Print your name neatly. If you forget to write your name, or if the grader can’t read your writing, you can lose up to 100% of the total points. Answer all the questions that you can.

No partial credit will be given for these problems. However, you can miss one of the 11 problems without penalty. Your grade will be based on your best 10 problems. You will not receive extra credit for getting all 11 right.

Your grade on the concept test will be based entirely on the answers that you circle on this sheet. If you have no answer or a wrong answer there, the grader will not look at the page with the problem to see if the right answer appears there. Illegible or ambiguous answers will be graded as wrong. You are responsible for copying your answers clearly, correctly, and in the right place.

Although there is no partial credit on this test, you must show your work in the space provided on the test. If need scratch paper ask for it: do not use it unless you have filled all the scratch space provided on the page with the problem. If you answer a difficult problem without doing any written work, the grader will assume that you got the answer by guessing or by copying from someone else, and will not give you credit for the problem even though you’ve indicated the correct solution on the answer sheet.

Notation for this test: For all problems assume the following notational conventions:

- \( u = u(x, y) \), where \( u \) is a dependent variable and \( x \) and \( y \) are independent variables.
- \( y = y(x) \), where \( y \) is a dependent variable and \( x \) is the independent variable.
- \( x = x(t) \), where \( x \) is a dependent variable and \( t \) is the independent variable.

Circle your answers here. Do not detach this sheet from the homework set.
Classification by type

Problem 1. (Classification by type)
Consider the following two equations:

(i) \( yy'' + y' = \sin(x) \),
(ii) \( u_{tt} = u_{xx} \).

Circle the correct answer below.

(a) (i) is an ODE; (ii) is a PDE
(b) (i) is an PDE; (ii) is a ODE
(c) (i) is an ODE; (ii) is a ODE
(d) (i) is an PDE; (ii) is a PDE
(e) None of these

Problem 2. (Classification by type)
Consider the following two equations:

(i) \( u_x + u = \sin(x) \),
(ii) \( u_t = u_{xx} \).

Circle the correct answer below.

(a) (i) is an ODE; (ii) is a PDE
(b) (i) is an PDE; (ii) is a ODE
(c) (i) is an ODE; (ii) is a ODE
(d) (i) is an PDE; (ii) is a PDE
(e) None of these

Classification by order

Problem 3. (Classification by order)
Consider the following two equations:

(i) \( \frac{d^2 y}{dx^2} + \left( \frac{dy}{dx} \right)^4 - 7y^5 = 0 \),
(ii) \( x^5y^2 \frac{d^2 y}{dx^2} = \frac{d^3 y}{dx^3} \).

Circle the correct answer below.

(a) order of eqn (i) = 5; order of eqn (ii) = 3
(b) order of eqn (i) = 4; order of eqn (ii) = 2
(c) order of eqn (i) = 2; order of eqn (ii) = 1
(d) order of eqn (i) = 2; order of eqn (ii) = 3
(e) None of these
Problem 4. (Classification by order)
Consider the following two equations:

(i) \( y'/y'' + y = 0 \),
(ii) \( \ln(y') = y^{1/2} \).

Circle the correct answer below.
(a) order of eqn (i) = 1; order of eqn (ii) = 1
(b) order of eqn (i) = 1; order of eqn (ii) = 1/2
(c) order of eqn (i) = 2; order of eqn (ii) = 1
(d) order of eqn (i) = 2; order of eqn (ii) = 1/2
(e) None of these

Classification as linear or nonlinear

Problem 5. (Classification as linear or nonlinear)
Consider the following two equations:

(i) \( y'' = yy' \),
(ii) \( y'' + \sin(x)y = 0 \).

Circle the correct answer below.
(a) (i) is linear; (ii) is linear
(b) (i) is linear; (ii) is nonlinear
(c) (i) is nonlinear; (ii) is linear
(d) (i) is nonlinear; (ii) is nonlinear
(e) None of these

Problem 6. (Classification as linear or nonlinear)
Consider the following two equations:

(i) \( y' = y^2 \),
(ii) \( y'' = \sin(xy) \).

Circle the correct answer below.
(a) (i) is linear; (ii) is linear
(b) (i) is linear; (ii) is nonlinear
(c) (i) is nonlinear; (ii) is linear
(d) (i) is nonlinear; (ii) is nonlinear
(e) None of these
Determine if you have a solution

Problem 7. (Determine if you have a solution)

Consider the following two equations:

(i) Yes or No. Is \( y = e^{-2x} \) a solution to the ODE \( y' = -4y \)?
(ii) Yes or No. Is \( y = \sin(x) \) a solution to the ODE \( y'' + y = 0 \)?

Circle the correct answer below.
(a) (i) yes; (ii) yes  (b) (i) yes; (ii) no
(c) (i) no; (ii) yes  (d) (i) no; (ii) no
(e) None of these

Problem 8. (Determine if you have a solution)

Consider the following two equations:

(i) Yes or No. Is \( y = e^{-x^2} \) a solution to the ODE \( y' = -xy \)?
(ii) Yes or No. Is \( y = e^{-4x} \) a solution to the ODE \( y' = -4y \)?

Circle the correct answer below.
(a) (i) yes; (ii) yes  (b) (i) yes; (ii) no
(c) (i) no; (ii) yes  (d) (i) no; (ii) no
(e) None of these
Family of curves

Problem 9. (family of curves)
Eliminate the arbitrary constant $c$ by converting the family of curves to a differential equation. That is, find the lowest-order ODE corresponding to the family of curves $y = cx^n$, where $c$ is the parameter to the family of curves and $n \in \mathbb{N}$ is fixed. Circle the correct answer below.

(a) $y' = nxy$       (b) $y' = (n - 1)xy$
(c) $y' = ny/x$       (d) $y' = nx/y$
(e) None of these

Problem 10. (family of curves)
Eliminate the arbitrary constant $c$ by converting the family of curves to a differential equation. That is, find the lowest-order ODE corresponding to the family of curves $y = c\sinh x$, where $c$ is the parameter to the family of curves. Circle the correct answer below.

(a) $y' = y$       (b) $y'' = xy$
(c) $y' = \sqrt{x} y$       (d) $y'' = y$
(e) None of these
Problem 11. (family of curves)

Eliminate the arbitrary constants $c_1$ and $c_2$ by converting the family of curves to a differential equation. That is, find the lowest-order ODE corresponding to the family of curves $y = c_1 \sin x + c_2 \cos x$, where $c_1$ and $c_2$ are the parameters to the family of curves. Circle the correct answer below.

(a) $y''' = -y' 
(b) \quad y'' = 2y'$
(c) $y' = y 
(d) \quad y'' = -y$
(e) None of these