

(a) 37 m (b) 159 m
(c) 339 m (d) 647 m
(e) None of these
Problem 1478. **(Similarity Problem)** You are driving at a speed of $V$ on a straight and level road when you see a deer in front of you and lock up the brakes. You travel a distance $D$ before coming to a complete stop. If your original speed had been $2V$ when you hit the brakes, what distance would you have travelled before stopping?

(a) $D$  
(b) $\sqrt{2}D$  
(c) $2D$  
(d) $4D$  
(e) None of these

14.5.2 Work-Potential-Energy Theorem

**Special case 2:** Work-Potential-Energy Theorem: $W_{ext} = \Delta PE$.

Problem 1479. **(The potential energy in a spring)** A giant spring with spring constant 50 N/m is 10 m long in its unstretched position. Suppose a force acts on the spring and stretches it to 12 m long. How much energy is stored in the spring?

(a) 50 J  
(b) 100 J  
(c) 150 J  
(d) 200 J  
(e) None of these

Problem 1480. A spring is hanging from the ceiling in the physics lab. When there is no weight hanging from it, the spring is 42 cm long; it has a spring constant of 8300 N/m. You hang a lead block with a mass of 87 kg from the spring, stretching it. What is the length of the spring with the weight hanging from it? Round your answer to the nearest centimeter.

(a) 10 cm  
(b) 21 cm  
(c) 32 cm  
(d) 52 cm  
(e) None of these

Problem 1481. An unstretched spring is 15.0 cm long. If you hang an object with a mass of 8.8 kg from it, it stretches to a length of 19.2 cm. What is the spring constant of the spring? Round your answer to two significant figures.

(a) 4.5 N/m  
(b) 22 N/m  
(c) 210 N/m  
(d) 2100 N/m  
(e) None of these

Problem 1482. An unstretched spring has a length of 22.8 cm. When an object with a mass of 12.0 kg is hung from it, it stretches to a length of 35.5 cm. What is the spring constant of the spring? Round your answer to the nearest N/m.

(a) 34 N/m  
(b) 94 N/m  
(c) 331 N/m  
(d) 926 N/m  
(e) None of these
**Problem 1483.** A box has a mass of 22.2 kg. It is sitting on an icy sidewalk, which constitutes a horizontal frictionless surface. A spring with a spring constant of 103 N/m is attached to the box and pulled horizontally, so that the box accelerates at 1.31 m/s². How much does the spring stretch? Round your answer to the nearest 0.01 m.

(a) 0.28 m  
(b) 1.38 m  
(c) 2.11 m  
(d) 2.77 m  
(e) None of these

**Problem 1484.** A spring has a spring constant of 1700 N/m. You pull on it with a force that increases to 120 N until it stops stretching. How much work have you done on the spring? Round your answer to two significant digits.

(a) 4.2 J  
(b) 8.5 J  
(c) 14 J  
(d) 60 J  
(e) None of these

**Problem 1485.** A spring has a spring constant of 680 N/m. You pull on it with a force that increases to 170 N until it stops stretching. How much work have you done on the spring? Round your answer to two significant digits.

(a) 21 J  
(b) 43 J  
(c) 5400 J  
(d) 11000 J  
(e) None of these

**Problem 1486.** With no force applied to it, a spring is 24 cm long. You pull on it with a force of 330 N, stretching it to a length of 30 cm. How much work have you done in stretching the spring?

(a) 9.9 J  
(b) 49.5 J  
(c) 990 J  
(d) 4950 J  
(e) None of these

**Problem 1487.** An unstretched spring is 0.24 m long. When a force that increases to 190 N is applied, it stretches until its total length is 0.30 m. How much work does it take to stretch the spring to this length?

(a) 5.7 J  
(b) 11.4 J  
(c) 28.5 J  
(d) 57 J  
(e) None of these

**Problem 1488.** An unstretched spring has a length of 23.8 cm. When an object with a mass of 18.2 kg is hung from it, it stretches to a length of 32.2 cm. What is the spring constant of the spring? Round your answer to the nearest N/m.

(a) 217 N/m  
(b) 554 N/m  
(c) 2123 N/m  
(d) 9435 N/m  
(e) None of these
Problem 1489. With no force applied to it, a spring hanging from the ceiling is 24 cm long. You hang a weight of 330 N from it, stretching it to a length of 30 cm. How much work was done by gravity in stretching the spring?

(a) 9.9 J  
(b) 49.5 J  
(c) 990 J  
(d) 4950 J  
(e) None of these
Problem 1490. (Derive Problem-Requires Calculus)

[Approximating the strength of gravity as a function of height] The magnitude of the gravitational attraction between two particles of mass $m_1$ and $m_2$ is given by

$$|F_g(r)| = G \frac{m_1 m_2}{r^2},$$

where $r$ is the distance between the two particles and the force that each particle exerts on the other is directed along the radial direction between the two particles, and $G$ is the universal gravitational constant.

(a) What is the potential energy function $U = U_g(r)$, if we take $U_0 = \lim_{r \to \infty} U_g(r) = 0$? That is, we take our reference point out at $r = \infty$.

(b) How much work $W_g$ must be done against the conservative gravitational force to increase the separation distance from $r = R$ to $r = R + h$?

(c) Let $m_1 = M_e$ the mass of the earth and $m_2 = m$ the mass of a small point particle on the earth, where we assume that $m \ll M_e$. Let $R = R_e$ be the mean radius of the earth and $h$ be the height that the particle is above the ground, where $h \ll R_e$. Compute the work done against gravity in moving the object from the earth’s surface at $r = R_e$ to a height $h$ above the ground $r = R_e + h$. Denote this work by $W_g$. Express your answer in terms of the parameter $g = \frac{G M_e}{R_e^2}$, and the variable $\frac{h}{R_e}$. Since $R_e, M_e, m,$ and $G$ are fixed, the work will be a function of $h$. However, it will turn out to be advantageous to think of the work as a function of $\frac{h}{R_e}$.

(d) We are now going to repeat the calculation in part (c) except this time we are going to approximate the force of gravity as a constant force $F_{grav} = -mg$, where $g = \frac{G M_e}{R_e^2}$ is the constant magnitude of gravity that is assumed to be independent of the separation distance between the two attracting masses. This is nothing new, when we apply Newton’s second law to trajectory problems involving gravity on the earth’s surface, such as computing the path of a cannon ball launched from the ground, we take the force of gravity as a constant.

Compute the work $W_{grav}$ done in moving the same object of mass $m$ in part (c) from the ground at $y = 0$ to height $y = h$ against the constant force of gravity $F_{grav} = -mg$.

(e) We now want to compare the exact result for the work found in part (c) with the approximate value of work found in part (d). What is the size of the relative error between the work found in part (c) and (d)? **Hint:** W.L.O.G. we can take $W_{grav}$ as our reference measure. Show that the relative error is

$$\frac{W_g - W_{grav}}{W_{grav}} = O\left(\frac{h}{R_e}\right).$$

(f) The mean radius of the earth is approximately 6370 km. Use the relative error that you find to predict how high you can go above the earth’s surface and safely ignore the fact that gravity is changing with height when you compute the work done in lifting an object from the earth’s surface.
Figure 7: Earth-mass system.
14.6 Conservation of mechanical energy (M.E.)

Problem 1491. Planet X has no air, hence no air resistance. An astronaut standing atop a cliff on Planet X drops a rock down into the lava lake below. If the gravitational potential energy of the rock just before being dropped is $U_{\text{grav}}$, and its kinetic energy just as it hits the lava is $K$, what is the relationship between $U_{\text{grav}}$ and $K$? Use the level of the lava lake as $y = 0$.

(a) $U_{\text{grav}} < K$
(b) $U_{\text{grav}} = K$
(c) $U_{\text{grav}} > K$
(d) Not enough information: it depends on Planet X’s gravity $g_X$.

Problem 1492. Two identical stone blocks are transported from the bottom to the top of a cliff. The first block is fastened to a rope and pulled straight up through the air to the top. The second block is pulled up a frictionless ramp to the top of the cliff. If the work done on the first block is $W_1$ and the work done on the second block is $W_2$, what is the relationship between $W_1$ and $W_2$?

(a) $W_1 < W_2$
(b) $W_1 = W_2$
(c) $W_1 > W_2$
(d) Not enough information: any of these could be true

Problem 1493. Two frictionless inclined planes have the same vertical height $Y$. Plane 1 makes an angle of $60^\circ$ to the horizontal; plane 2 makes an angle of $30^\circ$ to the horizontal. Two objects with the same mass $M$ are released from the tops of the planes to slide down. The object that slides down plane 1 has speed $v_1$ when it reaches the bottom; the object that slides down plane 2 has speed $v_2$ when it reaches the bottom. What is the relationship between $v_1$ and $v_2$?

(a) $v_1 = v_2$
(b) $v_1 = 2v_2$
(c) $v_1 \sin 30^\circ = v_2 \sin 60^\circ$
(d) $v_1 \cos 60^\circ = v_2 \cos 30^\circ$
(e) None of these

Problem 1494. Ricky, the 5 kg snowboarding raccoon, is on his snowboard atop a frictionless ice-covered quarter-pipe with radius 3 meters (see figure at right). If he starts from rest at the top of the quarter-pipe, what is his speed at the bottom of the quarter-pipe? For ease of calculation, assume that $g = 10$ m/s$^2$. Round your answer to the nearest 0.1 m/s.

(a) 7.0 m/s
(b) 7.7 m/s
(c) 8.5 m/s
(d) 9.3 m/s
(e) None of these
Problem 1495. The surface gravity on Planet X is $g_X$. An astronaut standing on the surface of the planet fires a bullet with mass $m$ straight upward with kinetic energy $K$. What is the maximum height $h_{\text{max}}$ reached by the bullet?

(a) $\sqrt{\frac{2K}{mg_X}}$  
(b) $\frac{K}{mg_X}$  
(c) $\sqrt{\frac{2K}{m}}$  
(d) $\frac{K^2}{mg_X}$  
(e) None of these

Problem 1496. Planet X has a surface gravity of 13.6 m/s$^2$ and no atmosphere. An astronaut on Planet X fires a potato gun straight upward; the potato has a mass of 0.73 kg, and leaves the gun with a speed of 34.3 m/s. What is the maximum height that the potato reaches? Round your answer to the nearest meter.

(a) 7 m  
(b) 24 m  
(c) 34 m  
(d) 43 m  
(e) None of these

Problem 1497. (Lab Problem) A spring cannon shoots a ball with a mass of 28 g vertically into the air from ground level. In this situation, air resistance can be ignored. If the spring is compressed by a distance of 1.2 cm, the ball reaches a maximum height of 2.4 m. What is the spring constant of the spring in the cannon? Round your answer to the nearest 100 N/m.

(a) 6700 N/m  
(b) 7400 N/m  
(c) 8200 N/m  
(d) 9100 N/m  
(e) None of these

Problem 1498. (Lab Problem) A spring cannon shoots a ball with a mass of 9.6 g vertically into the air from ground level. In this situation, air resistance can be ignored. The spring is compressed by a distance of 3.4 cm below the mouth of the cannon. If the ball reaches a maximum height of 96 cm above the mouth of the cannon, then what is the spring constant of the spring in the cannon? Round your answer to the nearest 10 N/m. Take $y = 0$ to be at the mouth of the cannon.

(a) 140 N/m  
(b) 150 N/m  
(c) 160 N/m  
(d) 170 N/m  
(e) None of these

Problem 1499. (Lab Similarity Problem) A spring cannon is used to launch two balls straight upward: first, a ball with mass $M$; then a ball with mass $2M$. The spring in the cannon is compressed by the same amount in both launchings. Which of the following statements is true?

(a) The two balls reach the same maximum height.
(b) The larger ball will leave the spring cannon at a higher speed than the smaller ball.
(c) The two balls leave the spring cannon with the same kinetic energy.
(d) At their maximum heights, the smaller ball has greater potential energy than the larger ball.
Problem 1500. A baseball with mass \( M \) and a cannonball with mass \( 9M \) are thrown vertically upward from ground level at the same initial speed \( V \), with no air resistance. The initial kinetic energy of the baseball is \( K_b \); it reaches a maximum height of \( Y_b \); and at this height, its gravitational potential energy is \( U_b \). The corresponding values for the cannonball are \( K_c, Y_c, \) and \( U_c \). Which of the following is true?

(a) \( Y_b = 3Y_c \)  
(b) \( U_c = 9U_b \)  
(c) \( K_b = K_c \)  
(d) \( K_c = 81K_b \)  
(e) None of these

Problem 1501. (Lab Problem) A frictionless horizontal air track has a horizontal spring at one end, with a spring constant of 800 N/m. A glider with a mass of 500 g is pressed against the spring, compressing it by 4 cm. When the glider is released, the spring pushes it down the track. How fast is the glider moving when it leaves the spring?

(a) 20 cm/s  
(b) 40 cm/s  
(c) 80 cm/s  
(d) 160 cm/s  
(e) None of these

Problem 1502. (Lab Problem) A horizontal spring with spring constant \( k_1 \) is at one end of a frictionless horizontal air track. A glider is pushed against the spring, compressing it by a distance of \( x_1 \), so that the energy stored in the spring is 100 J. After the glider is released, it reaches a speed of \( v_1 \) and a kinetic energy of \( K_1 \). The procedure is then repeated, with the spring replaced by one with spring constant \( k_2 > k_1 \); the new spring is compressed by a distance of \( x_2 \), so that the energy stored in it is once again 100 J. When the glider is released this time, it reaches a speed of \( v_2 \) and a kinetic energy of \( K_2 \). Which of the following is true?

(a) \( x_1 = x_2 \)  
(b) \( K_1 < K_2 \)  
(c) \( v_1 = v_2 \)  
(d) \( k_1x_1^2 < k_2x_2^2 \)  
(e) None of these
Problem 1503. (Lab Problem) A frictionless horizontal air track has a spring at either end. The spring on the left has a spring constant of $k_L$; the one on the right has a spring constant of $k_R$. A glider with a mass of $m$ is pressed against the left-hand spring, compressing it by a distance of $x_L$ from its equilibrium position. The glider is then released, so that the spring propels it rightward. It slides along the track and into the right-hand spring. Find a formula for the maximum compression of the right-hand spring $x_R$ as a function of the given parameters: $x_L$, $k_L$, $k_R$, $m$. (The diagram following the answers shows the situation before the glider is released, when the left-hand spring is compressed and the glider is not moving.)

(a) $x_R = \sqrt{\frac{k_L}{k_R}} x_L$
(b) $x_R = \sqrt{\frac{k_R}{k_L}} x_L$
(c) $x_R = \frac{k_L}{k_R} x_L$
(d) $x_R = x_L$
(e) None of these

Problem 1504. (Lab Problem) A frictionless horizontal air track has a spring at either end. The spring on the left has a spring constant of 1800 N/m; the one on the right has a spring constant of 1300 N/m. A glider with a mass of 4.30 kg is pressed against the left-hand spring, compressing it by 3.9 cm. The glider is then released, so that the spring propels it rightward. It slides along the track and into the right-hand spring. What is the maximum compression of the right-hand spring? Round your answer to the nearest 0.1 cm. (The diagram following the answers shows the situation before the glider is released, when the left-hand spring is compressed and the glider is not moving.)

(a) 2.0 cm  (b) 2.8 cm
(c) 4.6 cm  (d) 5.4 cm
(e) None of these
**Problem 1505.** You drop a rock off the Navajo Bridge and let it fall 467 feet to the Colorado River. This is a situation in which air resistance is too large to be ignored. If the gravitational potential energy of the rock just before being dropped is $U_{\text{grav}}$, and its kinetic energy just as it hits the water is $K$, what is the relationship between $U_{\text{grav}}$ and $K$? Use the level of the river as $y = 0$.

(a) $U_{\text{grav}} < K$  
(b) $U_{\text{grav}} = K$  
(c) $U_{\text{grav}} > K$  
(d) Not enough information: any of these could be true

**Problem 1506.** A frozen pond acts as a frictionless horizontal surface. A spring with a spring constant of 32,000 N/m is attached to a wall at the edge of the pond, and a piano with a mass of 450 kg is pushed against the spring, compressing it by 0.18 m. When the piano is released, how fast does it slide across the ice? Round your answer to the nearest 0.1 m/s.

(a) 1.5 m/s  
(b) 2.3 m/s  
(c) 4.6 m/s  
(d) 12.8 m/s  
(e) None of these
Problem 1507. (Derive Problem [Ball in the bucket]) You want to shoot a ball of mass $m$ out of a spring cannon into a bucket. The spring cannon is aimed to fire the ball in the horizontal direction. The bucket’s radius is just a little larger than the ball’s radius. The bucket is designed to capture the ball so long as the ball’s center of mass hits the center of the top of the bucket. The cannon’s manufacturer gives the spring constant as $k$. You measure the following distances: $H$, the height from the top of the bucket to the center of mass of the ball as it sits in the cannon; and $R$, the distance from the end of the muzzle of the cannon (where the ball exits the cannon) to the center of the top of the bucket. Neglecting friction and air drag, and treating the ball as a point particle, derive a formula for the distance $d$ that the spring must be compressed in order to shoot the ball into the bucket (see figure below). Your final answer should be an expression for $d$ as a function of $m$, $k$, $g$, $H$, and $R$. Justify your answer.

(a) $\sqrt{\frac{mg}{2Hk}}$  
(b) $R\sqrt{\frac{mg}{2Hk}}$

(c) $R\sqrt{\frac{Hk}{mg}}$  
(d) $R\sqrt{\frac{mg}{Hk}}$

(e) None of these

Figure 8: Ball in the bucket.
Problem 1508. (Derive Problem) A small block of mass $m$ slides along a frictionless loop-the-loop track. The radius of the loop is $R$. At what height $H$ above the bottom of the track should the block be released from so that it just makes it through the loop-the-loop without losing contact with the track? **Hint:** We want the maximum height such that the normal force exerted by the track on the block at the top of the loop is zero. That is, if we release the block from a height of $H + \epsilon$, where $\epsilon$ is arbitrarily small, then the normal force at the top of the loop will not be zero. In order to receive any credit for this problem, you must show all the work necessary to arrive at the answer. Simply writing down the final formula will earn you a grade of zero.

(a) $\frac{2}{3}R$  
(b) $\frac{3}{4}R$  
(c) $\frac{5}{2}R$  
(d) $\frac{3}{2}$  
(e) None of these

![Figure 9: The loop-the-loop.](image-url)
Problem 1509. (Derive Problem) A small block of mass $m$ sits at rest at the top of a frictionless hemispherical mound of ice of radius $R$. The block is given a slight tap by a very small bug and begins to slide down the side of the mound. Find the height $H$ above the ground where the block leaves the ice. Express this height as a function of the radius $R$.

Hint: Look for the position where the normal force first vanishes. This will be the point where the block leaves the ice.

(a) $\frac{2}{3}R$  
(b) $\frac{3}{4}R$

(c) $\frac{5}{2}R$  
(d) $\frac{3}{2}$

(e) None of these

Figure 10: Block sliding off a hemispherical piece of ice.
15 Conservation of Linear Momentum

15.1 Computing Linear Momentum $p = mv$

**Problem 1510.** A skateboarder with a mass of 60 kg is moving at a speed of 3 m/s. What is his momentum?
(a) 180 kg m/s  
(b) 270 kg m/s  
(c) 360 kg m/s  
(d) 540 kg m/s  
(e) None of these

**Problem 1511.** A shopping cart with a mass of 13.4 kg is rolling across a parking lot at 4.0 m/s. What is the cart’s momentum?
(a) 26.8 kg m/s  
(b) 53.6 kg m/s  
(c) 107.2 kg m/s  
(d) 214.4 kg m/s  
(e) None of these

**Problem 1512.** Your potato gun launches a potato with a mass of 370 g at a speed of 21 m/s. What is the magnitude of the potato’s momentum? Round your answer to two significant digits.
(a) 3.9 kg m/s  
(b) 7.8 kg m/s  
(c) 82 kg m/s  
(d) 160 kg m/s  
(e) None of these

**Problem 1513.** A flying duck with a mass of 2 kg has a momentum of 40 kg m/s. What is the duck’s speed?
(a) 10 m/s  
(b) 20 m/s  
(c) 40 m/s  
(d) 80 m/s  
(e) None of these

**Problem 1514.** An artillery shell has a mass of 5 kg and a momentum of 4000 kg m/s. What is the shell’s speed?
(a) 40 m/s  
(b) 80 m/s  
(c) 160 m/s  
(d) 800 m/s  
(e) None of these
15.2 Applying conservation of momentum to isolated systems

Problem 1515. You drop an object from rest on a planet with no atmosphere. As the object falls, which of the quantities of the object listed below is conserved?
(a) Momentum
(b) Mechanical energy
(c) Potential energy
(d) Kinetic energy

Problem 1516. You drop a wrench from the top of a bridge. Which property of the wrench is conserved as it falls? Assume that there is no air resistance.
(a) Momentum (b) Kinetic energy (c) Potential energy (d) Mechanical energy (e) None of these

Problem 1517. A cannon has a mass of 1100 kg. It fires a cannonball with a mass of 5.3 kg at a muzzle velocity of 390 m/s. How fast does the cannon move backward when the ball is fired? Round your answer to two significant digits.
(a) 1.9 m/s (b) 2.8 m/s (c) 18 m/s (d) 27 m/s (e) None of these

Problem 1518. A cannon has a mass of 1500 kg. It fires a cannonball with a mass of 5.5 kg at a muzzle velocity of 370 m/s. How fast does the cannon move backward when the ball is fired? Round your answer to two significant digits.
(a) -1.4 m/s (b) -2.7 m/s (c) -11 m/s (d) -22 m/s (e) None of these

Problem 1519. An astronaut with a mass of 80 kg is adrift in space outside of his space station. Fortunately, he is holding a space wrench with a mass of 2 kg. How fast must he throw the wrench to give himself a recoil speed of 20 cm/s?
(a) 8 m/s (b) 16 m/s (c) 20 m/s (d) 40 m/s (e) None of these
**Problem 1520.** A cannon with a mass of $m_c$ is sitting on a frictionless frozen lake. It fires a cannonball with a mass of $m_b$ horizontally at a speed of $v_b$. After it has fired the ball, what is the cannon’s backward recoil speed?

(a) $\sqrt{\frac{m_b v_b^2}{m_b + m_c}}$  
(b) $\frac{m_b v_b}{m_b + m_c}$

(c) $\sqrt{\frac{m_b v_b^2}{m_c}}$  
(d) $\frac{m_b v_b}{m_c}$

(e) None of these

**Problem 1521. (Lab Problem: The spring-cannon glider - a discrete rocket)**
A spring cannon is fastened to a glider at rest on a horizontal frictionless air track. The spring-cannon-glider assembly has a mass of $m_c$. It has been previously determined that the cannon can launch a ball with a mass of $m_b$ horizontally at a speed of $v_b$. Use the conservation of momentum to determine the recoil speed of the cannon after it launches the ball. Take the positive $x$-axis in the direction of the ball.

(a) $-\sqrt{\frac{m_b}{m_g}} v_b$  
(b) $-\left(\frac{m_b}{m_b + m_g}\right) v_b$

(c) $-\frac{m_b v_b}{m_g}$  
(d) $-v_b$

(e) None of these

**Problem 1522. (Lab Problem)** A spring cannon is fastened to a glider at rest on a horizontal frictionless air track. The combined mass of the cannon and the glider is 400 g. The cannon fires a ball with a mass of 20 g horizontally at a speed of 8 m/s. How fast does the cannon-glider assembly move backward after the cannon is fired?

(a) 20 cm/s  
(b) 25 cm/s

(c) 40 cm/s  
(d) 50 cm/s

(e) None of these
15.3 Impulse

Problem 1523. An irate constituent throws a pie at a congressman. The pie has a mass of 1.2 kg and hits the congressman with a horizontal speed of 14 m/s. How large is the impulse that the pie gives to the congressman? Round your answer to two significant figures.

(a) 8.4 kg m/s  (b) 17 kg m/s
(c) 120 kg m/s  (d) 140 kg m/s
(e) None of these

Problem 1524. An irate constituent throws a pie at a congressman. The pie has a mass of 1.6 kg and hits the congressman with a horizontal speed of 14 m/s. How large is the impulse that the pie gives to the congressman? Round your answer to two significant figures.

(a) 22 kg m/s  (b) 45 kg m/s
(c) 160 kg m/s  (d) 310 kg m/s
(e) None of these

Problem 1525. A golf ball has a mass of 0.071 kg. It is placed on a tee and hit, giving it a speed of 42 m/s. Golf scientists determine that the club is in contact with the ball for 0.046 s. Assuming constant acceleration, what is the average force exerted on the ball by the club? Round your answer to the nearest newton.

(a) 63 N  (b) 65 N
(c) 125 N  (d) 1361 N
(e) None of these

Problem 1526. A ball weighing 120 g and moving horizontally at 9.2 m/s strikes a wall and rebounds horizontally at 8.4 m/s. What is the magnitude of the impulse that the wall gives to the ball? Round your answer to two significant digits.

(a) 0.096 kg m/s  (b) 0.94 kg m/s
(c) 2.1 kg m/s  (d) 21 kg m/s
(e) None of these

Problem 1527. A ball weighing 290 g and moving horizontally at 14 m/s strikes a wall and rebounds horizontally at 10 m/s. What is the magnitude of the impulse that the wall gives to the ball? Round your answer to two significant digits.

(a) 1.2 kg m/s  (b) 7.0 kg m/s
(c) 14 kg m/s  (d) 43 kg m/s
(e) None of these
Problem 1528. A large truck travelling down the highway collides with a small stationary deer. Which of the following statements is true?

(a) The average force acting on the deer during the collision is greater than the average force acting on the car.
(b) After the collision, the magnitude of the deer’s velocity relative to the car is the same as it was before the collision.
(c) The change in the car’s momentum is greater than the change in the deer’s momentum.
(d) The magnitude of the impulse given to the car by the deer is the same as the magnitude of the impulse given to the deer by the car.

Problem 1529. A large freight train traveling down the tracks collides with a small, cute, cuddly stationary squirrel burying a nut. Which of the following statements is true?

(a) The average force acting on the squirrel during the collision is greater than the average force acting on the train.
(b) After the collision, the magnitude of the squirrel’s velocity relative to the train is the same as it was before the collision.
(c) The change in the train’s momentum is greater than the change in the squirrel’s momentum.
(d) The magnitude of the impulse given to the train by the squirrel is the same as the magnitude of the impulse given to the squirrel by the train.

Problem 1530. (Impulse over long-time period) A crate with a mass of 30 kg slides down a ramp. Starting from rest, it reaches a speed of 3 m/s after 10 s. What is the magnitude of the net force on the crate? Assume the net force on the crate is constant.

(a) 3 N (b) 9 N (c) 27 N (d) 30 N (e) None of these

Problem 1531. A golf ball has a mass of 0.071 kg. It is placed on a tee and hit, giving it a speed of 42 m/s. Golf scientists determine that the club is in contact with the ball for 0.046 s. Assuming constant acceleration, what is the average force exerted on the ball by the club? Round your answer to the nearest newton.

(a) 63 N (b) 65 N (c) 125 N (d) 1361 N (e) None of these

Problem 1532. (Similarity Problem) Two railroad cars are standing on two horizontal frictionless tracks. The first car has mass $M$; the second car has mass $3M$. Each car is pulled along its track with a force of $F$ for a time of $T$. At the end of this time, the momentum of the first car is $P$. What is the momentum of the second car?

(a) $P$ (b) $P/3$ (c) $3P$ (d) $\sqrt{3}P$ (e) None of these
Problem 1533. (Similarity Lab Problem) Two gliders are at rest on parallel horizontal frictionless air-tracks. Glider 1 has mass $m_1$; glider 2 has mass $m_2$. Each glider is pushed with a horizontal force of $F_{\text{push}}$ for the same total time $T$. At the end of this time, glider 1 has kinetic energy $K_1$. What is the relationship between the kinetic energy of glider 2 in terms of the kinetic energy of glider 1?

(a) $K/2$  
(b) $K$

(c) $\sqrt{2}K$  
(d) $4K$

(e) None of these

Problem 1534. (Similarity Problem) Two railroad cars are at rest on two horizontal frictionless tracks. One car is three times as heavy as the other. Each car is pushed with the same horizontal force of $F$ for the same time $T$. At the end of this time, the kinetic energy of the lighter car is $K$. What is the kinetic energy of the heavier car?

(a) $3K$  
(b) $\sqrt{3}K$

(c) $K/\sqrt{3}$  
(d) $K/3$

(e) None of these

Problem 1535. (Similarity Problem) Two carts are at rest on parallel horizontal frictionless tracks. One cart has mass $M$; the second has mass $2M$. Each cart is pushed with a horizontal force of $F$ for a time $T$. At the end of this time, the magnitude of the first cart’s momentum is $P$. What is the magnitude of the second cart’s momentum?

(a) $P/2$  
(b) $P$

(c) $2P$  
(d) $4P$

(e) None of these
15.4 Collisions

Problem 1536. (Lab Problem) Two gliders collide on a horizontal frictionless air track. Which of the following statements must be true regardless of whether the collision is elastic or inelastic?

(a) Neither momentum nor kinetic energy is necessarily conserved.
(b) Kinetic energy is conserved; momentum is not necessarily conserved.
(c) Momentum is conserved; kinetic energy is not necessarily conserved.
(d) Momentum and kinetic energy are both conserved.

Problem 1537. (Lab Problem) Two gliders collide on a frictionless horizontal air track. If the collision is elastic, which of the following statements must be true?

(a) Momentum and kinetic energy are both conserved.
(b) Momentum is conserved; kinetic energy is not conserved.
(c) Kinetic energy is conserved; momentum is not conserved
(d) Neither momentum nor kinetic energy is conserved.

Problem 1538. (Lab Problem) Two gliders collide on a horizontal frictionless air track. If the collision is inelastic, which of the following statements must be true?

(a) Momentum and kinetic energy are both conserved.
(b) Momentum is conserved; kinetic energy is not conserved.
(c) Kinetic energy is conserved; momentum is not conserved
(d) Neither momentum nor kinetic energy is conserved.

Problem 1539. Two railroad cars are rolling toward one another on a frictionless horizontal track. The two cars have the same mass and are moving at the same speed. When they collide, their couplings engage so that they remain attached together. Which of the following statements is true?

(a) The kinetic energy of the system is conserved during the collision.
(b) The momentum of each car is the same before and after the collision.
(c) The two cars lose all of their kinetic energy during the collision.
(d) The momentum of the system decreases during the collision.
Problem 1540. (Lab Problem) A glider with a mass of 500 g is at rest on a frictionless horizontal air track. A second glider with a mass of 250 g is launched toward it at a speed of $V$. The second glider has a spring attached, which is compressed as it collides with the first glider. When the spring is at its maximum compression, which of the following statements is true?

(a) The second glider is at rest relative to the ground.
(b) Both gliders have the same velocity.
(c) Both gliders have the same momentum.
(d) Both gliders have the same kinetic energy.

Problem 1541. (Lab Problem) A glider with a mass of $M$ is at rest on a horizontal frictionless air track. A second glider with a mass of $m$, where $m < M$, is launched toward it at a speed of $V$. The second glider has a spring attached, which is compressed as it collides with the first glider. When the spring is at its maximum compression, which of the following statements is true?

(a) The second glider is at rest relative to the ground.
(b) Both gliders have the same velocity.
(c) Both gliders have the same momentum.
(d) Both gliders have the same kinetic energy.

Problem 1542. (Lab Problem) Two gliders collide on a frictionless horizontal air track. The first glider has a mass of 2.0 kg; before the collision, it is moving rightward at 5.0 m/s. The second glider has a mass of 3.0 kg; before the collision, it is at rest. After the collision, the second glider is moving rightward at 4.0 m/s. What is the velocity of the first glider after the collision?

(a) 1.0 m/s leftward  
(b) 1.0 m/s rightward
(c) 3.0 m/s leftward  
(d) 3.0 m/s rightward
(e) None of these
Problem 1543. (Lab Problem) A glider with a mass of 2.0 kg is moving rightward at a speed of 3.0 m/s on a frictionless horizontal air track. It collides with a stationary glider whose mass is 4.0 kg. If the collision is elastic, what is the speed of the heavier glider after the collision? Round your answer to the nearest 0.1 m/s.

(a) 0.0 m/s  
(b) 1.0 m/s  
(c) 2.0 m/s  
(d) 3.0 m/s  
(e) None of these

Problem 1544. (Lab Problem) Two gliders collide on a frictionless horizontal air track. Glider 1 has a mass of 0.88 kg; glider 2 has a mass of 1.13 kg. Before the collision, glider 1 is moving rightward at 3.3 m/s; glider 2 is at rest. After the collision, glider 1 is moving rightward at 1.1 m/s. How fast is glider 2 moving? Round your answer to the nearest 0.1 m/s.

(a) 1.4 m/s  
(b) 1.7 m/s  
(c) 2.7 m/s  
(d) 4.4 m/s  
(e) None of these

Problem 1545. A block of wood with a mass of 2300 g is sitting on a frictionless horizontal tabletop. A rifle bullet with a mass of 44 g is fired horizontally into the block at a speed of 270 m/s and stops inside the block. After the collision, what is the speed of the combined block and bullet? Round your answer to the nearest 0.1 m/s.

(a) 3.7 m/s  
(b) 4.1 m/s  
(c) 4.6 m/s  
(d) 5.1 m/s  
(e) None of these

Problem 1546. A block of wood with a mass of 1300 g is sitting on a horizontal tabletop. The coefficient of kinetic friction between the block and the table is $\mu_k = 0.74$. A rifle bullet with a mass of 39 g is fired horizontally into the block at a speed of 290 m/s and stops inside the block. How far does the block with the bullet embedded in it slide before coming to a stop? Round your answer to the nearest 0.1 m.

(a) 4.4 m  
(b) 4.9 m  
(c) 5.4 m  
(d) 6.0 m  
(e) None of these

Problem 1547. (Derive Problem) A bullet of mass $m_b$ with an initial velocity of $v_{b,i}$ strikes a wooden block of mass $m_w$ at rest on a frictionless horizontal surface. The bullet emerges with its speed reduced to $v_{b,f}$. Derive a formula for the resulting velocity $v_{w,f}$ of the block. Assume there is no loss of mass in the wooden block when the bullet passes through the block.
Problem 1548. (Lab Problem: Explosion problem) Two gliders, each with mass $M$, are at rest on a frictionless horizontal air track. The gliders are tied together with a horizontal spring between them; the spring has been compressed so that it stores a potential energy of $U$. When the string connecting the two gliders is cut, the spring pushes them apart. After the spring has fully expanded, what is the speed of each glider?

(a) $\frac{U}{2M}$  
(b) $\frac{U}{M}$  
(c) $\sqrt{\frac{2U}{M}}$  
(d) $\sqrt{\frac{U}{M}}$  
(e) None of these

Problem 1549. Two identical gliders have an elastic collision on a horizontal frictionless air track. Assume there are no external forces on the two-glider system, and the motion is one-dimensional. Before the collision, the initial velocity of first glider is $v_{1,i} \neq 0$, and the initial velocity of the second glider is $v_{2,i} = 0$. Suppose that after the collision the first glider is observed to be at rest. What is the velocity of the second glider after the collision?

(a) $v_{1,i}$  
(b) $-v_{1,i}$  
(c) 0  
(d) $-v_{1,i}/2$  
(e) None of these

Problem 1550. (Lab Derive Problem) At either end of a horizontal frictionless air track is a spring. The spring on the left has spring constant $k_L$. The spring on the right has spring constant $k_R$. A glider with a mass of $m_L$ is pressed against the left-hand spring, compressing it a distance $x_L$ from equilibrium. The glider is initially at rest and held in place by a trigger mechanism. The glider is then released, so that the spring propels it rightward. In the middle of the track is a second glider with a mass of $m_R$. The two gliders have small magnets attached, so when the first glider strikes the second, they stick together and slide into the right-hand spring. Let $x_R$ be the maximum compression of the right-hand spring. Derive a formula for $x_R$. (The diagram shows the situation before the left-hand glider is released, when the left-hand spring is compressed and neither glider is moving.)
Problem 1551. (Derive Problem [The Ballistic Pendulum])

Below is a sketch of the ballistic pendulum from Pima community college’s physics lab. The apparatus is designed to measure the speed of the steel ball as it leaves the mouth of the spring cannon. The ball is fired horizontally into a heavy bob attached to a light rod that is free to pivot at one end like a pendulum. The pendulum catches the ball as it leaves the muzzle of the cannon. We treat this as a perfectly-inelastic collision between the ball and the pendulum. The pendulum arm then swings away from the cannon and upward; a device on the apparatus allows one to measure the angle $\theta = \Theta_{\text{max}}$ that the rod swings through. When the arm swings, it lifts the ball to a height of $h_{\text{max}}$ above the ball’s initial height when it was in the spring cannon. Although it would be very hard to measure this height directly, we can use trigonometry to compute the maximum height of the ball in terms of the measured angle $\Theta_{\text{max}}$. In the lab we can measure the values of the mass of the ball $M_{\text{ball}}$; the mass of the pendulum $M_{\text{pend}}$; the distance between the pivot point of the pendulum and the center of mass of the ball-pendulum system, which we shall refer to as the length of the rod $L_{\text{rod}}$; and the angle $\Theta_{\text{max}}$ of the swing after the cannon is fired.

(a) Find the initial velocity of the ball as it leaves the muzzle $v_0$ as a function of the given data: $M_{\text{ball}}$, $M_{\text{pend}}$, $L_{\text{rod}}$, $\Theta_{\text{max}}$. Your answer should not explicitly contain $h_{\text{max}}$, since it can be expressed as a function of these data variables. Note: I have used capital letters to denote known parameters that can be measured in the lab prior to the experiment.

(b) If the spring is compressed from its equilibrium position by a distance $X$, what is the spring constant of the cannon as a function of the measured quantities?

Hint: This is a 3-step problem. In step 1, use conservation of mechanical energy to find the relationship between the spring constant and the velocity of the ball as it leaves the cannon. In step 2, use conservation of momentum over the inelastic collision. Approximate the collision as a one-dimension collision in the horizontal direction. In step 3, apply conservation of mechanical energy to the ball-pendulum system. Take the initial state of the system just after the collision, and the final state of the system at the point where the pendulum is momentarily at rest at the highest point of its upswing.
Figure 12: Ballistic Pendulum: Stage 1

Figure 13: Ballistic Pendulum: Stage 2

Figure 14: Ballistic Pendulum: Stage 3
16 Rotational kinetic energy and angular momentum

Problem 1552. Two physics students are on a merry-go-round that is rotating frictionlessly. Each student has a mass of 80 kg. Initially, one student is standing at the center, while the other is standing at the outer edge. The two change places, walking past one another at the same speed. How does the angular speed of the merry-go-round change as the students move?

(a) The speed of the merry-go-round doesn’t change.
(b) The speed decreases until the students meet halfway between the edge and the center; then it increases.
(c) The speed increases until the students meet halfway between the edge and the center; then it decreases.
(d) The speed increases during the whole time that the students move, because they are adding energy to the system.

Problem 1553. A physics instructor has a mass of $M$; his wife has a mass of $M/2$. The two are on a merry-go-round that is rotating frictionlessly. Initially, the instructor is at the center and his wife is at the outer edge. The two trade places, passing one another halfway from the center to the edge. At which of the following three points is the merry-go-round’s angular speed the greatest?

(a) When the instructor is at the center and the wife is at the edge.
(b) When the wife is at the center and the instructor is at the edge.
(c) When the two are both halfway from the center to the edge.
(d) The merry-go-round’s speed doesn’t change.

Problem 1554. A physics instructor has a mass of $M$; his wife has a mass of $M/2$. The two are on a merry-go-round that is rotating frictionlessly. Initially, the instructor is at the center and his wife is at the outer edge. The two trade places, passing one another halfway from the center to the edge. At which of the following three points is the merry-go-round’s angular speed the smallest?

(a) When the instructor is at the center and the wife is at the edge.
(b) When the wife is at the center and the instructor is at the edge.
(c) When the two are both halfway from the center to the edge.
(d) The merry-go-round’s speed doesn’t change.

Problem 1555. A flywheel’s moment of inertia is 80 kg·m². Starting at rest, it is subjected to a torque of 15 N·m for 6 s. At the end of this time, what is its kinetic energy? Round your answer to two significant figures.

(a) 41 J  (b) 46 J
(c) 51 J  (d) 56 J
(e) None of these
Problem 1556. A wind turbine has three blades. Each blade has a mass of 1000 kg and a length of 20 m. Each blade can be regarded as a thin rod connected at one end to the axis. If the turbine is turning at a rate of one revolution every 3.0 s, what is its kinetic energy? Round your answer to two significant figures.

(a) $8.8 \times 10^5$ J  
(b) $9.7 \times 10^5$ J  
(c) $1.1 \times 10^6$ J  
(d) $1.2 \times 10^6$ J  
(e) None of these

Problem 1557. A shaft can be regarded as a solid cylinder with radius 4 cm and mass 120 kg. If the shaft is turning at 20 revolutions per second, what is its kinetic energy? Round your answer to two significant figures.

(a) 760 J  
(b) 830 J  
(c) 890 J  
(d) 960 J  
(e) None of these

Problem 1558. A windmill whose moment of inertia is $I$ is initially at rest. The wind begins to blow, producing a torque of $\tau$ about the windmill’s axis. After a time $t$, what is the windmill’s kinetic energy?

(a) $\frac{\tau^2 t^2}{2I}$  
(b) $\frac{I\tau^2 t^2}{2}$  
(c) $\frac{\tau t^2}{2I}$  
(d) $\frac{I\tau t^2}{2}$  
(e) None of these

Problem 1559. Two flywheels, one with moment of inertia 240 kg·m² and one with moment of inertia 150 kg·m², turn on the same shaft. The first flywheel is fixed on the shaft; the second one is connected by a clutch, so that it can be made to turn either with the first or independently of it. Initially, the first wheel is turning at 30 rad/s; the second is at rest. The clutch is then operated, so that both wheels turn together. What is their common angular speed? Round your answer to two significant figures.

(a) 17 rad/s  
(b) 18 rad/s  
(c) 20 rad/s  
(d) 22 rad/s  
(e) None of these

Problem 1560. A door’s moment of inertia is 8 kg·m². A bullet with a mass of 25 g moving at 300 m/s strikes the door perpendicularly at a distance of 75 cm from the hinges. The bullet remains embedded in the door. What is the door’s angular speed after the collision? Assume that the embedded bullet does not change the door’s moment of inertia. Round your answer to two significant figures.

(a) 0.63 rad/s  
(b) 0.70 rad/s  
(c) 0.77 rad/s  
(d) 0.85 rad/s  
(e) None of these
Problem 1561. A door’s moment of inertia is 8 kg·m². A bullet with a mass of 25 g, moving horizontally at 300 m/s, strikes the door at an angle of 60° to the door’s surface, at a distance of 75 cm from the hinges. The bullet remains embedded in the door. What is the door’s angular speed after the collision? Assume that the embedded bullet does not change the door’s moment of inertia. Round your answer to two significant figures.

(a) 0.55 rad/s  
(b) 0.61 rad/s  
(c) 0.67 rad/s  
(d) 0.74 rad/s  
(e) None of these

Problem 1562. A door’s moment of inertia is 10 kg·m². A ball with a mass of 40 g moving at 30 m/s strikes the door perpendicularly at a distance of 70 cm from the hinges; the ball bounces backward at 25 m/s. What is the door’s angular speed after the collision? Round your answer to two significant figures.

(a) 0.15 rad/s  
(b) 0.17 rad/s  
(c) 0.19 rad/s  
(d) 0.21 rad/s  
(e) None of these

Problem 1563. A merry-go-round’s moment of inertia is 900 kg·m²; its radius is 2 m. It is initially at rest. A physics student with a mass of 60 kg is standing on the outer edge. The student jumps off at 2.5 m/s, in a direction tangent to the outer edge. After the student has jumped off, how long does it take for the merry-go-round to make one complete revolution? Round your answer to two significant figures.

(a) 19 s  
(b) 21 s  
(c) 23 s  
(d) 25 s  
(e) None of these

Problem 1564. A merry-go-round’s moment of inertia is 6000 kg·m². It is initially turning frictionlessly at a rate of one revolution per 15 s. A physics instructor with a mass of 100 kg is initially standing at the center of the merry-go-round. The instructor then moves to the outer edge, 4 m from the center. When he has reached the outer edge, how long does it take for the merry-go-round to make one revolution? Round your answer to two significant figures.

(a) 15 s  
(b) 17 s  
(c) 19 s  
(d) 21 s  
(e) None of these

Problem 1565. A small wind turbine has a moment of inertia of 400 kg·m², and blades 2 m long. The turbine is at rest, with one blade pointing upward at an angle of 30° to the vertical. A goose with a mass of 8 kg, flying horizontally in the plane of the turbine at 20 m/s, strikes the end of that blade. Upon colliding with the blade, the goose momentarily comes to a stop before falling to the ground. What is the turbine’s angular speed after the collision? Round your answer to two significant figures.

(a) 0.69 rad/s  
(b) 0.76 rad/s  
(c) 0.84 rad/s  
(d) 0.92 rad/s  
(e) None of these
Problem 1566. A hollow pipe has radius $R$. It is initially at rest on top of a hill with height $y$. The pipe is given a nudge so that it rolls down the hill. How fast is it going when it reaches the bottom?

(a) $v = \sqrt{2g(y + R)}$  
(b) $v = \sqrt{g(y + R)}$

(c) $v = \left[2g \left(y + \frac{R}{2}\right)\right]^{1/2}$  
(d) $v = \sqrt{gy}$

(e) None of these

Problem 1567. A thin cylindrical pipe filled with concrete has radius $R$. It is initially at rest on top of a hill with height $y$. The pipe is given a nudge so that it rolls down the hill. How fast is it going when it reaches the bottom?

(a) $v = \sqrt{2g(y + R)}$  
(b) $v = \left[2g \left(y + \frac{R}{2}\right)\right]^{1/2}$

(c) $v = \sqrt{\frac{4gy}{3}}$  
(d) $v = \sqrt{\frac{3gy}{2}}$

(e) None of these

Problem 1568. (This problem requires calculus.) A physics instructor has mass $m_i$; his wife has mass $m_w$. The two are standing on a merry-go-round that is rotating frictionlessly. Initially, the instructor is standing at the outer edge, at a distance $R$ from the center; the wife is standing at the center. The two change places, walking past one another at the same speed in opposite directions. How far from the center will the instructor be when the merry-go-round is rotating at its maximum speed?

(a) $\frac{m_iR}{m_i + m_w}$  
(b) $\frac{m_wR}{m_i + m_w}$

(c) $\sqrt{\frac{m_iR^2}{m_i + m_w}}$  
(d) $\sqrt{\frac{m_wR^2}{m_i + m_w}}$

(e) None of these
Part VI

Applications of Mechanics
17 Applications of Mechanics

17.1 Harmonic Motion

Problem 1569. A pendulum goes through 20 complete cycles in 60 seconds. What is its frequency?

(a) \( \frac{1}{3} \) Hz  
(b) \( \frac{2\pi}{3} \) Hz 
(c) 3 Hz  
(d) \( \frac{3}{2\pi} \) Hz 
(e) None of these

Problem 1570. An oscillator is made using a 10 kg block sliding on a frictionless horizontal surface, attached to a horizontal spring with a force constant of 4000 N/m. The spring is initially stretched by 10 cm and held for a moment, then released. As the block oscillates, what is its maximum speed?

(a) 1 m/s  
(b) 2 m/s 
(c) 4 m/s  
(d) 8 m/s 
(e) None of these

Problem 1571. An oscillator is made using a 5 kg mass sliding on a frictionless horizontal surface, attached to a horizontal spring with a force constant of 20 N/m. What is the period of the oscillator in seconds?

(a) 1 s  
(b) \( \pi \) s 
(c) 2 s  
(d) 2\( \pi \) s 
(e) None of these

Problem 1572. An astronaut moves his furniture to Planet X. On Earth, the pendulum in his grandfather clock has a period of 1.0 s. On Planet X, the pendulum’s period is 0.5 s. What is the gravitational acceleration \( g_X \) of Planet X in terms of Earth’s gravity \( g_E \)?

(a) \( \frac{g_E}{4} \)  
(b) \( \frac{g_E}{2} \) 
(c) 2\( g_E \)  
(d) 4\( g_E \) 
(e) None of these
17.2 Mechanical waves and sound

Problem 1573. A tsunami consists of a series of waves travelling at a speed of 800 km/hr, with a wavelength of 200 km. What is the period of the waves?

(a) 15 s (b) 4 min (c) 15 min (d) 4 hr (e) None of these

Problem 1574. A cable with a linear density of 3 kg/m is stretched between two poles 30 m apart. If the cable is plucked, the resulting transverse wave travels from one pole to the other in 1 second. What is the tension in the cable?

(a) 100 N (b) 300 N (c) 900 N (d) 2700 N (e) None of these

Problem 1575. A violin string is tuned so that its third harmonic has a frequency of 1200 Hz. What is the fundamental frequency of the string?

(a) 150 Hz (b) 400 Hz (c) 3600 Hz (d) 9600 Hz (e) None of these

Problem 1576. An obscure stringed instrument has two strings of identical length and composition, called the P and Q strings. The P string has a tension of $T_P$, so that its fundamental frequency is $f_{P1}$. The Q string is tuned so that its fundamental frequency $f_{Q1}$ is the same as the second harmonic of the P string. What is the tension of the Q string?

(a) $T_P/2$ (b) $\sqrt{2}T_P$ (c) $2T_P$ (d) $4T_P$ (e) None of these

Problem 1577. A violinist is playing a note with a frequency of 480 Hz. The violinist next to him is playing a note with a frequency of 500 Hz. What is the beat frequency produced by the two notes?

(a) 20 Hz (b) 460 Hz (c) 490 Hz (d) 520 Hz (e) None of these
Problem 1578. Two wires of equal length $L$ are stretched between two supports. Both wires are subjected to the same tension $F_T$. The first wire has a mass of $M$; the second, of $3M$. If the fundamental frequency of the first wire is $f_A$ and that of the second wire is $f_B$, which of the following equations is true?

(a) $f_B = \frac{f_A}{3}$  
(b) $f_B = \frac{f_A}{\sqrt{3}}$

(c) $f_B = \sqrt{3} f_A$  
(d) $f_B = 3 f_A$

(e) None of these

Problem 1579. At a distance of 4 m from an omnidirectional speaker, the sound intensity is 9 W/m$^2$. At what distance is the intensity 1 W/m$^2$?

(a) 8 m  
(b) 12 m

(c) 18 m  
(d) 36 m

(e) None of these

Problem 1580. A physics professor is in a car stalled on a railroad track. An oncoming train sounds its horn, and the professor notes that he hears it at a frequency of $f_p$. He knows that when the train is standing still, its horn has a frequency of $f_t$; and that the local speed of sound is $v_0$. How fast is the train approaching him?

(a) $\frac{f_p - f_t}{f_t} v_0$  
(b) $\frac{f_p - f_t}{f_p} v_0$

(c) $\frac{f_t}{f_p - f_t} v_0$  
(d) $\frac{f_p}{f_p - f_t} v_0$

(e) None of these
17.3 Elasticity

Problem 1581. A mop handle makes an angle of $\theta$ to the horizontal. A force of $F$ is applied along the mop handle; the mop contacts the floor over an area of $A$. What is the pressure exerted by the mop on the floor?

(a) $\frac{F}{A \cos \theta}$  
(b) $\frac{F}{A \sin \theta}$  
(c) $\frac{F \cos \theta}{A}$  
(d) $\frac{F \sin \theta}{A}$  
(e) None of these

Problem 1582. Two wires hang from the ceiling. The wires are identical, except that one is twice as long as the other. If a 20 kg mass is attached to the shorter wire, it stretches by 1 mm. If the same mass is attached to the longer wire, how much will it stretch?

(a) 1 mm  
(b) $\sqrt{2}$ mm  
(c) 2 mm  
(d) 4 mm  
(e) None of these

Problem 1583. Two balls made of different materials are dropped from the deck of a ship and sink to the bottom of an ocean trench. On the surface, the first ball has a volume of 60 cm$^3$ and the second ball a volume of 120 cm$^3$. At the bottom of the trench, the first has a volume of 50 cm$^3$ and the second a volume of 110 cm$^3$. If the bulk modulus of the first ball is $B_1$ and that of the second is $B_2$, which of the following is true?

(a) $B_1 < B_2$  
(b) $B_1 = B_2$  
(c) $B_1 > B_2$  
(d) Not enough information  
(e) None of these

Problem 1584. A cable has a cross-sectional area of 4 cm$^2$. It will break if subjected to a tensile stress greater than $2 \times 10^8$ Pa. How much force must be applied to the cable in order to break it?

(a) $8 \times 10^4$ N  
(b) $5 \times 10^7$ N  
(c) $8 \times 10^8$ N  
(d) $5 \times 10^{12}$ N  
(e) None of these
17.4 Static Fluids

17.4.1 Pascal’s Law and Archimedes’s principle

Problem 1585. Which of the following is a statement of Pascal’s law?

(a) If a gas is maintained at a constant temperature, then its volume will be inversely proportional to its pressure.
(b) Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the container.
(c) A body immersed in a fluid is buoyed up with a force equal to the weight of the fluid displaced by the body.
(d) One mole of any ideal gas at standard temperature and pressure will have a volume of 22.4 liters.

Problem 1586. Which of the following is a statement of Archimedes’s principle?

(a) If a gas is maintained at a constant pressure, then its volume will be proportional to its absolute temperature.
(b) Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the container.
(c) A body immersed in a fluid is buoyed up with a force equal to the weight of the fluid displaced by the body.
(d) Don’t be rude to armed Romans.

17.4.2 Density, specific volume, specific weight

Problem 1587. The density of air is 1.3 kg/m$^3$. A room is 5.6 m long by 4.9 m deep by 2.4 m high. What is the mass of the air in the room? Round your answer to the nearest km.

(a) 62 kg  (b) 69 kg
(c) 77 kg  (d) 86 kg
(e) None of these

Problem 1588. The density of air at sea level is 1.20 kg/m$^3$. The living room in your seaside cottage is 5.0 m wide, 4.0 m deep, and 3.0 m high. What is the weight of the air in the room? Give your answer to the nearest pound. Recall: 1 lb = 4.448 N.

(a) 18 lb  (b) 39 lb
(c) 159 lb  (d) 1752 lb
(e) None of these
Problem 1589. Your new roommate has left the water running in the sink of your rectangular water-tight dorm room, and the room is now full of water. If the dimensions of the room are: length = 5.0 m, width = 4.0 m, and height = 3.0 m, and the density of water is $1.0 \times 10^3$ kg/m$^3$, then what is the weight of the water in the room? Round your answer to two significant figures.

(a) $6.5 \times 10^6$ N  (b) $5.9 \times 10^5$ N  
(c) $6.0 \times 10^4$ N  (d) $5.4 \times 10^3$ N  
(e) None of these

Problem 1590. Your new roommate has filled his room with nitrous oxide gas, which has a density of $1.96$ kg/m$^3$. The room is 4.0 m long by 5.0 m wide by 3.0 m high. What is the mass of the N$_2$O in the room? Round your answer to the nearest kilogram.

(a) 6 kg  (b) 15 kg  
(c) 58 kg  (d) 564 kg  
(e) None of these

Problem 1591. Your new roommate has filled his room with helium, which has a density of $0.18$ kg/m$^3$. The room is 4.0 m long by 5.0 m wide by 3.0 m high. What is the mass of the helium in the room? Round your answer to two significant figures.

(a) 0.71 kg  (b) 1.8 kg  
(c) 7.0 kg  (d) 68 kg  
(e) None of these

Problem 1592. The mass of an unknown gas mixture in a room that is $3 \text{ m} \times 4 \text{ m} \times 5 \text{ m}$ is known to be 500 kg. What is the density of the gas ($\rho = \frac{m}{V}$)?

17.4.3 Static Pressure

Problem 1593. A solar-water heating system uses solar panels on the roof of a large build, 40.0 m above the storage tank. The pressure at the panels is 1 atmosphere. What is the absolute pressure at the top of the tank?

Problem 1594. A solar-water heating system uses solar panels on the roof of a large build, 40.0 m above the storage tank. The pressure at the panels is 1 atmosphere. What is the gage pressure at the top of the tank?

Problem 1595. A town in the middle-of-nowhere is built up around a large 100 meter hill. Since the hill is in the center of a town, it is proposed at a town-hall meeting that a water tower be placed at the top of the hill. However, the town barber points out that when water flows through a pipe there is a great amount of resistance owing to viscosity and turbulent mixing of the fluid. He estimates that the gauge pressure in the pipe at the bottom of the hill must be at least 100 kPa in order for water to make it to the outskirts of town. Find what the gauge pressure would be at the bottom of the hill.

Problem 1596. A submarine has a circular top hatch with a .5 m radius. Determine the pressure on the hatch when the sub dives down to a depth of 100 m. The relative density of sea water is 1.03.
Problem 1597. A vertical, frictionless piston-cylinder device contains a gas that is in equilibrium with the weight of the piston balanced by the internal pressure for the gas. Suppose the mass of the piston is $m_{piston}$, the area of the piston cylinder is $A_{cyl}$, and the atmospheric pressure outside the cylinder is $P_{atm}$. Derive a formula for the gas contained in the cylinder $P_{gas}$ in terms of the given information.

Problem 1598. The basic barometer can be used to measure the height of a building. To do this a measurement is made at the bottom of the building at ground level and a second measurement is taken at the top of the building on the roof. One can then relate these readings to pressure using the equation for a mercury-filled barometer: $P_{atm}(h_{Hg}) = \rho_{Hg}gh_{Hg}$, where $\rho_{Hg} = 13,600$ kg/m$^3$ is the density of mercury, $g$ is the acceleration due to gravity, and $h_{Hg}$ is the height of the mercury column in the barometer measured in millimeters. The approximate height of the building can then be found by making a few simplifying assumptions. For moderate sized buildings a reasonable assumption for the average density of air under fair weather conditions is approximately $\rho_{air} \approx 1.18$ Kg/m$^3$.

If the barometric readings at the top and bottom of a certain building are 730 mmHg and 755 mmHg respectively, what is the height of the building?

Problem 1599. A gas is contained in a vertical, frictionless piston-cylinder device. The piston has a mass of 4 kg and cross-sectional area of 35 cm$^2$. A compressed spring above the piston exerts a force of 60 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.

Problem 1600. High-altitude balloons are often filled with helium gas because it weighs only about one-seventh of what air weighs under identical conditions. The buoyancy force, which can be expressed as $F_b = \rho_{air}gV_{balloon}$, will push the balloon upward. If we approximate the shape of the balloon as a sphere with a radius of 5 m and a total payload of 140kg (including the ropes and cage), determine the acceleration of the balloon when it is first released. Assume the density of air is $\rho_{air} = 1.16$ kg/m$^3$.

Problem 1601. A glass tube is attached to a water pipe as shown in the figure below. If the water pressure at the bottom of the tube is 115 kPa and the local atmospheric pressure is $P_{atm} = 92$ kPa, determine how high the water will rise in the tube, in m. Assume $g = 9.8$ m/s$^2$ at that location and take the density of water to be $10^3$ kg/m$^3$. 
Problem 1602. A pressure cooker cooks a lot faster than an ordinary pan by maintaining a higher pressure and temperature inside the cooker. The lid of a pressure cooker is well sealed, and the steam can escape only through an opening in the middle of the lid. A separate piece of metal, the petcock, sits on top of this opening and prevents steam from escaping until the pressure inside the cooker provides enough force on the petcock to overcome the weight of the petcock allowing the hot gas to escape. This mechanism provides a safety device that limits the maximum pressure that can form in the pressure cooker by providing a periodic release of the high-pressure gas in the container. This in turn prevents extremely high pressures from building up in the cooker that could lead to a potentially deadly explosion of hot gas and metal shards.

Assuming that once the cooker reaches the maximum pressure that the gas remains constant in the pressure cooker, determine the mass of the petcock needed for a pressure cooker that is designed to have an operation gage pressure of 100 kPa and has an opening cross-sectional area of 4 mm$^2$. Assume an atmospheric pressure of 101 kPa. Be sure and draw a free-body diagram of the petcock to accompany your solution.
18 Introduction to Thermodynamics

18.1 Introductory Concepts and Definitions

18.1.1 What is the study of thermodynamics?

**Problem 1603.** Which of the following items does **not** describe the main focus of the study of thermodynamics.
(a) energy storage  
(b) the transfer of energy through heat and work  
(c) how energy transforms from one form of energy, such as kinetic, into another form  
(d) how energy is bought and sold on the open market

18.1.2 Defining Systems

**Problem 1604.** Which statement best describes the concept of a system?
(a) the subject of the analysis  
(b) an object with fixed set of molecules  
(c) a fluid together with some sort of solid  
(d) an object that radiates heat into its environment

**Problem 1605.** Consider a refrigerator in a kitchen. Take the refrigerator and everything in it to be our system. Which best describes the system’s surroundings?
(a) All of the air in the kitchen.  
(b) Any one standing in the kitchen.  
(c) The air inside the refrigerator.  
(d) Everything in the universe external to the system.

**Problem 1606.** Consider a refrigerator in a kitchen. Take the refrigerator and everything in it to be our system. Which best describes the system’s boundary?
(a) All of the air in the kitchen.  
(b) The thin region separating the system from everything else.  
(c) The air inside the refrigerator.  
(d) The walls of the refrigerator.
18.1.3 Closed systems

Problem 1607. Circle all of the true statements:
(a) A closed system is necessarily an isolated system
(b) An isolated system is necessarily a closed system
(c) A system cannot be both closed and isolated.
(d) The concepts of closed and isolated in regards to a system are independent concepts.

18.1.4 Control volume

Problem 1608. Circle all of the true statements:
(a) closed system = control mass
(b) closed system = control volume
(c) open system = control mass
(d) open system = control volume

Problem 1609. Which of the following situations would be well suited for using a control volume in the thermodynamic analysis of the system? Do not worry about the details of the analysis.
(a) compression of air in a cylinder
(b) expansion of gases in a cylinder after a combustion
(c) the air in a balloon
(d) filling a bike tire with air from a compressor

Problem 1610. Which of the following situations would be well suited for using a control mass in the thermodynamic analysis of the system? Do not worry about the details of the analysis.
(a) compression of air in a sealed cylinder
(b) a window unit air conditioner
(c) a jet engine
(d) a fire hose spurting out water

18.1.5 Property, state, and process

Problem 1611. Consider the following two statements:
(i) A system is in steady state if its properties are independent of space.
(ii) A system is in steady state if its properties are independent of time.
Which of the following statements is true:
(a) Statement (i) is true and statement (ii) is true
(b) Statement (i) is true and statement (ii) is false
(c) Statement (i) is false and statement (ii) is true
(d) Statement (i) is false and statement (ii) is false
18.1.6 Extensive and intensive properties

**Problem 1612.** Which of the following is not an extensive property?
(a) Kinetic Energy  
(b) Momentum  
(c) Mass  
(d) Density  
(e) None of these

**Problem 1613.** Which of the following is not an intensive property?
(a) Velocity  
(b) Volume  
(c) Pressure  
(d) Temperature  
(e) None of these

18.1.7 Equilibrium, quasi-equilibrium, and processes

**Problem 1614.** Two metal blocks, one at 50°, and the other at 0° are set next to each other in a perfectly insulated box at time $t = 0$. At this instant are the blocks in thermal equilibrium? Now suppose you wait a long time. Are they in equilibrium now?

**Problem 1615.** In a quasi-equilibrium process, the pressure in a system
(a) is held constant throughout the entire process  
(b) is approximately spatially uniform throughout the system at each moment in time  
(c) increases if volume increases  
(d) always varies with temperature

**Problem 1616.** Which of the following process is a quasi-equilibrium process?
(a) the stirring and mixing of cold creamer in hot coffee  
(b) a balloon bursting  
(c) combustion  
(d) the slow and steady compression of air in a cylinder
18.1.8 Base SI units for mass, length, time, Force, energy, and pressure

Problem 1617. Express kinetic energy in terms of the base SI units: the kilogram, meter, and second. Write your answer in terms of the abbreviations (\(\{\text{kg, m, s}\}\)).

Problem 1618. Express work (force \(\times\) distance) in terms of the base SI units: the kilogram, meter, and second. Express your answer in terms of the abbreviations (kg, m, s).

Problem 1619. Express \(PV\), where \(P\) is pressure and \(V\) is volume, in terms of the base SI units: the kilogram, meter, and second. Express your answer in terms of the abbreviations (kg, m, s). Compare this answer to the previous one. What do you notice?

Problem 1620. Express power in terms of the base SI units: the kilogram, meter, and second. Write your answer in terms of the abbreviations (\(\{\text{kg, m, s}\}\)).

Problem 1621. Express specific weight in terms of the base SI units: the kilogram, meter, and second. Express your answer in terms of the abbreviations (\(\{\text{kg, m, s}\}\)).

Problem 1622. Express specific volume in terms of the base SI units: the kilogram, meter, and second. Express your answer in terms of the abbreviations (\(\{\text{kg, m, s}\}\)).
Problem 1623. Which one of the following expressions can be converted to the unit of a Joule?
(a) \( \text{Pa} \cdot \text{m}^2 \)
(b) \( \text{Pa} \cdot \text{m}^3 \)
(c) \( \text{Pa}/\text{m}^2 \)
(d) \( \text{N}/\text{kg} \)
(e) None of these

Problem 1624. Which of the following is not an acceptable “extended” SI unit? Recall: The SI system is the MKS system (\textbf{M}eter, \textbf{K}ilogram, \textbf{S}econd), but we’ll allow the extended SI system to include the cgs system (centimeter, gram, second), but you can’t mix these two systems.
(a) distance measured in centimeters
(b) pressure measured in newtons per square meter
(c) volume measured in cubic meters
(d) density measured in grams per cubic meter
18.2 Temperature Scales

Use the equations $T_C = T_K - 273$ and $T_F = \frac{9}{5}T_C + 32$ to answer the following questions. Round your answers to the nearest integer.

Problem 1625. If heat is added to a system and the temperature of a system increases, without knowing anything else, which form of energy will be definitely increase?
(a) The kinetic energy of the system
(b) The potential energy of the system
(c) The work done by the system
(d) The internal energy (i.e., the molecular energy) of the system

Problem 1626. Convert 98°F to °C.
(a) 32°C  (b) 208°C  
(c) 20°C  (d) 37°C  
(e) None of these

Problem 1627. Convert 68°F to °C.
(a) 12°C  (b) 17°C  
(c) 20°C  (d) 37°C  
(e) None of these

Problem 1628. Convert 110°F to °C.
(a) 50°C  (b) 43°C  
(c) 20°C  (d) 37°C  
(e) None of these

Problem 1629. Convert 20°C to °F.
(a) 68°F  (b) 98°F  
(c) 120°F  (d) 212°F  
(e) None of these
Problem 1630. Convert 100°C to °F.
(a) 68°F  (b) 98°F
(c) 120°F  (d) 212°F
(e) None of these

Problem 1631. Convert 20°C to K.
(a) −253 K  (b) 293 K
(c) 68 K  (d) 0 K
(e) None of these

Problem 1632. Convert 10 K to °C.
(a) −263 K  (b) 293 K
(c) 68 K  (d) 0 K
(e) None of these

Problem 1633. Convert 98°F to °R (degrees rankine).
(a) 460°F  (b) −558°F
(c) 558°F  (d) 200°F
(e) None of these

Note: Some books say do not use the degree symbol with the Rankine scale, since it’s an absolute scale like the Kelvin scale.
Problem 1634. (Measuring wind chill) It is well-known that cold air feels much colder in windy weather than what the thermometer reading indicates because of the “chilling effect” of the wind. This effect is due to the increase in the convection heat transfer coefficient with increasing air velocities. The equivalent wind chill temperature in °F is given by (all variables with an asterisk superscript (*) are dimensional quantities)

\[ T_{\text{equiv},F}^{*} (\text{°F}) = T_{\text{adj},F}^{*} (\text{°F}) + (T_{\text{ambient},F}^{*} (\text{°F}) - T_{\text{adj},F}^{*} (\text{°F})) f(v_b) \]

where \( f \) is the dimensionless function (which must have dimensionless arguments in order to be dimensionally consistent! - for any smooth function this can be proved via a Taylor series expansion)

\[ f(v_b) = 0.475 - 0.0203v_b + 0.304\sqrt{v_b}, \]

and \( T_{\text{adj},F}^{*} = 91.4 \text{°F} \) is a temperature adjustment term, \( v_b \) is a dimensionless wind speed that has been non-dimensionalized by miles per hour (i.e. \( v_b^* = v_b \), (miles/hour and \( T_{\text{ambient}}^{*} \) is the ambient air temperature in °F in calm air, which is taken to be light winds at speeds of no more than 4 mph. The constant 91.4°F in the above equation is the mean temperature of a resting person in a comfortable environment. Windy air at temperature \( T_{\text{ambient}} \) and velocity \( v \) will feel as cold as the calm air at temperature \( T_{\text{equiv}} \). Notice that the units of the coefficients in the second factor of the second term are, in order: dimensionless, hours per mile, and the square root of hours per mile so that the resulting second factor is dimensionless after the dimensional variables are substituted into the equation.

Using proper conversion factors, obtain an equivalent relation in SI units where \( v_{SI} \) is the dimensionless wind speed based on m/s and \( T_{\text{ambient}} \) is ambient air temperature in °C.

**Hint:** When working with physical equations all terms must be dimensionally consistent. To preserve the dimensional consistency whatever we do to one side of the equation, we must do to the other, just as you learned in algebra.