Compound Interest Primer Problem Set Solutions

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Round all answers to the nearest whole dollar, or to the nearest 0.01%.

NOTE: In the problems involving compound interest, $F(t) = Pe^{rt}$ (continuous), $F(t) = P(1 + \frac{r}{n})^{nt}$ (discrete). To find $F$, we need $P$, $r$, $n$, and $t$. Yield is given by $y = e^r - 1$ (continuous), $y = \left(1 + \frac{r}{n}\right)^n - 1$ (discrete).

Problem 1. (i) $56,000 is invested at 3.56%, compounded quarterly. What is the account’s value after 5 years and 3 months? (ii) What is the effective annual yield on the investment?

Solution: Given: $P = 56,000$, $t = 5.25$, $r = .0356$, and $n = 4$ (since compounded quarterly). Want

\[
(i)\quad F(5.25) = 56,000 \left(1 + \frac{.0356}{4}\right)^{4(5.25)}
\]

\[
(ii)\quad y = \left(1 + \frac{.0356}{4}\right)^4 - 1
\]

Problem 2. (i) $110,000 is invested at 5.25%, compounded monthly. What is the value of the account after 4 years? (ii) What is the effective annual yield on the investment?

Solution: Given: $P = 110,000$, $t = 4$, $r = .0525$, and $n = 12$ (since compounded monthly). Want

\[
(i)\quad F(4) = 110,000 \left(1 + \frac{.0525}{12}\right)^{12(4)}
\]

\[
(ii)\quad y = \left(1 + \frac{.0525}{12}\right)^{12} - 1
\]

Problem 3. (i) $74,000 is invested at 4.8%, compounded daily. What is the value of the account after 6 years and 6 months? (ii) What is the effective annual yield on the investment?

Solution: Given: $P = 74,000$, $t = 6.5$, $r = .048$, and $n = 365$ (since compounded daily). Want

\[
(i)\quad F(6.5) = 74,000 \left(1 + \frac{.048}{365}\right)^{365(6.5)}
\]

\[
(ii)\quad y = \left(1 + \frac{.048}{365}\right)^{365} - 1
\]
Problem 4. A certificate of deposit pays interest at 3.60%, compounded quarterly. What is the effective annual yield?

Solution: Given: $n = 4$, $r = .036$. Want $y =$?

$$y = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.036}{4}\right)^4 - 1$$

Problem 5. A certificate of deposit pays interest at 4.25%, compounded monthly. What is the effective annual yield?

Solution: Given: $n = 12$, $r = .0425$. Want $y =$?

$$y = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.0425}{12}\right)^{12} - 1$$

Problem 6. A certificate of deposit pays interest at 5.10%, compounded daily. What is the effective annual yield?

Solution: Given: $n = 365$, $r = .051$. Want $y =$?

$$y = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.051}{365}\right)^{365} - 1$$

Problem 7. A certificate of deposit pays interest at rate $r$, compounded quarterly. If the effective annual yield is 3.35%, what is the value of $r$?

Solution: Given: $n = 4$, $y = .0335$. Want $r$.

$$r = n \cdot \left[(1 + y)^{1/n} - 1\right] = 4 \cdot \left[(1 + .0335)^{1/4} - 1\right]$$

Problem 8. A certificate of deposit pays interest at rate $r$, compounded monthly. If the effective annual yield is 5.80%, what is the value of $r$?

Solution: Given: $n = 12$, $y = .058$. Want $r$.

$$r = 12 \cdot \left[(1 + .058)^{1/12} - 1\right]$$

Problem 9. A certificate of deposit pays interest at rate $r$, compounded daily. If the effective annual yield is 3.58%, what is the value of $r$?

Solution: Given: $n = 365$, $y = .0358$. Want $r$.

$$r = 365 \cdot \left[(1 + .0358)^{1/365} - 1\right]$$

Problem 10. A certificate of deposit pays interest at rate $r$, compounded annually. If the effective annual yield is 4.15%, what is the value of $r$?
Solution: Given: \( n = 1, \ y = .0415 \). Want \( r \).

\[
r = 1 \left[ (1 + .0415)^{1/1} - 1 \right] = [(1 + .0415) - 1] = .0415 = y
\]

Notice that when \( n = 1 \), it’s always true that \( r = y \).

Problem 11. \((i)\) $35,000 is invested at 3.4%, compounded continuously. What is the value of the account after 7 years?
\((ii)\) What is the effective annual yield on the investment?

Solution: Given: \( P = 35,000, \ t = 7 \) years, \( r = .034 \).

\[
(i) \quad F = 35,000e^{(.034)(7)}
\]

\[
(ii) \quad y = e^r - 1 = e^{.034} - 1
\]

Problem 12. \((i)\) $120,000 is invested at 5.02%, compounded continuously. What is the value of the account after 2 years and six months?
\((ii)\) What is the effective annual yield on the investment?

Solution: Given: \( P = 120,000, \ t = 2.5 \) years, \( r = .0502 \).

\[
(i) \quad F = 120,000e^{(.0502)(2.5)}
\]

\[
(ii) \quad y = e^r - 1 = e^{.0502} - 1
\]

Problem 13. Two certificates of deposit have the same effective annual yield. The first pays a rate of \( r \), compounded monthly. The second pays 6.03%, compounded continuously. What is the value of \( r \)?

Solution: Want \( r \) so that

\[
P \left( 1 + \frac{r}{n} \right)^n = P(1 + y) = Pe^{.0603}.
\]

Cancelling \( P \) from both sides gives \( (1 + \frac{r}{n})^n = e^{.0603} \). Since we compound monthly, \( n = 12 \). The equation becomes \( (1 + \frac{r}{12})^{12} = e^{.0603} \). Taking the 12th root gives

\[
1 + \frac{r}{12} = \left(e^{.0603}\right)^{1/12} = e^{.0603/12}.
\]

Solving for \( r \) yields

\[
r = 12 \left[ e^{.0603/12} - 1 \right]
\]

Problem 14. Two certificates of deposit have the same effective annual yield. The first pays a rate of 4%, compounded quarterly. The second pays a rate of \( r \), compounded continuously. What is the value of \( r \)?
Solution: Want \( r \) so that
\[
Pe^r = P(1 + r) = P \left( 1 + \frac{.04}{n} \right)^n.
\]
Cancelling \( P \) from both sides gives \( e^r = (1 + \frac{.04}{n})^n \). Since we compound quarterly, \( n = 4 \). The equation becomes \( e^r = (1 + \frac{.04}{4})^4 = (1.01)^4 \). Taking the natural log of both sides gives
\[
r = \ln(e^r) = \ln((1.01)^4) = 4 \ln(1.01).
\]

Problem 15. $5000 is invested in an account paying 5.5%, compounded quarterly. How long will it take for the value of the account to reach $6000? Round your answer to the nearest hundredth of a year.

Solution: Given: \( P = 5000, F = 6000, r = .055, \) and \( n = 4 \). Solve for \( t \).

\[
F = P \left(1 + \frac{r}{n}\right)^{nt} \quad \frac{\downarrow}{\downarrow} \quad \left(1 + \frac{r}{n}\right)^{nt} = \frac{F}{P}
\]

\[
\ln \left(1 + \frac{r}{n}\right)^{nt} = \ln \left(\frac{F}{P}\right) \quad \text{(take the log of both sides)}
\]

\[
\Rightarrow \quad nt \ln \left(1 + \frac{r}{n}\right) = \ln \left(\frac{F}{P}\right) \quad \text{(property of logarithms)}
\]

\[
\Rightarrow \quad t = \frac{\ln(F/P)}{n \ln \left(1 + \frac{r}{n}\right)} \quad \text{(General formula for } t) \]

For our problem
\[
t = \frac{\ln(6000/5000)}{4 \ln \left(1 + \frac{.055}{4}\right)}.
\]

Problem 16. $12,000 is invested in an account paying 3.85%, compounded monthly. How long will it take for the value of the account to reach $16,000? Round your answer to the nearest hundredth of a year.

Solution: Given: \( P = 12,000, F = 16,000, r = .0385, \) and \( n = 12 \). Solve for \( t \). Apart from different numbers, this is just like Exercise 15. We use the same formula:

\[
t = \frac{\ln \left(\frac{16,000}{12,000}\right)}{12 \ln \left(1 + \frac{.0385}{12}\right)}.
\]

Problem 17. $3000 is invested in an account paying 3.2%, compounded daily. How long will it take for the value of the account to reach $5000? Round your answer to the nearest hundredth of a year.
**Solution:** Given: $P = 5000$, $F = 6000$, $r = .055$, and $n = 365$. Solve for $t$.
Apart from different numbers, this is just like Exercise 15. We use the same formula:

$$t = \frac{\ln \left( \frac{5000}{3000} \right)}{365 \ln \left( 1 + \frac{.032}{365} \right)}.$$

**Problem 18.** $6500$ is invested in an account paying $5.15\%$, compounded continuously. How long will it take for the value of the account to reach $10,000$? Round your answer to the nearest hundredth of a year.

**Solution:** Given: Compounded continuously, $P = 6500$, $F = 10,000$, $r = .0515$. Find $t$.
Recall that $F = Pe^{rt}$. Solve for $t$:

$$e^{rt} = \frac{F}{P} \Rightarrow \ln \left( e^{rt} \right) = \ln \left( \frac{F}{P} \right) \quad \text{(take } \log_e \text{ of both sides)}$$

$$\Rightarrow rt = \ln \left( \frac{F}{P} \right) \quad \text{(by property of logarithms)}$$

$$\Rightarrow t = \frac{1}{r} \ln \left( \frac{F}{P} \right)$$

For our problem, we want

$$t = \frac{1}{.0515} \ln \left( \frac{10,000}{6500} \right) = 8.36 \text{ years}$$

**Problem 19.** A deposit account pays interest of $r$, compounded continuously. If the effective annual yield is $3.91\%$, what is $r$?

**Solution:** Given: compounded continuously, $y = .0391$. Want $r$.

$$r = \ln(1 + y) = \ln(1 + .0391) = \ln(1.0391)$$

**Problem 20.** A deposit account pays interest of $r$, compounded continuously. If the effective annual yield is $7.12\%$, what is $r$?

**Solution:** Given: compounded continuously, $y = .0712$. Want $r$.

$$r = \ln(1 + y) = \ln(1 + .0712) = \ln(1.0712)$$

**Problem 21.** Suppose money earns at an annual rate of $3.65\%$, compounded quarterly. What is the present value of a $30,000$ payment $4$ years from now?

**Solution:** Given: $F = 30,000$, $r = .0365$, $n = 4$ (compounded quarterly), and $t = 4$. Want $P$.

$$P = F \left( 1 + \frac{r}{n} \right)^{-nt} = 30,000 \left( 1 + \frac{.0365}{4} \right)^{-4 \cdot 4}$$
Problem 22. Suppose money earns at an annual rate of 5%, compounded monthly. What is the present value of a $5000 payment 6 years from now?

Solution: Given: \( F = 5000, r = .05, n = 12 \) (compounded monthly), and \( t = 6 \). Want \( P \).

\[
P = F \left(1 + \frac{r}{n}\right)^{-nt} = 5000 \left(1 + \frac{.05}{12}\right)^{-(12)6}
\]

Problem 23. Suppose money earns at an annual rate of 4.5%, compounded daily. What is the present value of a $125,000 payment 3 years and 6 months from now?

Solution: Given: \( F = 125,000, r = .045, n = 365 \) (compounded quarterly), and \( t = 3.5 \). Want \( P \).

\[
P = F \left(1 + \frac{r}{n}\right)^{-nt} = 125,000 \left(1 + \frac{.045}{365}\right)^{-(365)3.5}
\]

Problem 24. Suppose money earns at an annual rate of 5.2%, compounded continuously. What is the present value of a $23,000 payment 5 years from now?

Solution: Given: Compounded continuously, \( F = 23,000, r = .052, \) and \( t = 5 \). Want \( P \).

\[
P = Fe^{-rt} = 23,000e^{-(.052)5}
\]

Problem 25. Suppose money earns at an annual rate of 7.15%, compounded monthly. What is the future value of $13,000 after 3 years?

Solution: Given: \( n = 12 \) (compounded monthly), \( P = 13,000, t = 3, \) and \( r = .0715 \). Find \( F \).

\[
F = P \left(1 + \frac{r}{n}\right)^{nt} = 13,000 \left(1 + \frac{.0715}{12}\right)^{(12)(3)}
\]

Problem 26. Suppose money earns at an annual rate of 4.6%, compounded continuously. What is the future value of $2000 after 7 years and 6 months?

Solution: Given: Compounded continuously, \( P = 2000, r = .046, \) and \( t = 7.5 \). Find \( F \).

\[
F = Pe^{rt} = 2000e^{(.046)(7.5)}
\]

Problem 27. What is the yearly ratio of future value to present value that corresponds to an annual rate of 7.6%, compounded continuously?

Solution: Given: Compounded continuously, \( r = .076 \). We want a yearly ratio \( R \):

\[
R \equiv \frac{F}{P} = e^r.
\]

(For \( t = 1 \) year, \( F(t)/P = e^{rt} = e^r \).) For \( r = .076 \), we have \( R = e^{.076} \).
Problem 28. What is the annual rate of interest \( r \), compounded continuously, that corresponds to a yearly ratio of future value to present value of 1.10?

Solution: Given: compounded continuously, \( R = 1.10 \). Want \( r \).

\[
r = \ln(R) = \ln(1.10).
\]

Problem 29. A mortgage of $120,000 is to be paid off in 360 equal monthly payments, beginning one month after the purchase date. The interest on the mortgage is 5.78%, compounded continuously.

(i) What is the amount of each payment?

(ii) What is the total amount of all the payments?

Solution: Given: Compounded continuously, \( P = 120,000 \), \( r = 0.0578 \). The payments will be \( F_1, \ldots, F_{360} \), where the \( i \)th payment, \( F_i \), will be made \( i \) months, or \( i/12 \) years from now.

(i) The present value of \( F_i \) is \( P_i = F_i e^{-r(i/12)} \). Thus

\[
P = \sum_{i=1}^{360} P_i
= \sum_{i=1}^{360} F_i e^{-r(i/12)}
\]

Since all payments \( F_i \) are equal, we can call that amount \( F = F_1 = F_2 = \cdots = F_{360} \), and factor it out of the sum:

\[
= F \sum_{i=1}^{360} e^{-r(i/12)}
= F \sum_{i=1}^{360} \left[ e^{-r(12)} \right]^i
\]

Next, factor out \( e^{-r/12} \) and re-order the sum with \( k = i - 1 \) to get a geometric series:

\[
= Fe^{-r/12} \sum_{k=0}^{359} \left[ e^{-r(12)} \right]^k
= Fe^{-r/12} \left[ 1 - \left( e^{-r(12)} \right)^{360} \right] \frac{1}{1 - e^{-r(12)}}
= Fe^{-r/12} \left[ \frac{1 - e^{-30r}}{1 - e^{-r/12}} \right]
\]
Then

$$F = P \left( e^{-r/12} \left[ \frac{1 - e^{-30r}}{1 - e^{-(r/12)}} \right] \right)^{-1} = Pe^{r/12} \left[ \frac{1 - e^{-(r/12)}}{1 - e^{-30r}} \right].$$

Finish by substituting in the values for the given parameters.

(ii) To find the total amount, multiply the value of one of the (equal) payments by the total number of payments: $360F$.

**Problem 30.** A sofa costing $899.99 can be paid for in 24 equal monthly payments, beginning one month after the purchase date. The interest on the unpaid balance is 21.5%, compounded continuously.

(i) What is the amount of each payment?

(ii) What is the total amount of all the payments?

**Solution:** Given: Compounded continuously, $P = 899.99$, $r = .215$. The payments will be $F_1, \ldots, F_{24}$, where the $i$th payment, $F_i$, will be made $i$ months, or $i/12$ years from now.

(i) We can modify the formula we used to find the solution to Problem 29:

$$F = Pe^{r/12} \left[ \frac{1 - e^{-(r/12)}}{1 - e^{-2r}} \right].$$

(ii) To find the total amount, multiply the value of one of the (equal) payments by the total number of payments: $24F$. 