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1 Algebra

1.1 Evaluating functions with numerical arguments

1.1.1 Non-calculator based problems

Problem 1. If \( f(x) = x^2 - 5 \), find \( f(5) \).

(a) \(-5\)  
(b) \(0\)  
(c) \(9\)  
*(d) \(20\)  
(e) None of these

Solution: \( f(5) = 5^2 - 5 = 25 - 5 = 20 \)

Problem 2. If \( f(x) = \frac{x+3}{7} \), find \( f(2) \).

*(a) \(\frac{5}{7}\)  
(b) \(\frac{10}{7}\)  
(c) \(\frac{17}{7}\)  
(d) \(\frac{23}{7}\)  
(e) None of these

Solution: \( f(2) = \frac{2+3}{7} = \frac{5}{7} \)

Problem 3. If \( f(x) = \frac{x+2}{3} \), find \( f(4) \).

(a) \(\frac{5}{7}\)  
(b) \(\frac{2}{3}\)  
*(c) \(2\)  
(d) \(\frac{10}{3}\)  
(e) None of these

Solution: \( f(4) = \frac{4+2}{3} = \frac{6}{3} = 2 \)

Problem 4. If \( f(x) = \frac{x+1}{2} \), find \( f(2) \).

(a) \(\frac{3}{4}\)  
(b) \(1\)  
(c) \(\frac{5}{4}\)  
*(d) \(\frac{3}{2}\)  
(e) None of these

Solution: \( f(2) = \frac{2+1}{2} = \frac{3}{2} \)
Problem 5. If \( f(x) = \frac{x^2 + 1}{3} \), find \( f(2) \).

(a) \( \frac{5}{9} \)  
(b) \( 1 \)  
* (c) \( \frac{5}{3} \)  
(d) \( 3 \)  
(e) None of these

Solution: \( f(2) = \frac{2^2 + 1}{3} = \frac{4 + 1}{3} = \frac{5}{3} \)

Problem 6. If \( f(x) = \frac{x^2 - 1}{3} \), find \( f(2) \).

(a) \( \frac{1}{9} \)  
(b) \( \frac{4}{9} \)  
(c) \( \frac{5}{3} \)  
* (d) \( 1 \)  
(e) None of these

Solution: \( f(2) = \frac{2^2 - 1}{3} = \frac{4 - 1}{3} = \frac{3}{3} = 1 \)

Problem 7. If \( f(x) = \frac{x^2 - 3}{3} \), find \( f(3) \).

(a) \( 0 \)  
(b) \( \frac{1}{9} \)  
(c) \( \frac{1}{3} \)  
* (d) \( 2 \)  
(e) None of these

Solution: \( f(3) = \frac{3^2 - 3}{3} = \frac{9 - 3}{3} = \frac{6}{3} = 2 \)

Problem 8. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(1) \).

(a) \( -2 \)  
(b) \( 2 \)  
(c) \( -\sqrt{6} \)  
* (d) No real value  
(e) None of these

Solution: \( f(1) = \sqrt{1^2 - 5} = \sqrt{1 - 5} = \sqrt{-4} \). Since the quantity under the square root sign is negative, there is no real value.

Problem 9. If \( f(x) = \sqrt{x^2 + 2} \), find \( f(3) \).

* (a) \( \sqrt{11} \)  
(b) \( 5 \)  
(c) \( 25 \)  
(d) No real value  
(e) None of these

Solution: \( f(3) = \sqrt{3^2 + 2} = \sqrt{9 + 2} = \sqrt{11} \)
Problem 10. If \( f(x) = \sqrt{x^2 - 3} \), find \( f(2) \).

(a) \(-1\)  
(b) \(1\)  
(c) \(\sqrt{5}\)  
(d) No real value  
(e) None of these

Solution: \( f(2) = \sqrt{2^2 - 3} = \sqrt{4 - 3} = \sqrt{1} = 1 \)

Problem 11. If \( f(x) = 3x + 2 \), find \( f(4) \).

(a) 12  
(b) 14  
(c) 16  
(d) 18  
(e) None of these

Solution: \( f(4) = 3(4) + 2 = 12 + 2 = 14 \)

Problem 12. If \( f(x) = 3x + 2 \), find \( f(7) \).

(a) 15  
(b) 21  
(c) 23  
(d) 27  
(e) None of these

Solution: \( f(7) = 3(7) + 2 = 21 + 2 = 23 \)

Problem 13. If \( f(x) = x^2 - 5 \), find \( f(5) \).

(a) -5  
(b) 0  
(c) 9  
(d) 20  
(e) None of these

Solution: \( f(5) = 5^2 - 5 = 25 - 5 = 20 \)

Problem 14. If \( f(x) = x^2 - 5 \), find \( f(2) \).

(a) -1  
(b) 1  
(c) 9  
(d) 49  
(e) None of these

Solution: \( f(2) = 2^2 - 5 = 4 - 5 = -1 \)

1.1.2 Calculator-based problems

Problem 15. If \( f(x) = 6.3x - 4.4 \), find \( f(3.9) \).

(a) 14.70  
(b) 16.34  
(c) 18.15  
(d) 20.17  
(e) None of these

Solution: \( f(3.9) = (6.3)(3.9) - 4.4 = 20.17 \)
Problem 16. If \( f(x) = 6.3x - 4.4 \), find \( f(2.1) \).

(a) 7.15  
(b) 7.95  
*(c) 8.83  
(d) 9.71  
(e) None of these

**Solution:** \( f(2.1) = (6.3)(2.1) - 4.4 = 8.83 \)

Problem 17. If \( f(x) = -2.3x + 7.5 \), find \( f(1.1) \).

*(a) 4.97  
(b) 5.47  
(c) 6.01  
(d) 6.62  
(e) None of these

**Solution:** \( f(1.1) = (-2.3)(1.1) + 7.5 = 4.97 \)

Problem 18. If \( f(x) = -2.3x + 7.5 \), find \( f(4.6) \).

(a) -2.77  
*(b) -3.08  
(c) -3.39  
(d) -3.73  
(e) None of these

**Solution:** \( f(4.6) = (-2.3)(4.6) + 7.5 = -3.08 \)

Problem 19. If \( f(x) = \frac{x+3}{7} \), find \( f(5) \). Round your answer to one decimal place.

*(a) 1.1  
(b) 3.7  
(c) 4.0  
(d) 5.4  
(e) None of these

**Solution:** \( f(5) = \frac{5+3}{7} = \frac{8}{7} = 1.1 \)

Problem 20. If \( f(x) = \frac{x+3}{7} \), find \( f(9) \). Round your answer to one decimal place.

(a) 0.9  
*(b) 1.7  
(c) 4.3  
(d) 9.4  
(e) None of these

**Solution:** \( f(9) = \frac{9+3}{7} = \frac{12}{7} = 1.7 \)

Problem 21. If \( f(x) = \frac{x^2 - 5}{4} \), find \( f(5) \).

(a) 0  
*(c) 5  
(b) 1.25  
(d) 23.75  
(e) None of these

**Solution:** \( f(5) = \frac{5^2 - 5}{4} = \frac{20}{4} = 5 \)
Problem 22. If \( f(x) = \frac{x^2 - 5}{4} \), find \( f(9) \).

(a) 4 (b) 15.25 *(c) 19 (d) 79.75 (e) None of these

**Solution:** \( f(9) = \frac{9^2 - 5}{4} = 19 \)

Problem 23. If \( f(x) = \frac{(x - 5)^2}{4} \), find \( f(2) \).

(a) -5.75 (b) -4.25 (c) -0.25 *(d) 2.25 (e) None of these

**Solution:** \( f(2) = \frac{(2 - 5)^2}{4} = 2.25 \)

Problem 24. If \( f(x) = \frac{(x - 5)^2}{4} \), find \( f(7) \).

(a) -4.5 (b) 0.75 *(c) 1 (d) 11 (e) None of these

**Solution:** \( f(7) = \frac{(7 - 5)^2}{4} = 1 \)

Problem 25. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(7) \). Round your answer to one decimal place.

(a) 4.0 (b) 4.8 *(c) 6.6 (d) No real value (e) None of these

**Solution:** \( f(7) = \sqrt{7^2 - 5} = 6.6 \)

Problem 26. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(9) \). Round your answer to one decimal place.

(a) 4.0 (b) 6.8 *(c) 8.7 (d) No real value (e) None of these

**Solution:** \( f(9) = \sqrt{9^2 - 5} = 8.7 \)

Problem 27. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(-4) \). Round your answer to one decimal place.

(a) -6.2 *(b) 3.3 (c) 9.0 (d) No real value (e) None of these

**Solution:** \( f(-4) = \sqrt{(-4)^2 - 5} = 3.3 \)
Problem 28. If \( f(x) = \sqrt{x^2 - 5} \), find \( f(2) \). Round your answer to one decimal place.

(a) \(-0.2\)  \hspace{1cm} (b) \( 3.0 \)  
(c) \( 9.0 \)  \hspace{1cm} *(d) \hspace{1cm} \text{No real value} 
(e) None of these

*Solution:* \( f(2) = \sqrt{2^2 - 5} \). Since the quantity under the square root sign is negative, there is no real value.
### 1.2 Evaluating functions with variable-expression arguments

**Problem 29.** If \( f(x) = 3x + 2 \), find \( f(a + 1) \).

(a) \( 3a + 1 \)  
(b) \( 3a + 2 \)  
(c) \( 3a + 3 \)  
* (d) \( 3a + 5 \)  
(e) None of these

**Solution:** \( f(a + 1) = 3(a + 1) + 2 = 3a + 3 + 2 = 3a + 5 \)

**Problem 30.** If \( f(x) = 3x + 2 \), find \( f(a - 5) \).

(a) \( 3a - 3 \)  
* (c) \( 3a - 13 \)  
(d) \( 3a - 15 \)  
(e) None of these

**Solution:** \( f(a - 5) = 3(a - 5) + 2 = 3a - 15 + 2 = 3a - 13 \)

**Problem 31.** If \( f(x) = x^2 \), find \( f(a + 4) \).

(a) \( a^2 + 4 \)  
(b) \( a^2 + 4a + 4 \)  
(c) \( a^2 + 16 \)  
* (d) \( a^2 + 8a + 16 \)  
(e) None of these

**Solution:** \( f(a + 4) = (a + 4)^2 = a^2 + 8a + 16 \)

**Problem 32.** If \( f(x) = x^2 \), find \( f(a - 3) \).

(a) \( a^2 + 3 \)  
(b) \( a^2 - 3 \)  
(c) \( a^2 + 9 \)  
* (d) \( a^2 - 6a + 9 \)  
(e) None of these

**Solution:** \( f(a - 3) = (a - 3)^2 = a^2 - 6a + 9 \)

**Problem 33.** If \( f(x) = x^2 \), find \( f(a - 2) \).

(a) \( a^2 - 2 \)  
* (c) \( a^2 - 4a + 4 \)  
(b) \( a^2 + 4 \)  
(d) \( a^2 - 2a - 4 \)  
(e) None of these

**Solution:** \( f(a - 2) = (a - 2)^2 = a^2 - 4a + 4 \)

**Problem 34.** If \( f(x) = x^2 - 2 \), find \( f(a + 3) \).

* (a) \( a^2 + 6a + 7 \)  
(b) \( a^2 + 7 \)  
(c) \( a^2 + 2a + 1 \)  
(d) \( a^2 - 10a + 25 \)  
(e) None of these

**Solution:** \( f(a + 3) = (a + 3)^2 - 2 = a^2 + 6a + 9 - 2 = a^2 + 6a + 7 \)
Problem 35. If \( f(x) = x^2 - 2 \), find \( f(a - 1) \).

* (a) \( a^2 - 2a - 1 \)  
(c) \( a^2 - 6a - 9 \)  
(e) None of these

S\olution: \( f(a - 1) = (a - 1)^2 - 2 = a^2 - 2a + 1 - 2 = a^2 - 2a - 1 \)

Problem 36. If \( f(x) = x^2 - 2 \), find \( f(a + 2) \).

(a) \( a^2 \)  
(c) \( a^2 + 4a + 2 \)  
* (c) \( a^2 + 4a + 2 \)  
(e) None of these

S\olution: \( f(a + 2) = (a + 2)^2 - 2 = a^2 + 4a + 4 - 2 = a^2 + 4a + 2 \)

Problem 37. If \( f(x) = \frac{x - 3}{4} \), find \( f(a + 3) \).

* (a) \( \frac{a}{4} \)  
(c) \( \frac{a + 9}{4} \)  
(e) None of these

S\olution: \( f(a + 3) = \frac{(a + 3) - 3}{4} = \frac{a}{4} \)

Problem 38. If \( f(x) = \frac{x - 3}{4} \), find \( f(a - 1) \).

(a) \( \frac{a - 1}{4} \)  
* (b) \( \frac{a - 4}{4} \)  
(c) \( \frac{a + 1}{4} \)  
(e) None of these

S\olution: \( f(a - 1) = \frac{(a - 1) - 3}{4} = \frac{a - 4}{4} \)

Problem 39. If \( f(x) = \frac{x - 3}{4} \), find \( f(a - 5) \).

(a) \( \frac{a - 23}{4} \)  
* (b) \( \frac{a - 8}{4} \)  
(c) \( \frac{a + 23}{4} \)  
(e) None of these

S\olution: \( f(a - 5) = \frac{(a - 5) - 3}{4} = \frac{a - 8}{4} \)
Problem 40. If \( f(x) = (x - 2)^2 \), find \( f(a + 2) \).

\[
\begin{align*}
\text{(a)} & \quad a^2 \\
\text{(b)} & \quad a^2 + 4 \\
\text{(c)} & \quad a^2 + 4a + 2 \\
\text{(d)} & \quad a^2 - 2a + 4 \\
\text{(e)} & \quad \text{None of these}
\end{align*}
\]

**Solution:** \( f(a + 2) = ((a + 2) - 2)^2 = (a)^2 = a^2 \)

Problem 41. If \( f(x) = (x - 2)^2 \), find \( f(a - 1) \).

\[
\begin{align*}
\text{(a)} & \quad a^2 - 2a - 1 \\
\text{(b)} & \quad a^2 - 6a + 9 \\
\text{(c)} & \quad a^2 - 4a + 3 \\
\text{(d)} & \quad a^2 + 4a + 4 \\
\text{(e)} & \quad \text{None of these}
\end{align*}
\]

**Solution:** \( f(a - 1) = ((a - 1) - 2)^2 = (a - 3)^2 = a^2 - 6a + 9 \)

Problem 42. If \( f(x) = (x - 2)^2 \), find \( f(a - 2) \).

\[
\begin{align*}
\text{(a)} & \quad a^2 \\
\text{(b)} & \quad a^2 - 4a + 2 \\
\text{(c)} & \quad a^2 - 4a - 6 \\
\text{(d)} & \quad a^2 - 8a + 16 \\
\text{(e)} & \quad \text{None of these}
\end{align*}
\]

**Solution:** \( f(a - 2) = ((a - 2) - 2)^2 = (a - 4)^2 = a^2 - 8a + 16 \)
1.3 Evaluating functions of multiple variables

1.3.1 Functions of two variables

Non-calculator based problems

Problem 43. The formula for the volume of a cylinder is: \( V = \pi r^2 l \), where \( r \) is the radius of the cylinder and \( l \) is its length. If a cylinder has radius 3 cm and length 2 cm, what is its volume?

(a) \( 9\pi \) cm\(^3\)  
(b) \( 12\pi \) cm\(^3\)  
(c) \( 18\pi \) cm\(^3\)  
(d) \( 36\pi \) cm\(^3\)  
(e) None of these

Solution: \( V = \pi r^2 l = \pi (3 \text{ cm})^2 (2 \text{ cm}) = 18\pi \) cm\(^3\)

Problem 44. The formula for the volume of a cylinder is: \( V = \pi r^2 l \), where \( r \) is the radius of the cylinder and \( l \) is its length. If a cylinder has radius 2 cm and length 3 cm, what is its volume?

(a) \( 9\pi \) cm\(^3\)  
(b) \( 12\pi \) cm\(^3\)  
(c) \( 18\pi \) cm\(^3\)  
(d) \( 36\pi \) cm\(^3\)  
(e) None of these

Solution: \( V = \pi r^2 l = \pi (2 \text{ cm})^2 (3 \text{ cm}) = 12\pi \) cm\(^3\)

Problem 45. The formula for the volume of a cylinder is: \( V = \pi r^2 l \), where \( r \) is the radius of the cylinder and \( l \) is its length. If a cylinder has radius 3 cm and length 4 cm, what is its volume?

(a) \( 9\pi \) cm\(^3\)  
(b) \( 12\pi \) cm\(^3\)  
(c) \( 18\pi \) cm\(^3\)  
(d) \( 36\pi \) cm\(^3\)  
(e) None of these

Solution: \( V = \pi r^2 l = \pi (3 \text{ cm})^2 (4 \text{ cm}) = 36\pi \) cm\(^3\)

Problem 46. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 6 at a speed of 9, what is its centripetal acceleration?

(a) \( \frac{9}{4} \)  
(b) 4  
(c) \( \frac{27}{2} \)  
(d) 54  
(e) None of these

Solution: \( a = \frac{v^2}{r} = \frac{9^2}{6} = \frac{81}{6} = \frac{27}{2} \).
Problem 47. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 4 at a speed of 6, what is its centripetal acceleration?

\[
\begin{array}{cc}
(a) & \frac{4}{9} \\
(b) & \frac{9}{4} \\
(c) & \frac{8}{3} \\
(d) & 9 \\
(e) & \text{None of these}
\end{array}
\]

**Solution:** \( a = \frac{v^2}{r} = \frac{6^2}{4} = \frac{36}{4} = 9 \).

Problem 48. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 6 at a speed of 4, what is its centripetal acceleration?

\[
\begin{array}{cc}
(a) & \frac{4}{9} \\
(b) & \frac{9}{4} \\
(c) & \frac{8}{3} \\
(d) & 9 \\
(e) & \text{None of these}
\end{array}
\]

**Solution:** \( a = \frac{v^2}{r} = \frac{4^2}{6} = \frac{16}{6} = \frac{8}{3} \).

Problem 49. If an object is dropped from a height \( h \), the speed with which it hits the ground is: \( v = \sqrt{2gh} \), where \( g \) is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 5, and the gravitational acceleration is 10, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

\[
\begin{array}{cc}
(a) & 5 \\
(b) & \sqrt{50} \\
(c) & 10 \\
(d) & \sqrt{200} \\
(e) & \text{None of these}
\end{array}
\]

**Solution:** \( v = \sqrt{2gh} = \sqrt{2 \cdot 10 \cdot 5} = \sqrt{100} = 10 \)
Problem 50. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 6, and the gravitational acceleration is 2, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) $\sqrt{12}$  (b) 3
(c) $\sqrt{24}$  (d) 9
(e) None of these

Solution:  $v = \sqrt{2gh} = \sqrt{2 \cdot 2 \cdot 6} = \sqrt{24}$

Problem 51. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 3, and the gravitational acceleration is 5, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) $\sqrt{30}$  (b) 30
(c) $\sqrt{60}$  (d) 60
(e) None of these

Solution:  $v = \sqrt{2gh} = \sqrt{2 \cdot 5 \cdot 3} = \sqrt{30}$

Calculator-based problems

Problem 52. The formula for the volume of a cylinder is: $V = \pi r^2 l$, where $r$ is the radius of the cylinder and $l$ is its length. If a cylinder has radius 5.22 and length 7.80, what is its volume? Round your answer to the nearest integer.

(a) 487  (b) 541
(c) 601  (d) 668
(e) None of these

Solution:  $V = \pi r^2 l = \pi (5.22)^2 (7.80) = 668$

Problem 53. The formula for the volume of a cylinder is: $V = \pi r^2 l$, where $r$ is the radius of the cylinder and $l$ is its length. If a cylinder has radius 3.03 and length 5.51, what is its volume? Round your answer to the nearest integer.

(a) 116  (b) 129
(c) 143  (d) 159
(e) None of these

Solution:  $V = \pi r^2 l = \pi (3.03)^2 (5.51) = 159$
Problem 54. The formula for the volume of a cylinder is: \( V = \pi r^2 l \), where \( r \) is the radius of the cylinder and \( l \) is its length. If a cylinder has radius 6.99 and length 2.43, what is its volume? Round your answer to the nearest integer.

*(a) 373 (b) 410 (c) 451 (d) 496 (e) None of these

**Solution:** \( V = \pi r^2 l = \pi (6.99)^2 (2.43) = 373 \)

Problem 55. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you’ve never heard of “centripetal acceleration”, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 4.8 at a speed of 13, what is its centripetal acceleration? Round your answer to the nearest integer.

(a) 32 *(b) 35 (c) 39 (d) 43 (e) None of these

**Solution:** \( a = \frac{v^2}{r} = \frac{(13)^2}{4.8} = 35 \)

Problem 56. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you don’t know what centripetal acceleration is, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 4.4 at a speed of 19, what is its centripetal acceleration? Round your answer to the nearest integer.

(a) 60 (b) 66 (c) 74 *(d) 82 (e) None of these

**Solution:** \( a = \frac{v^2}{r} = \frac{(19)^2}{4.4} = 82 \)

Problem 57. The formula for the centripetal acceleration of an object moving in a circle is: \( a = \frac{v^2}{r} \), where \( v \) is the object’s speed and \( r \) is the radius of the circle. (If you don’t know what centripetal acceleration is, don’t worry; you’ll learn about it during this course.) If an object is moving in a circle with radius 8.2 at a speed of 27, what is its centripetal acceleration? Round your answer to the nearest integer.

(a) 70 (b) 82 *(c) 89 (d) 98 (e) None of these

**Solution:** \( a = \frac{v^2}{r} = \frac{(27)^2}{8.2} = 89 \)
Problem 58. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 23, and the gravitational acceleration is 32, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 31  (b) 35  
*(c) 38  (d) 42  
(e) None of these

**Solution:** $v = \sqrt{2gh} = \sqrt{2(32)(23)} = 38$

Problem 59. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 39, and the gravitational acceleration is 9.8, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 20  (b) 22  
(c) 25  *(d) 28  
(e) None of these

**Solution:** $v = \sqrt{2gh} = \sqrt{2(9.8)(39)} = 28$

Problem 60. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 120, and the gravitational acceleration is 32, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

(a) 71  (b) 79  
*(c) 88  (d) 96  
(e) None of these

**Solution:** $v = \sqrt{2gh} = \sqrt{2(32)(120)} = 88$

Problem 61. If an object is dropped from a height $h$, the speed with which it hits the ground is: $v = \sqrt{2gh}$, where $g$ is the gravitational acceleration. (If you don’t know what gravitational acceleration is, don’t worry; you’ll learn about it during this course.) If an object is dropped from a height of 56, and the gravitational acceleration is 9.8, what is the speed with which the object hits the ground? Round your answer to the nearest integer.

*(a) 33  (b) 36  
(c) 40  (d) 44  
(e) None of these

**Solution:** $v = \sqrt{2gh} = \sqrt{2(9.8)(56)} = 33$
1.3.2 Functions of three variables

Non-calculator based problems

Problem 62. The speed of an object is given by the formula: $v = v_0 + at$, where $v_0$ is the initial speed, $a$ is the acceleration, and $t$ is the time. What is the speed of an object if $v_0 = 3$, $a = 2$, and $t = 5$?

(a) 10  *(b) 13
(c) 25  (d) 30
(e) None of these

Solution:  $v = v_0 + at = 3 + (2)(5) = 3 + 10 = 13$

Problem 63. The speed of an object is given by the formula: $v = v_0 + at$, where $v_0$ is the initial speed, $a$ is the acceleration, and $t$ is the time. What is the speed of an object if $v_0 = 2$, $a = 3$, and $t = 5$?

*(a) 17  (b) 19
(c) 25  (d) 27
(e) None of these

Solution:  $v = v_0 + at = 2 + (3)(5) = 2 + 15 = 17$

Problem 64. The speed of an object is given by the formula: $v = v_0 + at$, where $v_0$ is the initial speed, $a$ is the acceleration, and $t$ is the time. What is the speed of an object if $v_0 = 5$, $a = 2$, and $t = 3$?

*(a) 11  (b) 21
(c) 25  (d) 30
(e) None of these

Solution:  $v = v_0 + at = 5 + (2)(3) = 5 + 6 = 11$

Problem 65. The force exerted by a stretched spring is given by the formula: $F = k(x - x_0)$, where $k$ is the spring constant, $x$ is the stretched length of the spring, and $x_0$ is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force $F$ if $k = 10$, $x = 5$, and $x_0 = 4$?

*(a) 10  (b) 45
(c) 46  (d) 90
(e) None of these

Solution:  $F = k(x - x_0) = 10(5 - 4) = 10 \cdot 1 = 10$
Problem 66. The force exerted by a stretched spring is given by the formula: $F = k(x - x_0)$, where $k$ is the spring constant, $x$ is the stretched length of the spring, and $x_0$ is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force $F$ if $k = 5$, $x = 10$, and $x_0 = 4$?

(a) 10  *(b) 30
(c) 46  (d) 60
(e) None of these

*Solution:  $F = k(x - x_0) = 5(10 - 4) = 5 \cdot 6 = 30$

Problem 67. The force exerted by a stretched spring is given by the formula: $F = k(x - x_0)$, where $k$ is the spring constant, $x$ is the stretched length of the spring, and $x_0$ is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force $F$ if $k = 4$, $x = 10$, and $x_0 = 5$?

(a) 10  *(b) 20
(c) 35  (d) 40
(e) None of these

*Solution:  $F = k(x - x_0) = 4(10 - 5) = 4 \cdot 5 = 20$

Problem 68. The mass of a cylindrical weight is given by the formula: $m = \pi r^2 l \rho$, where $r$ is the weight’s radius, $l$ is its length, and $\rho$ is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including $\rho$, which is “rho”.) What is the mass of such a weight if its radius is 2, its length is 3, and its density is 5?

(a) $10\pi$  *(b) $30\pi$
*(c) $60\pi$  (d) $90\pi$
(e) None of these

*Solution:  $m = \pi r^2 l \rho = \pi(2^2)(3)(5) = \pi(4)(3)(5) = 60\pi$

Problem 69. The mass of a cylindrical weight is given by the formula: $m = \pi r^2 l \rho$, where $r$ is the weight’s radius, $l$ is its length, and $\rho$ is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including $\rho$, which is “rho”.) What is the mass of such a weight if its radius is 3, its length is 2, and its density is 5?

(a) $10\pi$  *(b) $30\pi$
(c) $60\pi$  *(d) $90\pi$
(e) None of these

*Solution:  $m = \pi r^2 l \rho = \pi(3^2)(2)(5) = \pi(9)(2)(5) = 90\pi$
Problem 70. The mass of a cylindrical weight is given by the formula: \( m = \pi r^2 l \rho \), where \( r \) is the weight’s radius, \( l \) is its length, and \( \rho \) is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including \( \rho \), which is “rho”.) What is the mass of such a weight if its radius is 3, its length is 5, and its density is 2?

(a) 10\(\pi\)  
(b) 30\(\pi\)  
(c) 60\(\pi\)  
*(d) 90\(\pi\)  
(e) None of these

**Solution:**  
\[ m = \pi r^2 l \rho = \pi (3^2)(5)(2) = \pi (9)(5)(2) = 90\pi \]

Calculator-based problems

Problem 71. The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 12 \), \( a = 1.8 \), and \( t = 14 \)? Round your answer to the nearest integer.

(a) 30  
(b) 33  
*(c) 37  
(d) 41  
(e) None of these

**Solution:**  
\[ v = v_0 + at = 12 + (1.8)(14) = 37 \]

Problem 72. The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 45 \), \( a = 2.9 \), and \( t = 8.3 \)? Round your answer to the nearest integer.

(a) 62  
*(b) 69  
(c) 76  
(d) 84  
(e) None of these

**Solution:**  
\[ v = v_0 + at = 45 + (2.9)(8.3) = 69 \]

Problem 73. The speed of an object is given by the formula: \( v = v_0 + at \), where \( v_0 \) is the initial speed, \( a \) is the acceleration, and \( t \) is the time. What is the speed of an object if \( v_0 = 28 \), \( a = -2.1 \), and \( t = 5.6 \)? Round your answer to the nearest integer.

(a) 15  
*(b) 16  
(c) 18  
(d) 20  
(e) None of these

**Solution:**  
\[ v = v_0 + at = 28 + (-2.1)(5.6) = 16 \]
Problem 74. The force exerted by a stretched spring is given by the formula: \( F = k(x - x_0), \) where \( k \) is the spring constant, \( x \) is the stretched length of the spring, and \( x_0 \) is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force \( F \) if \( k = 28, x = 19, \) and \( x_0 = 12? \) Round your answer to the nearest integer.

(a) 143  (b) 159
(c) 176  (d) 196
(e) None of these

Solution:  \( F = k(x - x_0) = (28)(19 - 12) = 196 \)

Problem 75. The force exerted by a stretched spring is given by the formula: \( F = k(x - x_0), \) where \( k \) is the spring constant, \( x \) is the stretched length of the spring, and \( x_0 \) is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force \( F \) if \( k = 1700, x = 0.23, \) and \( x_0 = 0.19? \) Round your answer to the nearest integer.

(a) 68  (b) 75
(c) 82  (d) 91
(e) None of these

Solution:  \( F = k(x - x_0) = (1700)(0.23 - 0.19) = 68 \)

Problem 76. The force exerted by a stretched spring is given by the formula: \( F = k(x - x_0), \) where \( k \) is the spring constant, \( x \) is the stretched length of the spring, and \( x_0 \) is its unstretched length. (You will learn about forces and spring constants during this course.) What is the force \( F \) if \( k = 2700, x = 0.41, \) and \( x_0 = 0.34? \) Round your answer to the nearest integer.

(a) 153  (b) 170
(c) 189  (d) 208
(e) None of these

Solution:  \( F = k(x - x_0) = (2700)(0.41 - 0.34) = 189 \)

Problem 77. The mass of a cylindrical weight is given by the formula: \( m = \pi r^2 l \rho, \) where \( r \) is the weight’s radius, \( l \) is its length, and \( \rho \) is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including \( \rho, \) which is “rho”..) What is the mass of such a weight if its radius is 1.7, its length is 5.1, and its density is 7.9? Round your answer to the nearest integer.

(a) 296  (b) 329
(c) 366  (d) 402
(e) None of these

Solution:  \( m = \pi r^2 l \rho = \pi(1.7)^2(5.1)(7.9) = 366 \)
Problem 78. The mass of a cylindrical weight is given by the formula: $m = \pi r^2 l \rho$, where $r$ is the weight’s radius, $l$ is its length, and $\rho$ is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including $\rho$, which is “rho”.) What is the mass of such a weight if its radius is 0.32, its length is 1.1, and its density is 8.8? Round your answer to one decimal place.

*(a) 3.1 (b) 3.5  
(c) 3.8 (d) 4.2  
(e) None of these

Solution: $m = \pi r^2 l \rho = \pi (0.32)^2 (1.1) (8.8) = 3.1$

Problem 79. The mass of a cylindrical weight is given by the formula: $m = \pi r^2 l \rho$, where $r$ is the weight’s radius, $l$ is its length, and $\rho$ is its density. (You will learn about mass and density in this course. You will also learn a number of Greek letters, including $\rho$, which is “rho”.) What is the mass of such a weight if its radius is 0.55, its length is 1.8, and its density is 19.3? Round your answer to the nearest integer.

*(a) 33 (b) 36  
(c) 40 (d) 44  
(e) None of these

Solution: $m = \pi r^2 l \rho = \pi (0.55)^2 (1.8) (19.3) = 33$
1.4 Solving linear equations

1.4.1 Linear equations with numerical solutions

Problem 80. If $7x + 11 = 8$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:**  
$7x + 11 = 8 \rightarrow 7x = -3 \rightarrow x = \frac{-3}{7}$

Hence $-1 \leq x < 0$

Problem 81. If $9x - 11 = 7$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:**  
$9x - 11 = 7 \rightarrow 9x = 18 \rightarrow x = \frac{18}{9} = 2$

Hence $x \geq 1$

Problem 82. If $5x + 3 = 10$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:**  
$5x + 3 = 10 \rightarrow 5x = 7 \rightarrow x = \frac{7}{5}$

Hence $x \geq 1$

Problem 83. If $3x - 2 = -9$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:**  
$3x - 2 = -9 \rightarrow 3x = -7 \rightarrow x = \frac{-7}{3}$

Hence $x < -1$

Problem 84. If $-4x + 5 = 8$, find $x$. Which of the following statements is true?

(a) $x < -1$  
(b) $-1 \leq x < 0$  
(c) $0 \leq x < 1$  
(d) $x \geq 1$  
(e) None of these

**Solution:**  
$-4x + 5 = 8 \rightarrow -4x = 3 \rightarrow x = \frac{-3}{4}$

Hence $-1 \leq x < 0$
Problem 85. If \(5x - 7 = 3x + 8\), find \(x\). Which of the following statements is true?

(a)  \(x < -1\)  
(b)  \(-1 \leq x < 0\)
(c)  \(0 \leq x < 1\)  
*(d)  \(x \geq 1\)
(e)  None of these

**Solution:** We need to collect all the terms with \(x\) on one side of the equation, and all of the terms with no \(x\) on the other.

\[
5x - 7 = 3x + 8 \quad \overset{-3x}{\longrightarrow} \quad 2x - 7 = 8 \quad \overset{+7}{\longrightarrow} \quad 2x = 15 \quad \overset{\div 2}{\longrightarrow} \quad x = \frac{15}{2}
\]

Hence \(x \geq 1\)

Problem 86. If \(2x - 4 = 5x + 7\), find \(x\). Which of the following statements is true?

*(a)  \(x < -1\)  
(b)  \(-1 \leq x < 0\)
(c)  \(0 \leq x < 1\)  
(d)  \(x \geq 1\)
(e)  None of these

**Solution:** We need to collect all the terms with \(x\) on one side of the equation, and all of the terms with no \(x\) on the other.

\[
2x - 4 = 5x + 7 \quad \overset{-5x}{\longrightarrow} \quad -3x - 4 = 7 \quad \overset{+4}{\longrightarrow} \quad -3x = 11 \quad \overset{\div (-3)}{\longrightarrow} \quad x = -\frac{11}{3}
\]

Hence \(x < -1\)

Problem 87. If \(-3x - 8 = x - 11\), find \(x\). Which of the following statements is true?

(a)  \(x < -1\)  
(b)  \(-1 \leq x < 0\)
*(c)  \(0 \leq x < 1\)  
(d)  \(x \geq 1\)
(e)  None of these

**Solution:** We need to collect all the terms with \(x\) on one side of the equation, and all of the terms with no \(x\) on the other.

\[
-3x - 8 = x - 11 \quad \overset{x}{\longrightarrow} \quad -4x - 8 = -11 \quad \overset{+8}{\longrightarrow} \quad -4x = -3 \quad \overset{\div (-4)}{\longrightarrow} \quad x = \frac{3}{4}
\]

Hence \(0 \leq x < 1\)

Problem 88. If \(-3x - 4 = 2x + 7\), find \(x\). Which of the following statements is true?

*(a)  \(x < -1\)  
(b)  \(-1 \leq x < 0\)
(c)  \(0 \leq x < 1\)  
(d)  \(x \geq 1\)
(e)  None of these

**Solution:** We need to collect all the terms with \(x\) on one side of the equation, and all of the terms with no \(x\) on the other.

\[
-3x - 4 = 2x + 7 \quad \overset{-2x}{\longrightarrow} \quad -5x - 4 = 7 \quad \overset{+4}{\longrightarrow} \quad -5x = 11 \quad \overset{\div (-5)}{\longrightarrow} \quad x = -\frac{11}{5}
\]

Hence \(x < -1\)
Problem 89. If \( x + 7 = -4x + 9 \), find \( x \). Which of the following statements is true?

(a) \( x < -1 \)  
(b) \( -1 \leq x < 0 \)  
*(c) \( 0 \leq x < 1 \)  
(d) \( x \geq 1 \)  
(e) None of these

**Solution:** We need to collect all the terms with \( x \) on one side of the equation, and all of the terms with no \( x \) on the other.

\[
x + 7 = -4x + 9 \quad \rightarrow \quad +4x \quad \rightarrow \quad 5x + 7 = 9 \quad \rightarrow \quad -7 \quad \rightarrow \quad 5x = 2 \quad \rightarrow \quad \frac{5}{5} \quad \rightarrow \quad x = \frac{2}{5}
\]

Hence \( 0 \leq x < 1 \)

1.4.2 Linear equations with variable-expression solutions

Problem 90. If \( PV = nRT \), find \( P \).

(a) \( P = \frac{V}{nRT} \)  
*(b) \( P = \frac{nRT}{V} \)  
(c) \( P = V - nRT \)  
(d) \( P = nRT - V \)  
(e) None of these

**Solution:** \( PV = nRT \quad \rightarrow \quad V \quad \rightarrow \quad P = \frac{nRT}{V} \)

Problem 91. If \( PV = nRT \), find \( T \).

*(a) \( T = \frac{PV}{nR} \)  
(b) \( T = \frac{nR}{PV} \)  
(c) \( T = PV - nR \)  
(d) \( T = nR - PV \)  
(e) None of these

**Solution:** \( PV = nRT \quad \rightarrow \quad \text{switch sides} \quad \rightarrow \quad nRT = PV \quad \rightarrow \quad nR \quad \rightarrow \quad T = \frac{PV}{nR} \)

Problem 92. If \( m = \pi r^2 l \rho \), find \( \rho \).

(a) \( \rho = m - \pi r^2 l \)  
(b) \( \rho = \pi r^2 l - m \)  
*(c) \( \rho = \frac{m}{\pi r^2 l} \)  
(d) \( \rho = \frac{\pi r^2 l}{m} \)  
(e) None of these

**Solution:** \( m = \pi r^2 l \rho \quad \rightarrow \quad \text{switch sides} \quad \rightarrow \quad \pi r^2 l \rho = m \quad \rightarrow \quad \frac{\pi r^2 l}{m} \quad \rightarrow \quad \rho = \frac{m}{\pi r^2 l} \)
Problem 93. If \( m = \pi r^2 l \rho \), find \( l \).

(a) \( l = m - \pi r^2 \rho \)  
(b) \( l = \pi r^2 \rho - m \)  
* (c) \( l = \frac{m}{\pi r^2 \rho} \)  
(d) \( l = \frac{\pi r^2 \rho}{m} \)  
(e) None of these

**Solution:** \( m = \pi r^2 l \rho \) \( \xrightarrow{\text{switch sides}} \pi r^2 l \rho = m \) \( \xrightarrow{\div \pi r^2 \rho} \) \( l = \frac{m}{\pi r^2 \rho} \)

Problem 94. If \( v = v_0 + at \), find \( v_0 \).

(a) \( v_0 = \frac{v}{at} \)  
(b) \( v_0 = -\frac{v}{at} \)  
(c) \( v_0 = v + at \)  
* (d) \( v_0 = v - at \)  
(e) None of these

**Solution:** \( v = v_0 + at \) \( \xrightarrow{\text{switch sides}} \) \( v_0 + at = v \) \( \xrightarrow{-at} \) \( v_0 = v - at \)

Problem 95. If \( v = v_0 + at \), find \( a \).

* (a) \( a = \frac{v - v_0}{t} \)  
(b) \( a = \frac{v}{t} - v_0 \)  
(c) \( a = \frac{vt - v_0}{t} \)  
(d) \( a = v - \frac{v_0}{t} \)  
(e) None of these

**Solution:** \( v_0 + at = v \) \( \xrightarrow{-v_0} \) \( at = v - v_0 \) \( \xrightarrow{\div t} \) \( a = \frac{v - v_0}{t} \)

Problem 96. If \( v = v_0 + at \), find \( t \).

(a) \( t = \frac{v}{a} - v_0 \)  
(b) \( t = \frac{v_a - v_0}{a} \)  
(c) \( t = v - \frac{v_0}{a} \)  
* (d) \( t = \frac{v - v_0}{a} \)  
(e) None of these

**Solution:** \( v_0 + at = v \) \( \xrightarrow{-v_0} \) \( at = v - v_0 \) \( \xrightarrow{\div a} \) \( t = \frac{v - v_0}{a} \)

Problem 97. If \( F = k(x - x_0) \), find \( k \).

(a) \( k = \frac{x_0 - x}{F} \)  
(b) \( k = \frac{x - x_0}{F} \)  
(c) \( k = \frac{F}{x_0 - x} \)  
* (d) \( k = \frac{F}{x - x_0} \)  
(e) None of these

**Solution:** \( k(x - x_0) = F \) \( \xrightarrow{\div (x - x_0)} \) \( k = \frac{F}{x - x_0} \)
Problem 98. If $F = k(x - x_0)$, find $x$.

(a) $x = \frac{k - F}{x_0}$
(b) $x = \frac{F + kx_0}{k}$
(c) $x = F - kx_0$
(d) $x = F + kx_0$
(e) None of these

**Solution:** Here, it’s a good idea to start by expanding the expression to get rid of the parentheses.

\[ F = k(x - x_0) = kx - kx_0 \implies kx = F + kx_0 \implies x = \frac{F + kx_0}{k} \]

Problem 99. If $F = k(x - x_0)$, find $x_0$.

* (a) $x_0 = \frac{kx - F}{k}$
(b) $x_0 = \frac{F + kx}{k}$
(c) $x_0 = \frac{F - x}{k}$
(d) $x_0 = \frac{F + x}{k}$
(e) None of these

**Solution:** Here, it’s a good idea to start by expanding the expression to get rid of the parentheses.

\[ F = k(x - x_0) = kx - kx_0 \implies -kx_0 = F - kx \implies x_0 = \frac{F - kx}{-k} = \frac{kx - F}{k} \]
1.5 Solving systems of linear equations

Problem 100. For the following pair of equations, find \( x \) and \( y \):

\[
\begin{align*}
5x - 2y &= 1 \\
-2x + y &= 2
\end{align*}
\]

What is the product \( xy \)?

\[
\begin{array}{ll}
(a) & xy = -48 \\
(b) & xy = -60 \\
(c) & xy = 48 \\
(d) & xy = 60 \\
(e) & \text{None of these}
\end{array}
\]

**Solution:** There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. In this case, since \( y \) has a coefficient of 1 in the second equation, it’s easy to solve it for \( y \):

\[
\begin{align*}
-2x + y &= 2 \\
\downarrow &+2x \\
y &= 2x + 2
\end{align*}
\]

Now substitute this into the first equation:

\[
\begin{align*}
5x - 2y &= 1 \\
y &= 2x + 2 \\
\downarrow &+2x \\
5x - 2(2x + 2) &= 1 \\
\text{simplify} \\
x - 4 &= 1 \\
\downarrow &+4 \\
x &= 5
\end{align*}
\]

Now that we know \( x \), we can substitute into the expression for \( y \):

\[
y = 2x + 2 = 2(5) + 2 = 12
\]

It’s always a good idea to check your solutions in the original equations:

\[
\begin{align*}
5x - 2y &= 5(5) - 2(12) = 25 - 24 = 1 \\
-2x + y &= -2(5) + 12 = -10 + 12 = 2
\end{align*}
\]

Then \( xy = (5)(12) = 60 \).

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, if we multiply the second equation by 2, we can eliminate \( y \).

\[
\begin{align*}
5x - 2y &= 1 & &\rightarrow & & 5x - 2y &= 1 \\
-2x + y &= 2 & &\times2 & & -4x + 2y &= 4 \\
\downarrow &\text{add} & & & & x + 0y = 5 \Rightarrow x = 5
\end{align*}
\]

Now we can substitute this value into either of the original two equations and solve for \( y \). Since the coefficient of \( y \) in the second equation is 1, it’s easiest to use that one:

\[
\begin{align*}
-2x + y &= 2 \\
\downarrow &+2x \\
y &= 2x + 2 = 2(5) + 2 = 12
\end{align*}
\]
Problem 101. For the following pair of equations, find $x$ and $y$:

$$2x + y = 4$$
$$3x + 2y = 12$$

What is the product $xy$?

* (a) $xy = -48$  
(b) $xy = -60$

(c) $xy = 48$  
(d) $xy = 60$

(e) None of these

**Solution:** There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. In this case, since $y$ has a coefficient of 1 in the first equation, it’s easy to solve it for $y$:

$$y = 4 - 2x$$

Now substitute this into the second equation:

$$3x + 2(4 - 2x) = 12$$
$$3x + 8 - 4x = 12$$
$$-x + 8 = 12$$
$$x = -4$$

Now that we know $x$, we can substitute into the expression for $y$:

$$y = -2x + 4 = -2(-4) + 4 = 12$$

It’s always a good idea to check your solutions in the original equations:

$$2x + y = 2(-4) + 12 = 4$$
$$3x + 2y = 3(-4) + 2(12) = 12$$

Then $xy = (-4)(12) = -48$.

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, if we multiply the first equation by 2, we can eliminate $y$.

$$2x + y = 4$$
$$3x + 2y = 12$$

$$\times 2$$

$$4x + 2y = 8$$
$$3x + 2y = 12$$

$$\text{subtract}$$

$$x + 0y = -4 \implies x = -4$$

Now we can substitute this value into either of the original two equations and solve for $y$. Since the coefficient of $y$ in the first equation is 1, it’s easiest to use that one:

$$2x + y = 4$$
$$-2x$$

$$y = -2x + 4 = -2(-4) + 4 = 12$$
Problem 102. For the following pair of equations, find $x$ and $y$:

\[
6x + y = -2 \\
-2x - y = 23
\]

What is the product $xy$?

(a) $xy = -48$  
(b) $xy = -60$  
(c) $xy = 48$  
(d) $xy = 60$  
(e) None of these

Solution: There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. In this case, it’s easy to solve the second equation for $x$:

\[
x - y = 23 \quad \Rightarrow \quad x = y + 23
\]

Now substitute this into the first equation:

\[
6x + y = -2 \quad \Rightarrow \quad 6(y + 23) + y = -2 \quad \Rightarrow \quad 7y + 138 = -2
\]

\[
\Rightarrow \quad 7y = -140 \quad \Rightarrow \quad y = -20
\]

Now that we know $y$, we can substitute into the expression for $x$:

\[
x = y + 23 = -20 + 23 = 3
\]

It’s always a good idea to check your solutions in the original equations:

\[
6x + y = 6(3) + (-20) = 18 - 20 = -2 \quad \Rightarrow \quad x - y = 3 - (-20) = 3 + 20 = 23
\]

Then $xy = (3)(-20) = -60$.

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, we don’t have to multiply by constants: we can add the two equations and eliminate $y$.

\[
6x + y = -2 \\
x - y = 23
\]

\[
\Rightarrow \quad 7x + 0y = 21 \quad \Rightarrow \quad 7x = 21 \quad \Rightarrow \quad x = 3
\]

Now we can substitute this value into either of the original two equations and solve for $y$. Since the coefficient of $y$ in the first equation is 1, it’s easiest to use that one:

\[
6x + y = -2 \quad \Rightarrow \quad y = -6x - 2 = -6(3) - 2 = -18 - 2 = -20
\]
Problem 103. For the following pair of equations, find $x$ and $y$:

$$-x + y = 2$$
$$3x - 2y = 2$$

What is the product $xy$?

(a) $xy = -48$  
(b) $xy = -60$

*(c) $xy = 48$  
(d) $xy = 60$

(e) None of these

**Solution:** There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. In this case, since $y$ has a coefficient of 1 in the first equation, it’s easy to solve it for $y$:

$$-x + y = 2 \quad \xrightarrow{+x} \quad y = x + 2$$

Now substitute this into the second equation:

$$3x - 2y = 2 \quad \xrightarrow{y=x+2} \quad 3x - 2(x + 2) = 2 \quad \xrightarrow{\text{simplify}} \quad x - 4 = 2 \quad \xrightarrow{+4} \quad x = 6$$

Now that we know $x$, we can substitute into the expression for $y$:

$$y = x + 2 = 6 + 2 = 8$$

It’s always a good idea to check your solutions in the original equations:

$$-x + y = -6 + 8 = 2 \quad 3x - 2y = 3(6) - 2(8) = 18 - 16 = 2$$

Then $xy = (6)(8) = 48$.

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, if we multiply the first equation by 2, we can eliminate $y$.

$$-x + y = 2 \quad \xrightarrow{\times 2} \quad -2x + 2y = 4$$
$$3x - 2y = 2 \quad \xrightarrow{\text{add}} \quad 3x - 2y = 2$$

$$x + 0y = 6 \quad \Rightarrow \quad x = 6$$

Now we can substitute this value into either of the original two equations and solve for $y$. Since the coefficient of $y$ in the first equation is 1, it’s easiest to use that one:

$$-x + y = 2 \quad \xrightarrow{+x} \quad y = x + 2 = 6 + 2 = 8$$
Problem 104. For the following pair of equations, find $x$ and $y$:

\[-x + y = 15 \]
\[2x + 3y = 0 \]

What is the product $xy$?

* (a) $xy = -54$

(b) $xy = -56$

(c) $xy = 54$

(d) $xy = 56$

(e) None of these

Solution: There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. In this case, since $y$ has a coefficient of 1 in the first equation, it’s easy to solve it for $y$:

\[-x + y = 15 \quad \rightarrow \quad y = x + 15 \]

Now substitute this into the first equation:

\[2x + 3y = 0 \quad \rightarrow \quad 2x + 3(x + 15) = 0 \quad \text{simplify} \quad 5x + 45 = 0 \]
\[-45 \quad 5x = -45 \quad \div 5 \quad x = -9 \]

Now that we know $x$, we can substitute into the expression for $y$:

\[y = x + 15 = -9 + 15 = 6 \]

It’s always a good idea to check your solutions in the original equations:

\[-x + y = -(9) + 6 = 9 + 6 = 15 \quad 2x + 3y = 2(-9) + 3(6) = -18 + 18 = 0 \]

Then $xy = (-9)(6) = -54$.

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, if we multiply the first equation by 2, we can eliminate $x$.

\[-x + y = 15 \quad \times 2 \quad -2x + 2y = 30 \]
\[2x + 3y = 0 \quad \rightarrow \quad 2x + 3y = 0 \]
\[\text{add} \quad 0x + 5y = 30 \quad \Rightarrow \quad 5y = 30 \quad \div 5 \quad y = 6 \]

Now we can substitute this value into either of the original two equations and solve for $y$. We’ll choose the second equation, because of the zero on the right:

\[2x + 3y = 0 \quad \div 3 \quad 2x = -3y \quad \div 2 \quad x = -\frac{3y}{2} = -\frac{3(6)}{2} = -9 \]
Problem 105. For the following pair of equations, find $x$ and $y$:

\[
\begin{align*}
  x + 3y &= 2 \\
  2x + 5y &= 8 
\end{align*}
\]

What is the product $xy$?

(a) $xy = -54$  
(b) $xy = -56$  
(c) $xy = 54$  
(d) $xy = 56$  
(e) None of these

**Solution:** There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. In this case, since $x$ has a coefficient of 1 in the first equation, it’s easy to solve it for $x$:

\[
x + 3y = 2 \quad \rightarrow \quad x = -3y + 2
\]

Now substitute this into the second equation:

\[
\begin{align*}
  2x + 5y &= 8 \\
  2(-3y + 2) + 5y &= 8 \\
  -6y + 4 + 5y &= 8 \\
  -y + 4 &= 8 \\
  y &= -4
\end{align*}
\]

Now that we know $y$, we can substitute into the expression for $x$:

\[
x = -3y + 2 = -3(-4) + 2 = 12 + 2 = 14
\]

It’s always a good idea to check your solutions in the original equations:

\[
\begin{align*}
  x + 3y &= 14 + 3(-4) = 14 - 12 = 2 \\
  2x + 5y &= 2(14) + 5(-4) = 28 - 20 = 8
\end{align*}
\]

Then $xy = (14)(-4) = -56$.

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, if we multiply the first equation by 2, we can eliminate $x$.

\[
\begin{align*}
  x + 3y &= 2 \quad \rightarrow \quad 2x + 6y = 4 \\
  2x + 5y &= 8 \\
  0x + y &= -4 \quad \Rightarrow \quad y = -4
\end{align*}
\]

Now we can substitute this value into either of the original two equations and solve for $y$. Since the coefficient of $x$ in the first equation is 1, it’s easiest to use that one:

\[
x + 3y = 2 \quad \rightarrow \quad x = -3y + 2 = -3(-4) + 2 = 12 + 2 = 14
\]
Problem 106. For the following pair of equations, find $x$ and $y$:

$$
5x - y = -3 \\
-3x + y = 9
$$

What is the product $xy$?

(a) $xy = -54$  
(b) $xy = -56$
*(c) $xy = 54$  
(d) $xy = 56$
(e) None of these

**Solution:** There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. In this case, since $y$ has a coefficient of 1 in the second equation, it’s easy to solve it for $y$:

$$-3x + y = 9 \quad \Rightarrow \quad y = 3x + 9$$

Now substitute this into the first equation:

$$5x - y = 3 \quad \Rightarrow \quad 5x - (3x + 9) = -3 \quad \Rightarrow \quad 2x - 9 = -3 \quad \Rightarrow \quad 2x = 6 \quad \Rightarrow \quad x = 3$$

Now that we know $x$, we can substitute into the expression for $y$:

$$y = 3x + 9 = 3(3) + 9 = 18$$

It’s always a good idea to check your solutions in the original equations:

$5x - y = 5(3) - 18 = 15 - 18 = -3$  
$-3x + y = -3(3) + 18 = -9 + 18 = 9$

Then $xy = (3)(18) = 54$.

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, we don’t have to multiply by constants: we can add the two equations and eliminate $y$.

$$
5x - y = -3 \\
-3x + y = 9
$$

$$\Rightarrow \quad 2x + 0y = 6 \quad \Rightarrow \quad 2x = 6 \quad \Rightarrow \quad x = 3
$$

Now we can substitute this value into either of the original two equations and solve for $y$. Since the coefficient of $y$ in the second equation is 1, it’s easiest to use that one:

$$-3x + y = 9 \quad \Rightarrow \quad y = 3x + 9 = 3(3) + 9 = 9 + 9 = 18$$
Problem 107. For the following pair of equations, find $x$ and $y$:
\[
\begin{align*}
x + y &= 15 \\
x - y &= 1
\end{align*}
\]

What is the product $xy$?
(a) $xy = -54$  
(b) $xy = -56$
(c) $xy = 54$  
(d) $xy = 56$
(e) None of these

Solution: There are two different approaches that we can use to solve this. We’ll go through both.

The first approach is to solve one of the equations for one of the variables, then to substitute that expression into the other equation. We’ll solve the second equation for $x$:
\[
x - y = 1 \quad \Rightarrow \quad x = y + 1
\]
Now substitute this into the first equation:
\[
\begin{align*}
x + y &= 15 \\
x &= y + 1
\end{align*}
\]
\[
\begin{align*}
x + (y + 1) &= 15 \quad \Rightarrow \quad 2y + 1 = 15 \\
2y &= 14 \quad \Rightarrow \quad y = 7
\end{align*}
\]
Now that we know $y$, we can substitute into the expression for $x$:
\[
x = y + 1 = 7 + 1 = 8
\]
It’s always a good idea to check your solutions in the original equations:
\[
\begin{align*}
x + y &= 8 + 7 = 15 \\
x - y &= 8 - 7 = 1
\end{align*}
\]
Then $xy = (8)(7) = 56$.

The second approach is to multiply one or both of the equations by constants so that the coefficient of one of the variables is the same in both equations (up to sign); then add or subtract the two equations to eliminate that variable. In this case, we don’t have to multiply by constants: we can add the two equations and eliminate $y$.
\[
\begin{align*}
x + y &= 15 \\
x - y &= 1
\end{align*} \quad \Rightarrow \quad 2x = 16 \quad \Rightarrow \quad x = 8
\]
Now we can substitute this value into either of the original two equations and solve for $y$. We’ll use the first equation:
\[
x + y = 15 \quad \Rightarrow \quad y = -x + 15 = -8 + 15 = 7
\]
1.6 Solving quadratic equations

1.6.1 Factoring quadratic equations

Problem 108. Solve the equation: \( x^2 - 2bx + b^2 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  
(b) \( x_1 - x_2 = 2b \)  
(c) \( x_1 - x_2 = -2b \)  
(d) \( x_1 - x_2 = 1 \)  
(e) None of these

Solution: We can solve this by factoring, then by setting each factor equal to zero.

\[
x^2 - 2bx + b^2 = 0
\]

Factor: \((x - b)(x - b) = (x - b)^2 = 0\) (a perfect square)

Set factors equal to zero: \( x - b = 0 \) (a double root)

Solve: \( x_1 = b \) and \( x_2 = b \)

Answer: \( x_1 - x_2 = b - b = 0 \)

Problem 109. Solve the equation: \( x^2 - 11x + 28 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  
(b) \( x_1 - x_2 = 3 \)  
(c) \( x_1 - x_2 = 8 \)  
(d) \( x_1 - x_2 = 11 \)  
(e) None of these

Solution: We can solve this by factoring, then by setting each factor equal to zero.

\[
x^2 - 11x + 28 = 0
\]

Factor: \((x - 7)(x - 4) = 0\)

Set factors equal to zero: \( x - 7 = 0 \) or \( x - 4 = 0 \)

Solve: \( x_1 = 7 \) and \( x_2 = 4 \)

Answer: \( x_1 - x_2 = 7 - 4 = 3 \)

Problem 110. Solve the equation: \( x^2 + 4x - 21 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  
(b) \( x_1 - x_2 = 4 \)  
*(c) \( x_1 - x_2 = 10 \)  
(d) \( x_1 - x_2 = 17 \)  
(e) None of these

Solution: We can solve this by factoring, then by setting each factor equal to zero.

\[
x^2 + 4x - 21 = 0
\]

Factor: \((x - 3)(x + 7) = 0\)

Set factors equal to zero: \( x - 3 = 0 \) or \( x + 7 = 0 \)

Solve: \( x_1 = 3 \) and \( x_2 = -7 \)

Answer: \( x_1 - x_2 = 3 - (-7) = 10 \)
Problem 111. Solve the equation: \( x^2 - 3x - 18 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  (b) \( x_1 - x_2 = 3 \)
(c) \( x_1 - x_2 = 6 \)  *(d) \( x_1 - x_2 = 9 \)
(e) None of these

**Solution:** We can solve this by factoring, then by setting each factor equal to zero.

\[
x^2 - 3x - 18 = 0
\]

Factor: \((x - 6)(x + 3) = 0\)

Set factors equal to zero: \( x - 6 = 0 \) or \( x + 3 = 0 \)

Solve: \( x_1 = 6 \) and \( x_2 = -3 \)

Answer: \( x_1 - x_2 = 6 - (-3) = 9 \)

Problem 112. Solve the equation: \( x^2 + 8x + 15 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  *(b) \( x_1 - x_2 = 2 \)
(c) \( x_1 - x_2 = 7 \)  (d) \( x_1 - x_2 = 8 \)
(e) None of these

**Solution:** We can solve this by factoring, then by setting each factor equal to zero.

\[
x^2 + 8x + 15 = 0
\]

Factor: \((x + 3)(x + 5) = 0\)

Set factors equal to zero: \( x + 3 = 0 \) or \( x + 5 = 0 \)

Solve: \( x_1 = -3 \) and \( x_2 = -5 \)

Answer: \( x_1 - x_2 = -3 - (-5) = 2 \)

Problem 113. Solve the equation: \( 2x^2 - 5x + 3 = 0 \). There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) What is the difference \( x_1 - x_2 \)?

(a) \( x_1 - x_2 = 0 \)  *(b) \( x_1 - x_2 = \frac{1}{2} \)
(c) \( x_1 - x_2 = \frac{3}{2} \)  (d) \( x_1 - x_2 = \frac{5}{2} \)
(e) None of these

**Solution:** We can solve this by factoring, then by setting each factor equal to zero.

\[
2x^2 - 5x + 3 = 0
\]

Factor: \((2x - 3)(x - 1) = 0\)

Set factors equal to zero: \( 2x - 3 = 0 \) or \( x - 1 = 0 \)

Solve: \( x_1 = \frac{3}{2} \) and \( x_2 = 1 \)

Answer: \( x_1 - x_2 = \frac{3}{2} - 1 = \frac{3}{2} - \frac{2}{2} = \frac{1}{2} \)
Problem 114. Solve the equation: \(2x^2 + 11x + 5 = 0\). There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) What is the difference \(x_1 - x_2\)?

(a) \(x_1 - x_2 = 0\)  
(b) \(x_1 - x_2 = \frac{5}{2}\)  
(c) \(x_1 - x_2 = 3\)  
*(d) \(x_1 - x_2 = \frac{9}{2}\)  
(e) None of these

**Solution:** We can solve this by factoring, then by setting each factor equal to zero.

\[2x^2 + 11x + 5 = 0\]

Factor: \((2x + 1)(x + 5) = 0\)

Set factors equal to zero: \(2x + 1 = 0\) or \(x + 5 = 0\)

Solve: \(x_1 = -\frac{1}{2}\) and \(x_2 = -5\)

Answer: \(x_1 - x_2 = -\frac{1}{2} - (-5) = -\frac{1}{2} + \frac{10}{2} = \frac{9}{2}\)

Problem 115. Solve the equation: \(x^2 - 4x = 5\). There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) What is the difference \(x_1 - x_2\)?

(a) \(x_1 - x_2 = 0\)  
(b) \(x_1 - x_2 = 1\)  
(c) \(x_1 - x_2 = 4\)  
*(d) \(x_1 - x_2 = 6\)  
(e) None of these

**Solution:** First, we need an equation with zero on one side. We can then solve by factoring, then by setting each factor equal to zero.

\[x^2 - 4x = 5\]

\[-5\] \[x^2 - 4x - 5 = 0\]

Factor: \((x - 5)(x + 1) = 0\)

Set factors equal to zero: \(x - 5 = 0\) or \(x + 1 = 0\)

Solve: \(x_1 = 5\) and \(x_2 = -1\)

Answer: \(x_1 - x_2 = 5 - (-1) = 6\)
Problem 116. Solve the equation: \(x^2 + 7x = -10\). There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) What is the difference \(x_1 - x_2\)?

(a) \(x_1 - x_2 = 0\)   *(b) \(x_1 - x_2 = 3\)  
(c) \(x_1 - x_2 = 5\)   (d) \(x_1 - x_2 = 7\)  
(e) None of these

**Solution:** First, we need an equation with zero on one side. We can then solve by factoring, then by setting each factor equal to zero.

\[
x^2 + 7x = -10
\]
\[
\frac{+10}{\quad} x^2 + 7x + 10 = 0
\]
Factor: \((x + 2)(x + 5) = 0\)
Set factors equal to zero: \(x + 2 = 0\) or \(x + 5 = 0\)
Solve: \(x_1 = -2\) and \(x_2 = -5\)
Answer: \(x_1 - x_2 = -2 - (-5) = -2 + 5 = 3\)

1.6.2 Quadratic formula

Problem 117. Use a calculator or equivalent to solve the equation:

\[1.1x^2 + 3.4x - 5.5 = 0\]

There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) Which of the following statements is true of the larger solution \(x_1\)?

(a) \(x_1 < 1\)   *(b) \(1 \leq x_1 < 1.5\)  
(c) \(1.5 \leq x_1 < 2\)   (d) \(2 \leq x_1\)  
(e) No real solution

**Solution:** We will need the quadratic formula to solve this equation:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

Since \(\sqrt{b^2 - 4ac} \geq 0\), if it’s not imaginary, and \(a > 0\), the larger solution is

\[x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-3.4 + \sqrt{(3.4)^2 - 4(1.1)(-5.5)}}{2(1.1)} \approx 1.2 \quad \Rightarrow \quad 1 \leq x_1 < 1.5\]
Problem 118. Use a calculator or equivalent to solve the equation:

\[2.4x^2 + 3.7x - 0.61 = 0\]

There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) Which of the following statements is true of the larger solution \(x_1\)?

(a) \(x_1 < -1\)  
(b) \(-1 \leq x_1 < 0\)  
*(c) \(0 \leq x_1 < 1\)  
(d) \(1 \leq x_1\)  
(e) No real solution

\textbf{Solution:} We will need the quadratic formula to solve this equation:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

Since \(\sqrt{b^2 - 4ac} \geq 0\), if it’s not imaginary, and \(a > 0\), the larger solution is

\[x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-3.7 + \sqrt{(3.7)^2 - 4(2.4)(-0.61)}}{2(2.4)} \approx 0.15 \Rightarrow 0 \leq x_1 < 1\]

Problem 119. Use a calculator or equivalent to solve the equation:

\[0.82x^2 - 2.9x + 0.32 = 0\]

There are two solutions, \(x_1\) and \(x_2\), with \(x_1 \geq x_2\). (It is possible that \(x_1 = x_2\).) Which of the following statements is true of the larger solution \(x_1\)?

(a) \(x_1 < -1\)  
(b) \(-1 \leq x_1 < -0.5\)  
(c) \(-0.5 \leq x_1 < 0\)  
*(d) \(0 \leq x_1\)  
(e) No real solution

\textbf{Solution:} We will need the quadratic formula to solve this equation:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

Since \(\sqrt{b^2 - 4ac} \geq 0\), if it’s not imaginary, and \(a > 0\), the larger solution is

\[x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{2.9 + \sqrt{(-2.9)^2 - 4(0.82)(0.32)}}{2(0.82)} \approx 3.4 \Rightarrow 0 \leq x_1\]
Problem 120. Use a calculator or equivalent to solve the equation:

\[ 1.7x^2 + 8.9x - 0.77 = 0 \]

There are two solutions, \( x_1 \) and \( x_2 \), with \( x_1 \geq x_2 \). (It is possible that \( x_1 = x_2 \).) Which of the following statements is true of the larger solution \( x_1 \)?

(a) \( x_1 < -2 \)  
(b) \( -2 \leq x_1 < 0 \)  
*(c) \( 0 \leq x_1 < 2 \)  
(d) \( 2 \leq x_1 \)  
(e) No real solution

**Solution:** We will need the quadratic formula to solve this equation:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Since \( \sqrt{b^2 - 4ac} \geq 0 \), if it’s not imaginary, the larger solution is

\[ x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-8.9 + \sqrt{(8.9)^2 - 4(1.7)(-0.77)}}{2(1.7)} \approx 0.085 \Rightarrow 0 \leq x_1 < 2 \]

1.7 Algebra word problems

Problem 121. The length of a carpet is 3 feet greater than its width. The area of the carpet is 80 square feet. Which of the following equations describes the carpet’s width?

*(a) \( w^2 + 3w - 80 = 0 \)  
(b) \( w^2 - 3w + 80 = 0 \)  
(c) \( w^2 - 20 = 0 \)  
(d) \( w^2 + 20 = 0 \)  
(e) None of these

**Solution:** If \( w \) is the carpet’s width, then the length is \( l = w + 3 \). Then

\[ A = lw = w(w + 3) = w^2 + 3w = 80 \Rightarrow w^2 + 3w - 80 = 0 \]

Problem 122. A window is twice as long as it is wide. The area of the window is 40 square feet. Which of the following equations describes the window’s length?

(a) \( 3l^2 - 40 = 0 \)  
(b) \( l^2 - 20 = 0 \)  
(c) \( l^2 - 120 = 0 \)  
*(d) \( l^2 - 80 = 0 \)  
(e) None of these

**Solution:** Let \( l \) be the window’s length and \( w \) its width. Since \( l = 2w \), we know that \( w = l/2 \). Hence

\[ A = lw = l \cdot \frac{l}{2} = \frac{l^2}{2} = 40 \Rightarrow l^2 = 80 \Rightarrow l^2 - 80 = 0 \]
**Problem 123.** In the physics lab, you find two circular pieces of sheet metal. The radius of one of the circles is 3 centimeters greater than the radius of the other. The area of the larger circle is twice the area of the smaller one. Which of the following equations describes the radius of the smaller circle?

*(a) $r^2 - 6r - 9 = 0$  
(b) $r^2 - 3r - 9 = 0$  
(c) $r^2 + 6r + 9 = 0$  
(d) $r^2 + 3r + 9 = 0$  
(e) None of these

**Solution:** Let $r$ be the radius of the smaller circle. Then $r + 3$ is the radius of the larger one. The area of the small circle is $\pi r^2$; the area of the large circle is $\pi (r + 3)^2$. Since the area of the large circle is twice the area of the small one,

\[
\pi (r + 3)^2 = 2\pi r^2 \Rightarrow (r + 3)^2 = 2r^2 \Rightarrow r^2 + 6r + 9 = 2r^2 \Rightarrow r^2 - 6r - 9 = 0
\]

**Problem 124.** A rectangular window is twice as long as it is wide. If its length were increased by 3 feet and its width were decreased by 1 foot, it would have the same area. Which of the following equations describes the window’s width?

(a) $w^2 - w + 3 = 0$  
(b) $w - 3 = 0$  
(c) $w^2 + w - 3 = 0$  
(d) $w^2 - w - 3 = 0$  
(e) None of these

**Solution:** Let $w$ be the window’s width. Then its length is $l = 2w$. Its area is $lw = (2w)w = 2w^2$. Decreasing the width by 1 and increasing the length by 3 gives an area of $(w - 1)(2w + 3)$. Hence

\[
2w^2 = (w - 1)(2w + 3) = 2w^2 + w - 3 \Rightarrow w - 3 = 0
\]

**Problem 125.** The distance from Tucson to Phoenix is 120 miles. You want to drive there and back at an average speed of 60 miles per hour. Because of traffic congestion, your average speed from Tucson to Phoenix is 40 mph. How fast do you have to drive on the return trip?

(a) 60 miles per hour  
(b) 80 miles per hour  
(c) 90 miles per hour  
(d) 120 miles per hour  
(e) None of these

**Solution:** The round-trip distance is 240 miles. If you drive that distance at an average speed of 60 miles per hour, then the time that it will take is $240/60 = 4$ hours. If your average speed from Tucson to Phoenix is 40 mph, then the time for that leg of the trip is $120/40 = 3$ hours. Hence you have $4 - 3 = 1$ hour for the return trip. This means you will have to cover 120 miles in 1 hour, so your speed must be 120 miles per hour.
Problem 126. You drive 40 miles at 60 miles per hour, then bicycle an additional 10 miles at 12 miles per hour. What is your average speed for the entire trip?

*(a) 33\(\frac{1}{3}\) miles per hour  (b) 36 miles per hour
(c) 50 miles per hour  (d) 50.4 miles per hour
(e) None of these

Solution: Your average speed for the trip is the total distance divided by the total time. The total distance is 40 + 10 = 50 miles. The total time is

\[
\frac{40 \text{ miles}}{60 \text{ miles/hr}} + \frac{10 \text{ miles}}{12 \text{ miles/hr}} = \frac{2}{3} + \frac{5}{6} = \frac{9}{6} = \frac{3}{2} \text{ hours}
\]

Hence the average speed is

\[
\frac{50 \text{ miles}}{\frac{3}{2} \text{ hours}} = 50 \cdot \frac{2}{3} = \frac{100}{3} = 33\frac{1}{3} \text{ miles per hour}
\]

Problem 127. A boat moves at 5 miles per hour in still water. It is launched in a river that flows at 3 miles per hour. From its launch point, it goes downstream for 4 miles, then turns around and comes back upstream to the launch point. How long does the round trip take?

(a) \(\frac{4}{5}\) hours  (b) \(\frac{8}{5}\) hours
(c) 2 hours  *(d) \(\frac{5}{2}\) hours
(e) None of these

Solution: The time for the round trip is the time that it takes to go downstream at 5 + 3 = 8 miles per hour, plus the time that it takes to come back upstream at 5 − 3 = 2 miles per hour. Thus

\[
t = \frac{4 \text{ miles}}{8 \text{ miles/hour}} + \frac{4 \text{ miles}}{2 \text{ miles/hour}} = \frac{1}{2} + 2 = \frac{5}{2} \text{ hours}
\]
Problem 128. An airplane flies at a speed of 80 miles per hour in still air. On a day when the wind is blowing from the north at 20 miles per hour, the airplane flies 200 miles straight north, then turns around and returns to its starting point. What is its average speed on the round trip?

(a) 64 miles per hour  
(b) 66\(\frac{2}{3}\) miles per hour  
(c) 75 miles per hour  
(d) 80 miles per hour  
(e) None of these

Solution:  The average speed for the round trip is the total distance divided by the total time. The total distance is \(2 \times 200 = 400\) miles. The total time is

\[
\frac{200}{80 - 20} + \frac{200}{80 + 20} = \frac{200}{60} + \frac{200}{100} = \frac{10}{3} + 2 = \frac{16}{3}\] hours

Hence the average speed is

\[
\frac{400 \text{ miles}}{\frac{16}{3} \text{ hours}} = 400 \cdot \frac{3}{16} = 75 \text{ miles/hour}
\]

Problem 129. A runner and a bicyclist start from the same point at the same time, with the runner going straight north and the bicyclist going straight south. The bicyclist is 7 miles per hour faster than the runner. At the end of two hours, the two are 60 miles apart. What is the bicyclist’s speed?

(a) 11\(\frac{1}{2}\) miles per hour  
(b) 14 miles per hour  
(c) 18\(\frac{1}{2}\) miles per hour  
(d) 23 miles per hour  
(e) None of these

Solution:  Let \(b\) be the bicyclist’s speed. Then the runner’s speed is \(b - 7\). Since the two are going in opposite directions, after two hours the distance between them is

\[
2b + 2(b - 7) = 4b - 14 = 60\] miles  \(\Rightarrow\)  \(b = \frac{60 + 14}{4} = \frac{37}{2} = 18\frac{1}{2}\) miles/hour

Problem 130. You drive from Smithtown to Jonesville at a speed of \(v\), making the trip in time \(t\). On the return trip, you are able to drive 10 miles per hour faster, which shortens your travel time by one hour. Which of the following equations is true?

(a) \((v - 10)t = v(t + 1)\)  
(b) \((v + 10)t = v(t - 1)\)  
(c) \((v - 10)(t + 1) = vt\)  
(d) \((v + 10)(t - 1) = vt\)  
(e) None of these

Solution:  On the first leg of the trip, your speed is \(v\) and your time is \(t\). On the return leg, your speed is \(v + 10\) and your time is \(t - 1\). The distance is the same in each direction; so

\[vt = (v + 10)(t - 1)\]
1.8 Graphs

1.8.1 Single points on graphs

Problem 131. The graph at right shows four points labelled with letters. The points are

\[(3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2).\]

Which of the four points is \((3, 3)\)?

(a) \(A\)  *(b) \(B\)
(c) \(C\)  (d) \(D\)

Solution: Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3, 3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4, -2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2, -5)\).

Problem 132. The graph at right shows four points labelled with letters. The points are

\[(3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2).\]

Which of the four points is \((2, -5)\)?

(a) \(A\)  (b) \(B\)
(c) \(C\)  *(d) \(D\)

Solution: Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3, 3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4, -2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2, -5)\).
Problem 133. The graph at right shows four points labelled with letters. The points are

\((3,3), (2, -5), (-6, 5), \) and \((-4, -2)\).

Which of the four points is \((-6, 5)\)?

*(a) A  \hspace{1cm} (b) B  
(c) C  \hspace{1cm} (d) D

Solution: Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3,3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4,-2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2,-5)\).

Problem 134. The graph at right shows four points labelled with letters. The points are

\((3,3), (2, -5), (-6, 5), \) and \((-4, -2)\).

Which of the four points is \((-4, -2)\)?

(a) A  \hspace{1cm} (b) B  
*(c) C  \hspace{1cm} (d) D

Solution: Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3,3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4,-2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2,-5)\).
Problem 135. The graph at right shows four points labelled with letters. The points are

\[(3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\].

Which of the four points is \(A\)?

(a) \((3, 3)\)  
(b) \((2, -5)\)  
*(c) \((-6, 5)\)  
(d) \((-4, -2)\)

**Solution:** Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3, 3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4, -2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2, -5)\).

Problem 136. The graph at right shows four points labelled with letters. The points are

\[(3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\].

Which of the four points is \(B\)?

*(a) \((3, 3)\)  
(b) \((2, -5)\)  
(c) \((-6, 5)\)  
(d) \((-4, -2)\)

**Solution:** Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3, 3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4, -2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2, -5)\).
Problem 137. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\).

Which of the four points is \(C\)?

(a) \((3, 3)\) \hspace{1cm} (b) \((2, -5)\) \hspace{1cm} (c) \((-6, 5)\) \hspace{1cm} (d) \((-4, -2)\)

**Solution:** Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3, 3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4, -2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2, -5)\).

Problem 138. The graph at right shows four points labelled with letters. The points are

\((3, 3), (2, -5), (-6, 5), \text{ and } (-4, -2)\).

Which of the four points is \(D\)?

(a) \((3, 3)\) \hspace{1cm} * (b) \((2, -5)\) \hspace{1cm} (c) \((-6, 5)\) \hspace{1cm} (d) \((-4, -2)\)

**Solution:** Each of the four points is in a different quadrant, so we can use that to identify them. Point \(A\) is in the second quadrant, with a negative \(x\)-coordinate and a positive \(y\)-coordinate. It must be \((-6, 5)\). Point \(B\) is in the first quadrant; its \(x\)- and \(y\)-coordinates are both positive. It must be \((3, 3)\). Point \(C\) is in the third quadrant; its \(x\)- and \(y\)-coordinates are both negative. It must be \((-4, -2)\). Point \(D\) is in the fourth quadrant; it has a positive \(x\)-coordinate and a negative \(y\)-coordinate. It must be \((2, -5)\).
Problem 139. The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \(A\)?

\[(a) \quad (2, 2) \quad \quad (b) \quad (3, 9)\]
\[(c) \quad (7, 2) \quad \quad (d) \quad (8, 8)\]

**Solution:** The point \((3, 9)\) is the only one where the \(x\)-coordinate is small and the \(y\)-coordinate is large. That must correspond to \(A\). The point \((7, 2)\) has a large \(x\)-coordinate and a small \(y\)-coordinate, so it must correspond to \(D\). For the point \((2, 2)\) the \(x\)- and \(y\)-coordinates are both small; that point must be \(C\). For the point \((8, 8)\), the \(x\)- and \(y\)-coordinates are both large; that point must be \(B\).

Problem 140. The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \(B\)?

\[(a) \quad (2, 2) \quad \quad (b) \quad (3, 9)\]
\[(c) \quad (7, 2) \quad \quad (d) \quad (8, 8)\]

**Solution:** The point \((3, 9)\) is the only one where the \(x\)-coordinate is small and the \(y\)-coordinate is large. That must correspond to \(A\). The point \((7, 2)\) has a large \(x\)-coordinate and a small \(y\)-coordinate, so it must correspond to \(D\). For the point \((2, 2)\) the \(x\)- and \(y\)-coordinates are both small; that point must be \(C\). For the point \((8, 8)\), the \(x\)- and \(y\)-coordinates are both large; that point must be \(B\).

Problem 141. The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \(C\)?

\[*(a) \quad (2, 2) \quad \quad (b) \quad (3, 9)\]
\[(c) \quad (7, 2) \quad \quad (d) \quad (8, 8)\]

**Solution:** The point \((3, 9)\) is the only one where the \(x\)-coordinate is small and the \(y\)-coordinate is large. That must correspond to \(A\). The point \((7, 2)\) has a large \(x\)-coordinate and a small \(y\)-coordinate, so it must correspond to \(D\). For the point \((2, 2)\) the \(x\)- and \(y\)-coordinates are both small; that point must be \(C\). For the point \((8, 8)\), the \(x\)- and \(y\)-coordinates are both large; that point must be \(B\).
**Problem 142.** The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \(D\)?

(a) \((2, 2)\)  
(b) \((3, 9)\)  
*(c) \((7, 2)\)  
(d) \((8, 8)\)

**Solution:** The point \((3, 9)\) is the only one where the \(x\)-coordinate is small and the \(y\)-coordinate is large. That must correspond to \(A\). The point \((7, 2)\) has a large \(x\)-coordinate and a small \(y\)-coordinate, so it must correspond to \(D\). For the point \((2, 2)\) the \(x\)- and \(y\)-coordinates are both small; that point must be \(C\). For the point \((8, 8)\), the \(x\)- and \(y\)-coordinates are both large; that point must be \(B\).

**Problem 143.** The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \((2, 2)\)?

(a) \(A\)  
(b) \(B\)  
*(c) \(C\)  
(d) \(D\)

**Solution:** The point \((3, 9)\) is the only one where the \(x\)-coordinate is small and the \(y\)-coordinate is large. That must correspond to \(A\). The point \((7, 2)\) has a large \(x\)-coordinate and a small \(y\)-coordinate, so it must correspond to \(D\). For the point \((2, 2)\) the \(x\)- and \(y\)-coordinates are both small; that point must be \(C\). For the point \((8, 8)\), the \(x\)- and \(y\)-coordinates are both large; that point must be \(B\).

**Problem 144.** The graph at right shows four points labelled with letters. The points are

\[(2, 2), (3, 9), (7, 2), \text{ and } (8, 8).\]

Which of the four points is \((3, 9)\)?

*(a) \(A\)  
(b) \(B\)  
(c) \(C\)  
(d) \(D\)

**Solution:** The point \((3, 9)\) is the only one where the \(x\)-coordinate is small and the \(y\)-coordinate is large. That must correspond to \(A\). The point \((7, 2)\) has a large \(x\)-coordinate and a small \(y\)-coordinate, so it must correspond to \(D\). For the point \((2, 2)\) the \(x\)- and \(y\)-coordinates are both small; that point must be \(C\). For the point \((8, 8)\), the \(x\)- and \(y\)-coordinates are both large; that point must be \(B\).
Problem 145. The graph at right shows four points labelled with letters. The points are

(2, 2), (3, 9), (7, 2), and (8, 8).

Which of the four points is (7, 2)?

(a) A  (b) B
(c) C  *(d) D

Solution: The point (3, 9) is the only one where the $x$-coordinate is small and the $y$-coordinate is large. That must correspond to A. The point (7, 2) has a large $x$-coordinate and a small $y$-coordinate, so it must correspond to D. For the point (2, 2) the $x$- and $y$-coordinates are both small; that point must be C. For the point (8, 8), the $x$- and $y$-coordinates are both large; that point must be B.

Problem 146. The graph at right shows four points labelled with letters. The points are

(2, 2), (3, 9), (7, 2), and (8, 8).

Which of the four points is (8, 8)?

(a) A  *(b) B
(c) C  (d) D

Solution: The point (3, 9) is the only one where the $x$-coordinate is small and the $y$-coordinate is large. That must correspond to A. The point (7, 2) has a large $x$-coordinate and a small $y$-coordinate, so it must correspond to D. For the point (2, 2) the $x$- and $y$-coordinates are both small; that point must be C. For the point (8, 8), the $x$- and $y$-coordinates are both large; that point must be B.
1.8.2 Matching graphs and equations

**Problem 147.** Which equation is shown on the graph at right?

(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = -2x$
(d) $y = -2x - 1$

**Solution:** If the equation of a line is written in the form: $y = mx + b$, then $m$ is the slope and $b$ is the $y$-intercept.

The line in the figure has a positive slope (as $x$ increases, $y$ increases) and a positive $y$-intercept (when $x = 0$, $y > 0$). Of the answers given, only $y = 2x + 1$ has $m > 0$ and $b > 0$.

**Problem 148.** Which equation is shown on the graph at right?

(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = -2x$
(d) $y = -2x - 1$

**Solution:** If the equation of a line is written in the form: $y = mx + b$, then $m$ is the slope and $b$ is the $y$-intercept.

The line in the figure has a positive slope (as $x$ increases, $y$ increases) and a $y$-intercept of 0 (when $x = 0$, $y = 0$). Of the answers given, only $y = 2x$ has $m > 0$ and $b = 0$.

**Problem 149.** Which equation is shown on the graph at right?

(a) $y = 2x$
(b) $y = 2x + 1$
(c) $y = -2x$
(d) $y = -2x - 1$

**Solution:** If the equation of a line is written in the form: $y = mx + b$, then $m$ is the slope and $b$ is the $y$-intercept.

The line in the figure has a negative slope (as $x$ increases, $y$ decreases) and a $y$-intercept of 0 (when $x = 0$, $y = 0$). Of the answers given, only $y = -2x$ has $m < 0$ and $b = 0$. 
Problem 150. Which equation is shown on the graph at right?

(a) \( y = 2x \)
(b) \( y = 2x + 1 \)
(c) \( y = -2x \)
* (d) \( y = -2x - 1 \)

Solution: If the equation of a line is written in the form: \( y = mx + b \), then \( m \) is the slope and \( b \) is the \( y \)-intercept.

The line in the figure has a negative slope (as \( x \) increases, \( y \) increases) and a negative \( y \)-intercept (when \( x = 0, y < 0 \)). Of the answers given, only \( y = -2x - 1 \) has \( m < 0 \) and \( b < 0 \).

Problem 151. Which equation is shown on the graph at right? (The scale is the same for the \( x \)- and \( y \)-axes.)

(a) \( y = 3x \)
(b) \( y = -3x \)
* (c) \( y = \frac{x}{3} \)
(d) \( y = -\frac{x}{3} \)

Solution: If the equation of a line is written in the form: \( y = mx + b \), then \( m \) is the slope and \( b \) is the \( y \)-intercept. In this case, all of the possible answers have \( b = 0 \), so we must focus on the slope \( m \).

The line in the figure has a positive slope (as \( x \) increases, \( y \) increases), so it must be \( m = 3 \) or \( m = 1/3 \). Since the value of \( y \) increases more slowly than the value of \( x \), \( m < 1 \). Hence \( m = 1/3 \); so \( y = x/3 \).

Problem 152. Which equation is shown on the graph at right? (The scale is the same for the \( x \)- and \( y \)-axes.)

* (a) \( y = 3x \)
(b) \( y = -3x \)
(c) \( y = \frac{x}{3} \)
(d) \( y = -\frac{x}{3} \)

Solution: If the equation of a line is written in the form: \( y = mx + b \), then \( m \) is the slope and \( b \) is the \( y \)-intercept. In this case, all of the possible answers have \( b = 0 \), so we must focus on the slope \( m \).

The line in the figure has a positive slope (as \( x \) increases, \( y \) increases), so it must be \( m = 3 \) or \( m = 1/3 \). Since the value of \( y \) increases more rapidly than the value of \( x \), \( m > 1 \). Hence \( m = 3 \); so \( y = 3x \).
Problem 153. Which equation is shown on the graph at right? (The scale is the same for the $x$- and $y$-axes.)

(a) $y = 3x$
(b) $y = -3x$
(c) $y = \frac{x}{3}$
(d) $y = -\frac{x}{3}$

Solution: If the equation of a line is written in the form: $y = mx + b$, then $m$ is the slope and $b$ is the $y$-intercept. In this case, all of the possible answers have $b = 0$, so we must focus on the slope $m$.

The line in the figure has a negative slope (as $x$ increases, $y$ decreases), so it must be $m = -3$ or $m = -1/3$. Since the value of $y$ decreases more slowly than the value of $x$ increases, $m > -1$. Hence $m = -1/3$; so $y = -x/3$.

Problem 154. Which equation is shown on the graph at right? (The scale is the same for the $x$- and $y$-axes.)

(a) $y = 3x$
(b) $y = -3x$
(c) $y = \frac{x}{3}$
(d) $y = -\frac{x}{3}$

Solution: If the equation of a line is written in the form: $y = mx + b$, then $m$ is the slope and $b$ is the $y$-intercept. In this case, all of the possible answers have $b = 0$, so we must focus on the slope $m$.

The line in the figure has a negative slope (as $x$ increases, $y$ decreases), so it must be $m = -3$ or $m = -1/3$. Since the value of $y$ decreases more rapidly than the value of $x$ increases, $m < -1$. Hence $m = -3$; so $y = -3x$. 
Problem 155. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = x + 1 \)?

\[ (a) \quad y \quad (b) \quad y \quad (c) \quad y \quad * (d) \quad y \]

Solution: The equation \( y = x + 1 \) is written in the form: \( y = mx + b \). Here \( m = 1 > 0 \) and \( b = 1 > 0 \). Since \( m > 0 \), the graph should have a positive slope (as \( x \) increases, \( y \) increases). Since \( b > 0 \), the graph should have a positive \( y \)-intercept (when \( x = 0, y > 0 \)). Only graph (d) slopes upward and crosses the \( y \)-axis above the \( x \)-axis.

Problem 156. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = x - 1 \)?

\[ (a) \quad y \quad (b) \quad y \quad * (c) \quad y \quad (d) \quad y \]

Solution: The equation \( y = x - 1 \) is written in the form: \( y = mx + b \). Here \( m = 1 > 0 \) and \( b = -1 < 0 \). Since \( m > 0 \), the graph should have a positive slope (as \( x \) increases, \( y \) increases). Since \( b < 0 \), the graph should have a negative \( y \)-intercept (when \( x = 0, y < 0 \)). Only graph (c) slopes upward and crosses the \( y \)-axis below the \( x \)-axis.
Problem 157. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = -x + 1 \)?

\[ \text{Solution:} \quad \text{The equation } y = -x + 1 \text{ is written in the form: } y = mx + b. \text{ Here } m = -1 < 0 \text{ and } b = 1 > 0. \text{ Since } m < 0, \text{ the graph should have a negative slope (as } x \text{ increases, } y \text{ decreases). Since } b > 0, \text{ the graph should have a positive } y\text{-intercept (when } x = 0, y > 0). \text{ Only graph (a) slopes downward and crosses the } y\text{-axis above the } x\text{-axis.} \]

Problem 158. The four graphs (a), (b), (c), and (d) below represent four different equations:

\[ y = x + 1 \quad y = x - 1 \quad y = -x + 1 \quad y = -x - 1 \]

Which of the four graphs represents \( y = -x - 1 \)?

\[ \text{Solution:} \quad \text{The equation } y = -x - 1 \text{ is written in the form: } y = mx + b. \text{ Here } m = -1 < 0 \text{ and } b = -1 < 0. \text{ Since } m < 0, \text{ the graph should have a negative slope (as } x \text{ increases, } y \text{ decreases). Since } b < 0, \text{ the graph should have a negative } y\text{-intercept (when } x = 0, y < 0). \text{ Only graph (b) slopes downward and crosses the } y\text{-axis below the } x\text{-axis.} \]
Problem 159. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

**Solution:** The graph has a negative \( y \)-intercept: it crosses the \( y \)-axis below the \( x \)-axis. This means that when \( x = 0 \), \( y < 0 \). That allows us to rule out equations (a) and (c). In the graph, when \( x \) has large absolute values, \( y > 0 \). This is consistent with (b), where the coefficient of \( x^2 \) is positive; but not with (d), where the coefficient of \( x^2 \) is negative.

Problem 160. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

**Solution:** The graph has a positive \( y \)-intercept: it crosses the \( y \)-axis above the \( x \)-axis. This means that when \( x = 0 \), \( y > 0 \). That allows us to rule out equations (b) and (d). In the graph, when \( x \) has large absolute values, \( y < 0 \). This is consistent with (c), where the coefficient of \( x^2 \) is negative, but not with (a), where the coefficient of \( x^2 \) is positive.

Problem 161. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)
(b) \( y = x^2 - 1 \)
(c) \( y = -x^2 + 1 \)
(d) \( y = -x^2 - 1 \)

**Solution:** The graph has a positive \( y \)-intercept: it crosses the \( y \)-axis above the \( x \)-axis. This means that when \( x = 0 \), \( y > 0 \). That allows us to rule out equations (b) and (d). In the graph, when \( x \) has large absolute values, \( y > 0 \). This is consistent with (a), where the coefficient of \( x^2 \) is positive, but not with (c), where the coefficient of \( x^2 \) is negative.
Problem 162. Which equation is shown on the graph at right?

(a) \( y = x^2 + 1 \)  
(b) \( y = x^2 - 1 \)  
(c) \( y = -x^2 + 1 \)  
*(d) \( y = -x^2 - 1 \)  

Solution: The graph has a negative \( y \)-intercept: it crosses the \( y \)-axis below the \( x \)-axis. This means that when \( x = 0 \), \( y < 0 \). That allows us to rule out equations (a) and (c). In the graph, when \( x \) has large absolute values, \( y < 0 \). This is consistent with (d), where the coefficient of \( x^2 \) is negative, but not with (b), where the coefficient of \( x^2 \) is positive.

Problem 163. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[
\begin{align*}
  y &= 2x^2 \\
  y &= \frac{x^2}{2} \\
  y &= -2x^2 \\
  y &= -\frac{x^2}{2}
\end{align*}
\]

Which of the four graphs represents \( y = 2x^2 \)?

Solution: All four equations have \( y \)-intercepts of zero, so that won’t help us. In graphs (a) and (b), \( y \leq 0 \) for all values of \( x \). They must represent the two equations where the coefficient of \( x^2 \) is negative. That leaves us with graphs (c) and (d), which must represent the two equations with positive coefficients of \( x^2 \). Of these two equations, the curve of \( y = 2x^2 \) will rise faster than the curve of \( y = x^2 / 2 \): for example, the first of these includes the point \((1, 2)\), whereas the second includes the point \((1, \frac{1}{2})\). Hence the graph of \( y = 2x^2 \) is (d).
Problem 164. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[
\begin{align*}
  y &= 2x^2 \\
  y &= \frac{x^2}{2} \\
  y &= -2x^2 \\
  y &= -\frac{x^2}{2}
\end{align*}
\]

Which of the four graphs represents \( y = \frac{x^2}{2} \)?

**Solution:** All four equations have \( y \)-intercepts of zero, so that won’t help us. In graphs (a) and (b), \( y \leq 0 \) for all values of \( x \). They must represent the two equations where the coefficient of \( x^2 \) is negative. That leaves us with graphs (c) and (d), which must represent the two equations with positive coefficients of \( x^2 \). Of these two equations, the curve of \( y = 2x^2 \) will rise faster than the curve of \( y = x^2/2 \): for example, the first of these includes the point \((1, 2)\), whereas the second includes the point \((1, \frac{1}{2})\). Hence the graph of \( y = x^2/2 \) is (c).

Problem 165. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[
\begin{align*}
  y &= 2x^2 \\
  y &= \frac{x^2}{2} \\
  y &= -2x^2 \\
  y &= -\frac{x^2}{2}
\end{align*}
\]

Which of the four graphs represents \( y = -2x^2 \)?

**Solution:** All four equations have \( y \)-intercepts of zero, so that won’t help us. In graphs (c) and (d), \( y \geq 0 \) for all values of \( x \). They must represent the two equations where the coefficient of \( x^2 \) is positive. That leaves us with graphs (a) and (b), which must represent the two equations with negative coefficients of \( x^2 \). Of these two equations, the curve of \( y = -2x^2 \) will fall faster than the curve of \( y = -x^2/2 \): for example, the first of these includes the point \((1, -2)\), whereas the second includes the point \((1, -\frac{1}{2})\). Hence the graph of \( y = -2x^2 \) is (b).
Problem 166. The four graphs (a), (b), (c), and (d) below are all drawn on the same scale. They represent four different equations:

\[ y = 2x^2 \quad y = \frac{x^2}{2} \quad y = -2x^2 \quad y = -\frac{x^2}{2} \]

Which of the four graphs represents \( y = -\frac{x^2}{2} \)?

\[ (a) \quad \quad (b) \quad \quad (c) \quad \quad (d) \]

Solution: All four equations have \( y \)-intercepts of zero, so that won’t help us. In graphs (c) and (d), \( y \geq 0 \) for all values of \( x \). They must represent the two equations where the coefficient of \( x^2 \) is positive. That leaves us with graphs (a) and (b), which must represent the two equations with negative coefficients of \( x^2 \). Of these two equations, the curve of \( y = -2x^2 \) will fall faster than the curve of \( y = -\frac{x^2}{2} \): for example, the first of these includes the point \((1, -2)\), whereas the second includes the point \((1, -\frac{1}{2})\). Hence the graph of \( y = -\frac{x^2}{2} \) is (a).